



DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

**PROVINSIALE VOORBEREIDENDE EKSAMEN/
PROVINCIAL PREPARATORY EXAMINATION**

GRAAD 12/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL 2/PAPER 2

SEPTEMBER 2025 *morephysics.com*

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

**Hierdie vraestel bestaan uit 13 bladsye, 'n inligtingsblad en 'n
23 bladsy- SPESIALE ANTWOORDEBOEK./
This question paper consists of 13 pages, an information sheet and a
23-page SPECIAL ANSWER BOOK.**

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

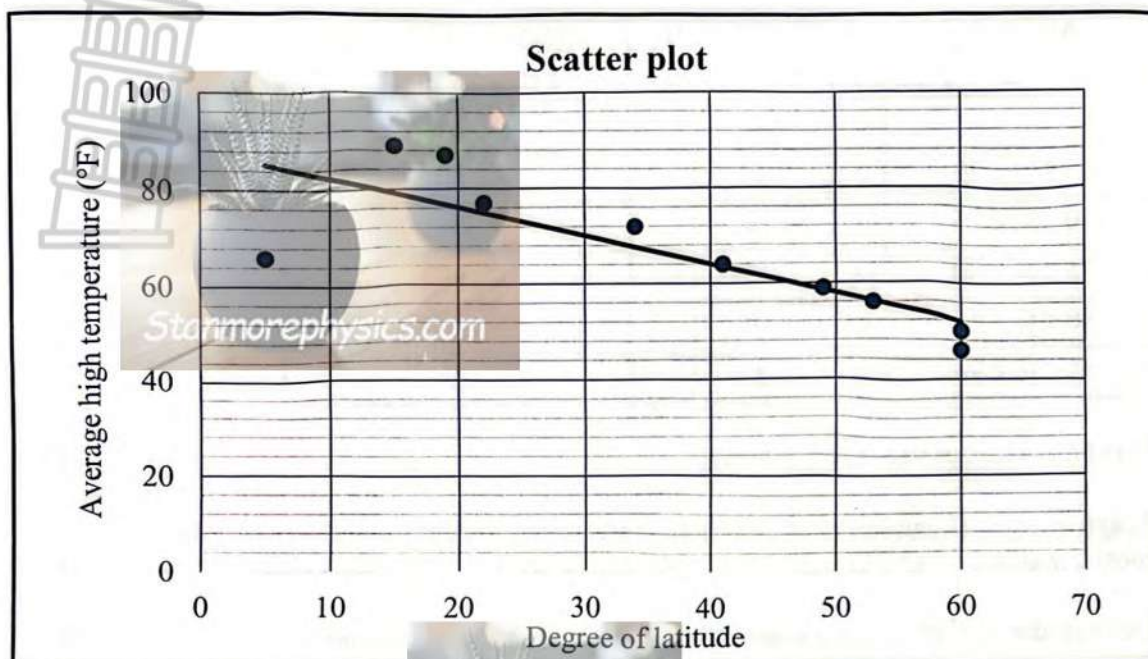
1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.



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QUESTION 1

The scatter plot below shows the degrees of latitude of 10 cities in the Northern Hemisphere and their average high temperatures (in °F). The least squares regression line is also shown.



- 1.1 Use the scatter plot to describe the strength of the correlation between the latitude of a city and its average high temperature. Motivate your answer. (2)

The table below shows the data that was used to draw the scatter plot above.

Latitude in degrees	5	19	34	53	22	41	60	15	60	49
Average high temperature (in °F)	66	87	72	56	77	64	46	89	50	59

- 1.2 Determine the equation of the least squares regression line for the data. (3)
- 1.3 Predict the average high temperature of a city with a latitude of 28 degrees. (2)
- 1.4 Write down the standard deviation of the average high temperature of the 10 cities. (1)
- 1.5 Determine the number of cities of which the average high temperature is greater than one standard deviation above the mean. (3)
- [11]**

QUESTION 2

Fifty boys were weighed and their mass recorded to the nearest kilogram. The results are shown in the table below.

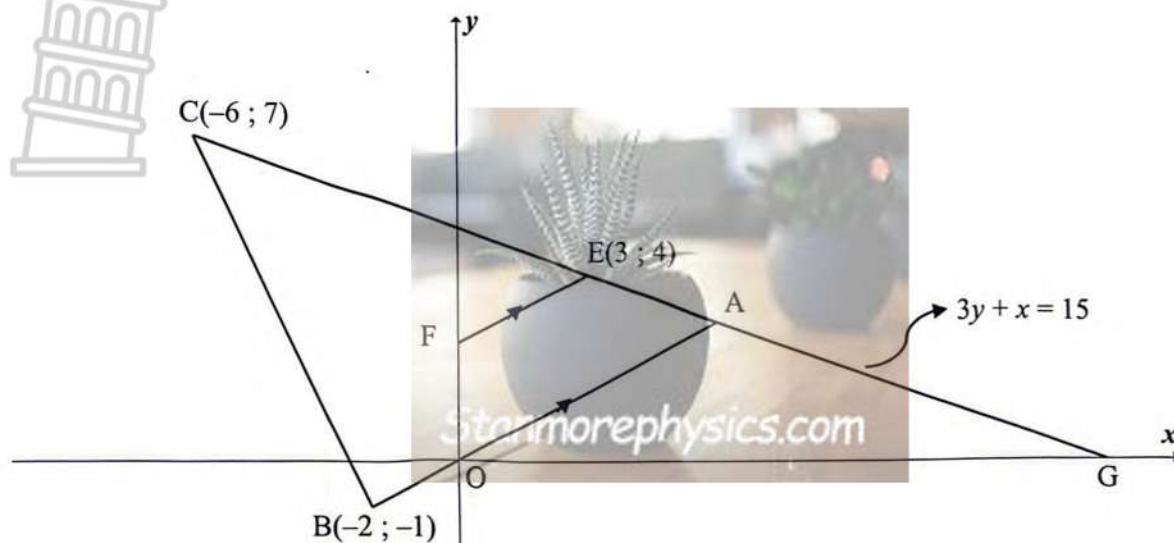
MASS (kg)	FREQUENCY	CUMULATIVE FREQUENCY
$60 \leq m < 65$	2	2
$65 \leq m < 70$	6	8
$70 \leq m < 75$	12	20
$75 \leq m < 80$	14	34
$80 \leq m < 85$	10	k
$85 \leq m < 90$	f	50

- 2.1 Write down the values of k and f . (2)
- 2.2 Draw an ogive (cumulative frequency graph) for the data on the grid provided in the ANSWER BOOK. (3)
- 2.3 Use the graph to determine the median mass for this data. (2)
- 2.4 What percentage of the boys weighed 83 kg or more? (2)
- [9]

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QUESTION 3

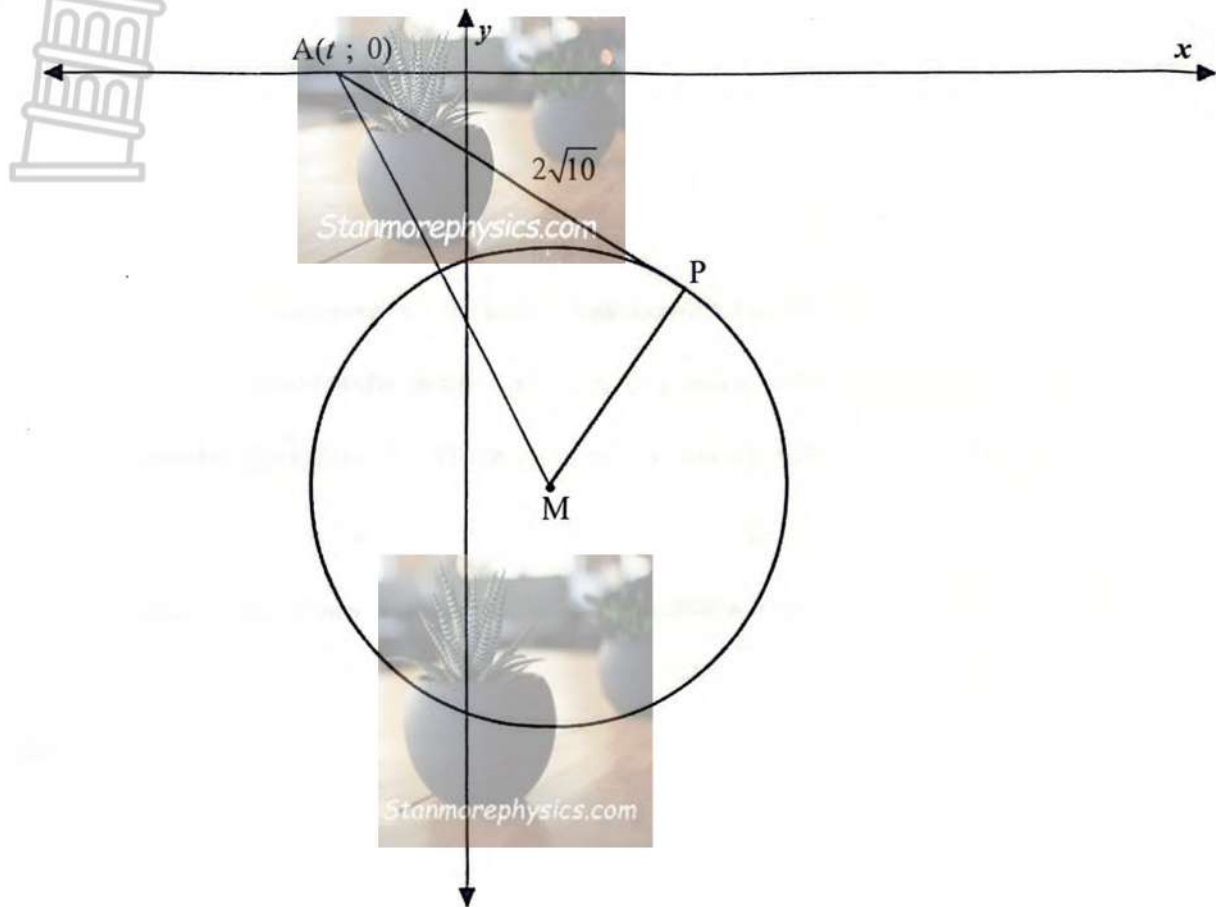
In the diagram below, A, B(-2 ; -1) and C(-6 ; 7) are vertices of $\triangle ABC$. CA is produced to cut the x -axis at G. F is a point on the y -axis and E(3 ; 4) a point on CA such that $FE \parallel BA$. Line AB passes through the origin. The equation of CG is $3y + x = 15$.



- 3.1 Calculate the gradient of AB. (2)
- 3.2 Prove that $CB \perp BA$. (3)
- 3.3 Determine the equation of FE in the form $y = mx + c$. (3)
- 3.4 Calculate the:
- 3.4.1 Size of $\angle OFE$ (3)
- 3.4.2 Area of quadrilateral OFEG (6)
- 3.5 A(6 ; 3), B, C and D form a rectangle with point D in the first quadrant.
- 3.5.1 Calculate the coordinates of D. (2)
- 3.5.2 A circle is drawn through points A, B and C.
- (a) Calculate the coordinates of the centre of the circle. (2)
- (b) Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- [24]

QUESTION 4

In the diagram below, a circle, centred at M, with the equation $x^2 - 4x + y^2 + 10y + 19 = 0$, is drawn. From $A(t; 0)$, a tangent is drawn to touch the circle at P. $\triangle APM$ is drawn and $AP = 2\sqrt{10}$.



- 4.1 Give a reason why $\hat{APM} = 90^\circ$. (1)
- 4.2 Calculate the: (3)
- 4.2.1 Coordinates of M (1)
- 4.2.2 Length of PM, the radius of the circle (4)
- 4.3 Show that $t = -3$. (5)
- 4.4 Another circle, centred at N, with the equation $(x - 5)^2 + (y - e)^2 = 40$, is drawn. Centre N lies on produced line MA, with the equation $y = -x - 3$. (2)
- 4.4.1 Write down the coordinates of N. (5)
- 4.4.2 Determine whether the two circles, centred at M and N, will intersect, touch externally, or do not intersect at all. [16]

QUESTION 5

5.1 Given: $\sqrt{13} \sin \alpha - 2 = 0$ and $x \in [90^\circ; 270^\circ]$

Determine, without the use of a calculator, the value of:

5.1.1 $\tan \alpha$ (4)

5.1.2 $\sin(90^\circ - \alpha)$ (2)

5.2 Consider: $\frac{\sin(180^\circ + \theta) \cdot \cos(90^\circ + \theta)}{\tan \theta \cdot \cos(-\theta)}$

Simplify the expression to a single trigonometric ratio. (5)

5.3 Determine the general solution of x in the equation $\sin x = 1 - \cos 2x$. (6)

5.4 Consider the following identity: $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$

5.4.1 Prove the identity. (2)

5.4.2 Hence, or otherwise, show, without using a calculator, that

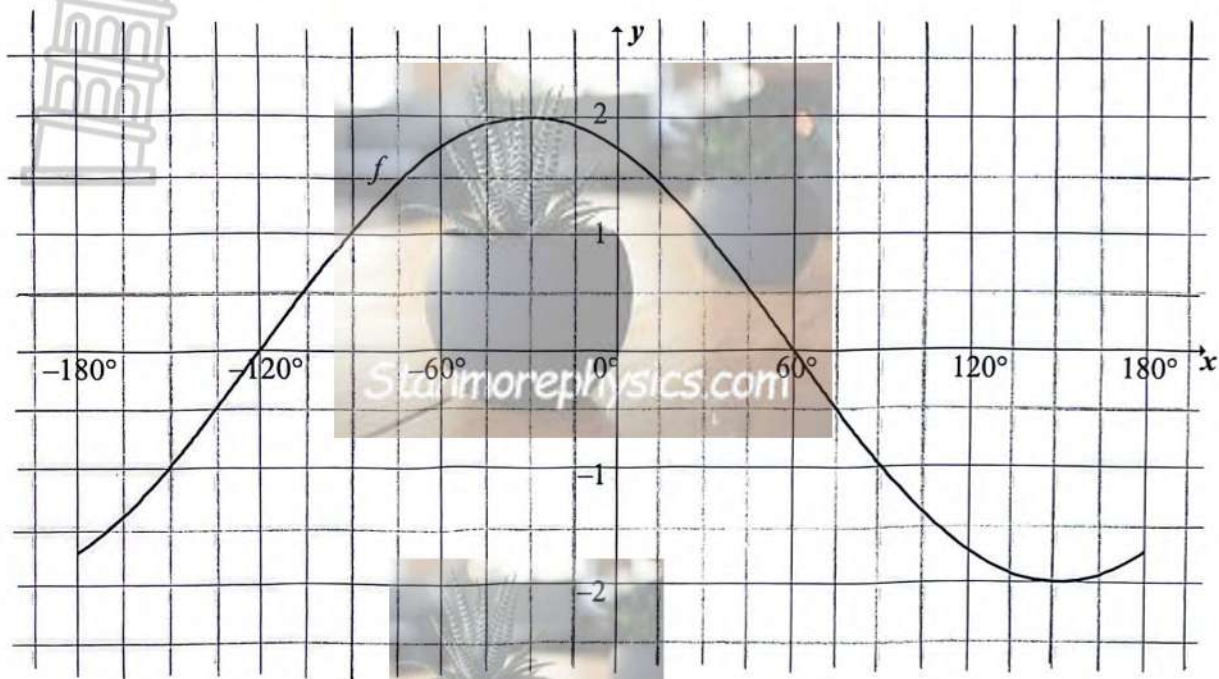
$$\frac{(\sin 7x + \sin 3x)}{(\cos 7x + \cos 3x)} = \tan 5x$$

(5)

[24]

QUESTION 6

In the diagram below, the graph of $f(x) = a \cos(x+b)$ is drawn for the interval $x \in [-180^\circ; 180^\circ]$.

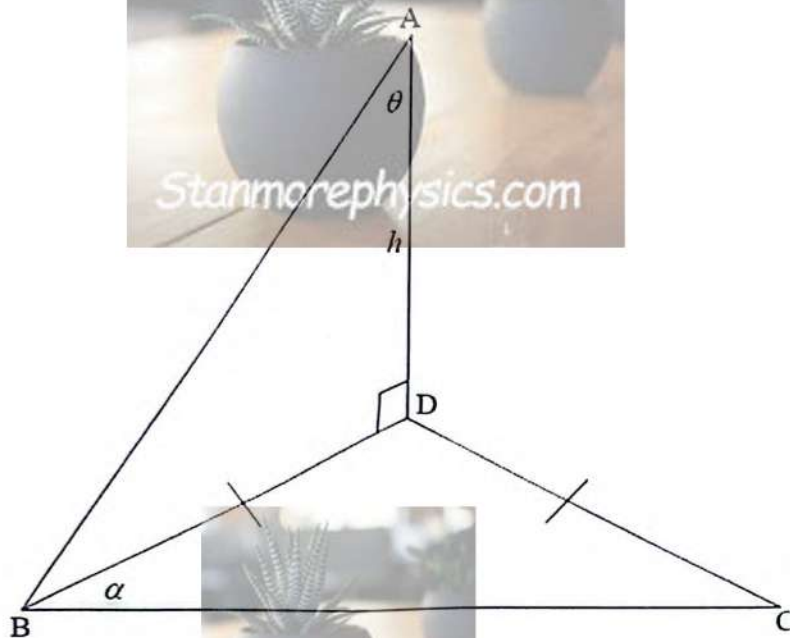


- 6.1 Use the graph to determine the values of a and b . (2)
- 6.2 Draw the graph of $g(x) = \sin 2x + 1$ for the interval $x \in [-180^\circ; 180^\circ]$ on the grid provided in the ANSWER BOOK. Clearly show the intercepts with the axes as well as the coordinates of the turning points. (3)
- 6.3 Write down the period of g . (1)
- 6.4 Determine the range of $2g(x)$. (3)
- 6.5 Use the graphs to determine the value(s) of x for which:
- 6.5.1 $f(x) < g(x)$, in the interval $x \in [-180^\circ; 0^\circ]$ (2)
- 6.5.2 $\tan(x+b)$ is undefined in the interval $x \in [-180^\circ; 180^\circ]$ (2)
- 6.6 Graph of g is shifted 45° to the left to obtain a new graph p . Determine the equation of p in its simplest form. (2)
- [15]

QUESTION 7

In the diagram, AD is a vertical pole such that $AD \perp BD$. B, D and C lie in the same horizontal plane. The pole is anchored at B.

$\hat{B}AD = \theta$, $\hat{D}BC = \alpha$, $AD = h$ and $DB = DC$.



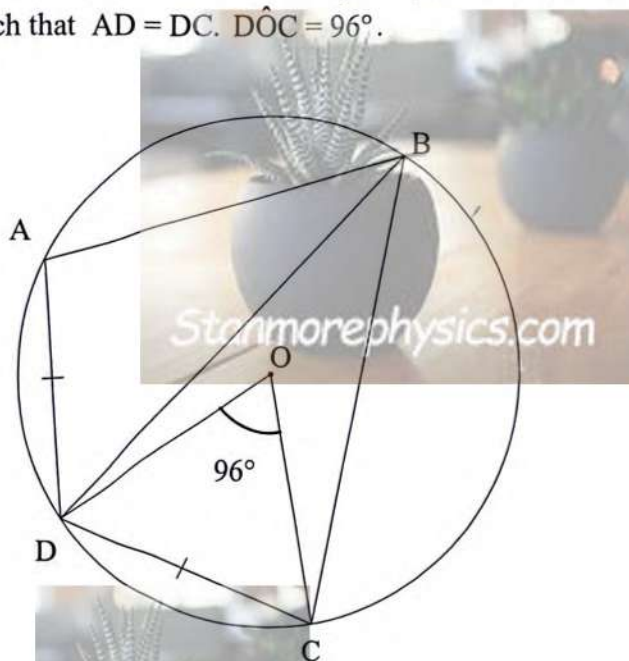
- 7.1 Determine BD in terms of h . (2)
- 7.2 Hence, show that $BC = 2h \tan \theta \cdot \cos \alpha$. (4)
- 7.3 If $\theta = 40^\circ$, $\alpha = 25^\circ$ and $h = 5$ metres, determine the area of $\triangle BDC$. (3)
- [9]

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Provide reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

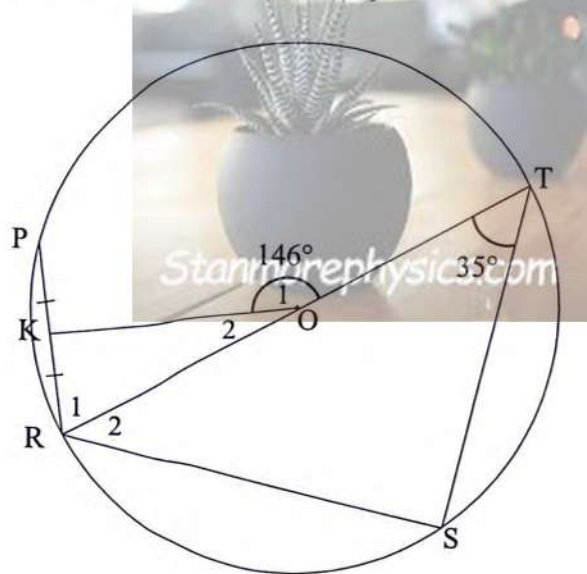
- 8.1 In the diagram, O is the centre of the circle. Cyclic quadrilateral ABCD and chord BD are drawn such that $AD = DC$. $\hat{DOC} = 96^\circ$.



Calculate, with reasons, the size of:

- | | | |
|-------|-------------|-----|
| 8.1.1 | $\hat{DBC}$ | (2) |
| 8.1.2 | $\hat{ABD}$ | (2) |
| 8.1.3 | $\hat{ADO}$ | (5) |

- 8.2 In the diagram, O is the centre of the circle. P, R, S and T are points on the circle. $\triangle RTS$ is drawn. OK bisects chord PR at K. $\hat{O}_1 = 146^\circ$ and $\hat{T} = 35^\circ$.



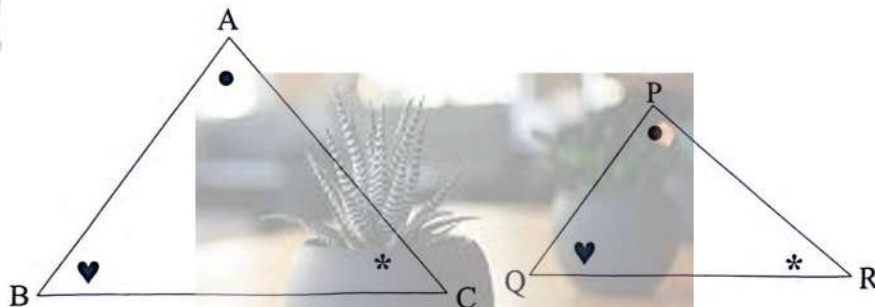
Calculate, with reasons, the size of $\widehat{PRS}$.

(7)
[16]



QUESTION 9

- 9.1 In the diagram below, $\triangle ABC$ and $\triangle PQR$ are drawn such that $\hat{A} = \hat{P}$; $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.

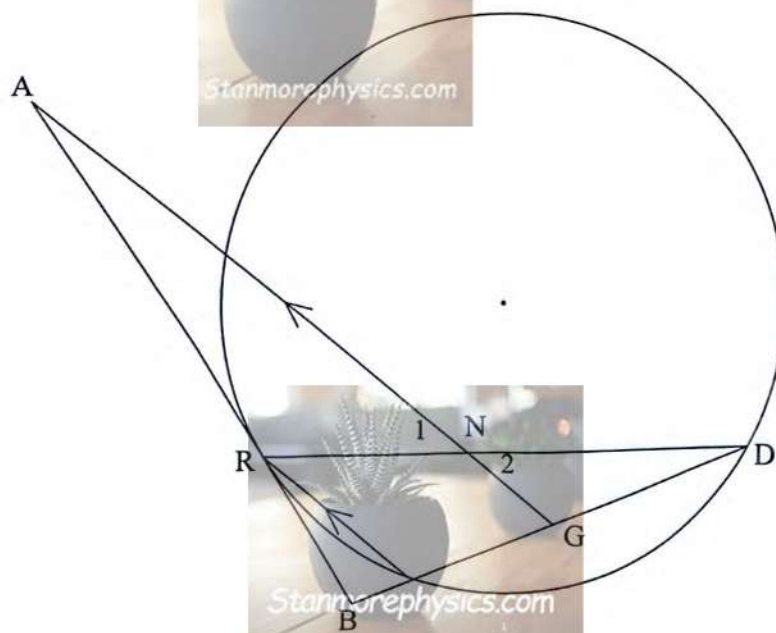


Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion,

i.e. $\frac{AB}{PQ} = \frac{AC}{PR}$

(6)

- 9.2 In the diagram below, AB is a tangent to the circle at R. C and D are points on the circle. $\triangle RCB$ and $\triangle RCD$ are drawn. G is a point on chord CD such that $AG \parallel RC$. Chord RD and AG intersect at N.



Prove, with reasons, that:

9.2.1 $\triangle DNG \parallel \triangle ANR$

(5)

9.2.2 $\hat{BRC} = \hat{A}$

(2)

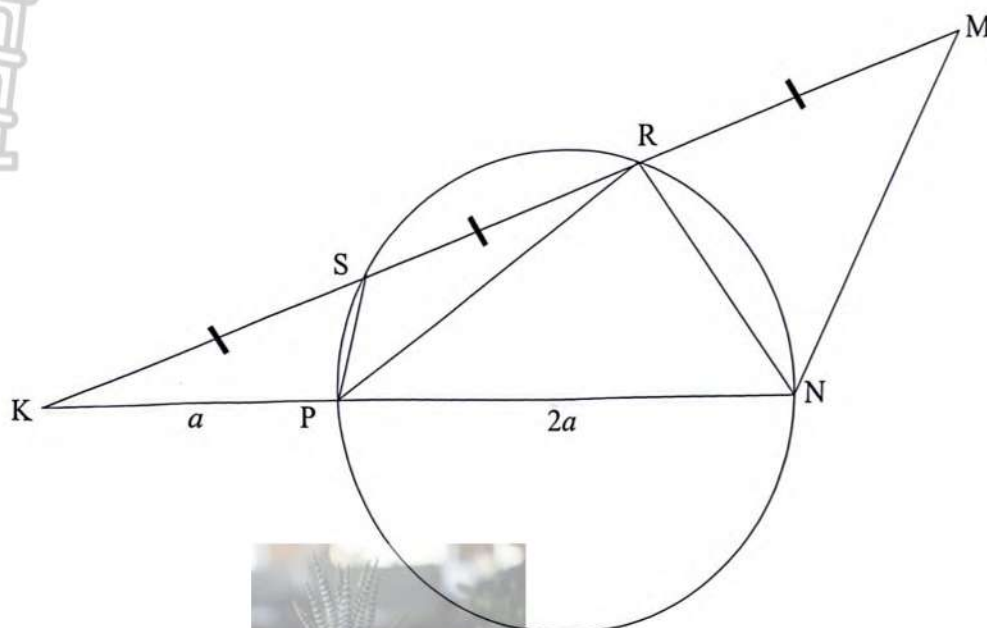
[13]

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QUESTION 10

In the diagram, PNRS is a cyclic quadrilateral of the circle. RS and NP are produced to meet at K. NM meets SR produced at M. Chord PR is drawn.

KS = SR = RM, KM = 18 units, KP = a units and PN = $2a$ units



10.1 Prove, with reasons, that:

10.1.1 PS $\parallel$ NM (2)

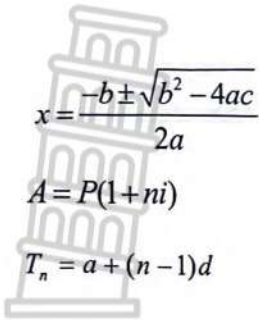
10.1.2 PN is a tangent to a circle passing through R, N and M, at N (4)

10.1.3 $\frac{NM}{NR} = \frac{1}{4}a$ (4)

10.2 If it is further given that $\triangle PNR \parallel \triangle NMR$, determine the length of NM in terms of a . (3)
[13]

TOTAL: 150

INLICHTINGSBLAD



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = (1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
 $a^2 = b^2 + c^2 - 2bc \cos A$
 $\text{area } \triangle ABC = \frac{1}{2} ab \sin C$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

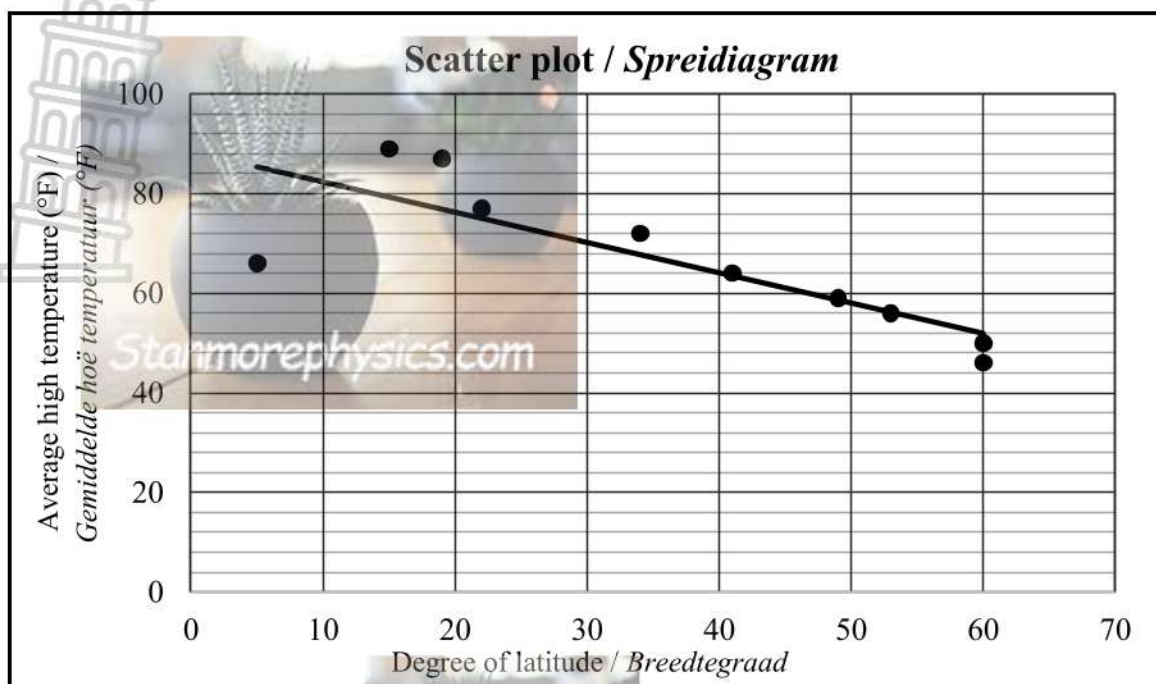
$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG HIERDIE INSTRUKSIES NOUKEURIG
- Clearly write your name, surname and class in the space provided. - Answer ALL the questions in the spaces provided. - No pages may be torn from this answer book. - Read the instructions in the question paper carefully. - Candidates may not retain any answer book or remove them from the examination room. - Answers must be written in blue ink as distinctly as possible. - Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded. - If you require additional space for your answers: - Use the additional space provided at the end of the answer book. - When answering a question in the additional space, clearly indicate the question number in the column on the left-hand side. - Draw a neat line through any work/rough work that must not be marked.	- Skryf jou naam, van en klas in die ruimtes soos verskaf. - Beantwoord ALLE vrae in die ruimtes wat voorsien word. - Geen bladsye mag uit hierdie antwoordeboek geskeur word nie. - Lees die instruksies op die eksamenvraestel sorgvuldig deur. - Kandidate mag nie antwoordeboeke hou of uit die eksamenlokaal verwyder nie. - Antwoorde moet so duidelik moontlik met blou ink geskryf word. - Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word. - Ingeval jy bykomende ruimte benodig vir jou antwoorde: - Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek voorsien word. - As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die linkerkant aan. - Trek 'n netjiese streep deur enige werk/rofwerk wat nie nagesien moet word nie.

QUESTION/VRAAG 1



	Solution Oplossing	Marks Punte
1.1		(2)

Degree of latitude / Breedtegraad	5	19	34	53	22	41	60	15	60	49
Average high temperature (in °F) / Gemiddelde hoë temperatuur (in °F)	66	87	72	56	77	64	46	89	50	59

	Solution Oplossing	Marks Punte
1.2		(3)
1.3		(2)

	Solution Oplossing	Marks Punte
1.4		(1)
1.5		(3)
		[11]



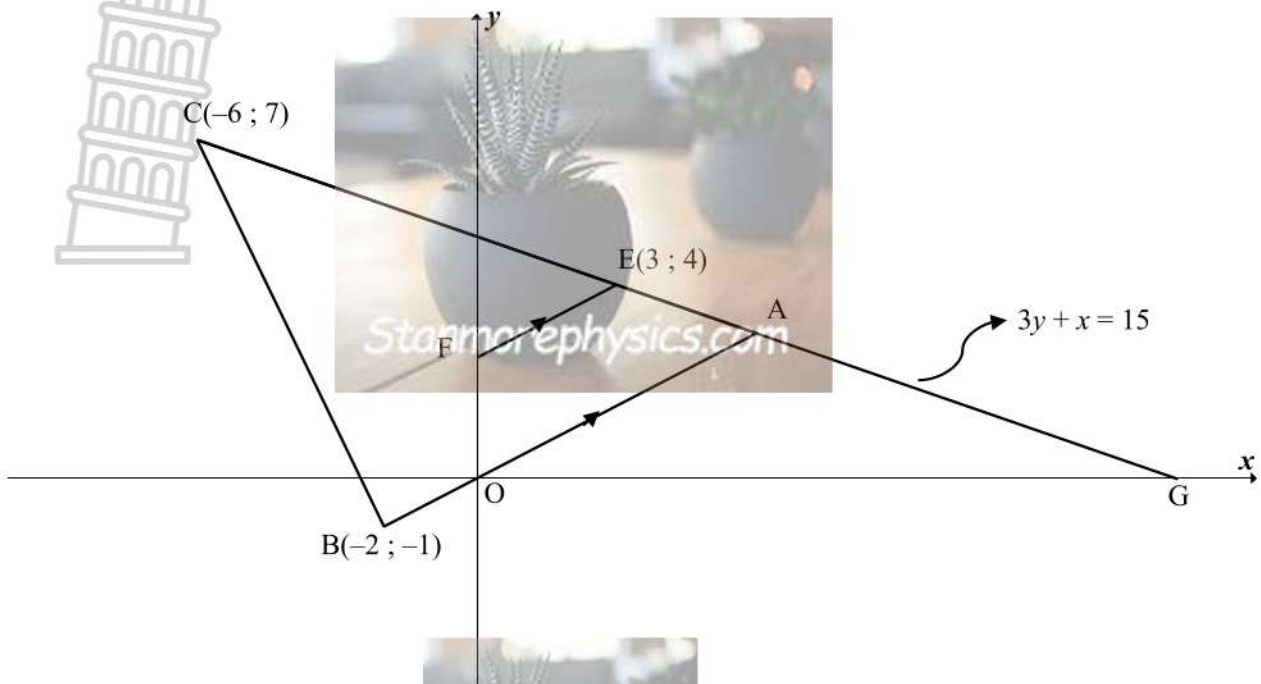
QUESTION/VRAAG 2

	Solution <i>Oplossing</i>	Marks <i>Punte</i>
2.1		(2)
2.2	OGIVE / OGIEF	(3)

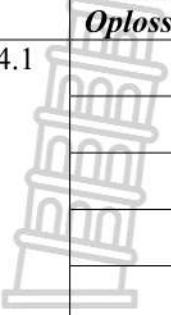
	Solution Oplossing	Marks Punte
2.3		(2)
2.4		(2)
		[9]



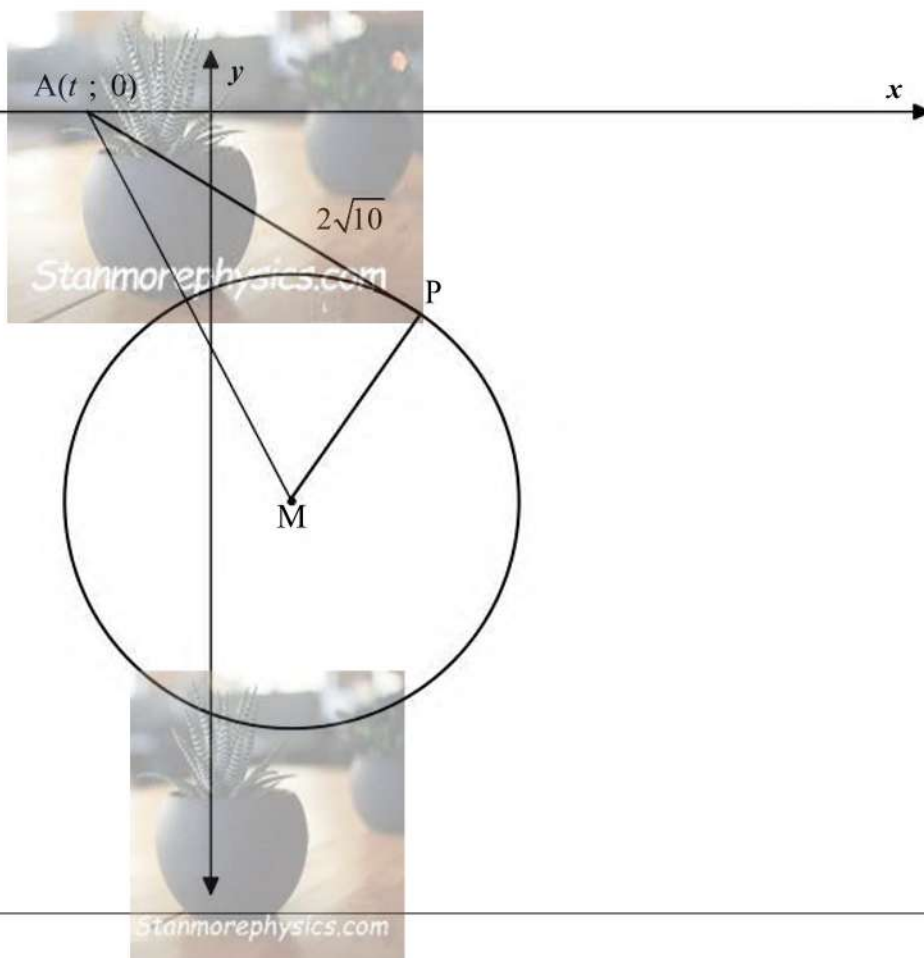
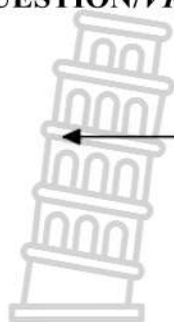
QUESTION/VRAAG 3



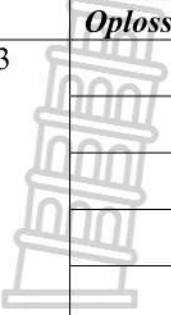
	Solution Oplossing	Marks Punte
3.1		(2)
3.2		(3)
3.3		(3)

	Solution Oplossing	Marks Punte
3.4.1		(3)
3.4.2		(6)
3.5.1		(2)
3.5.2 (a)		(2)
3.5.2 (b)		(3)
		[24]

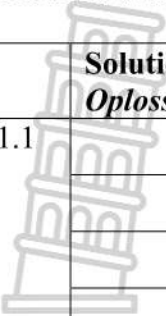
QUESTION/VRAAG 4

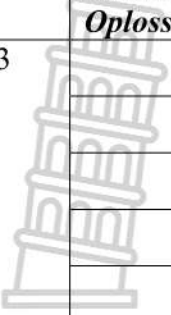
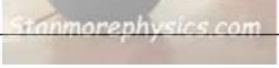


	Solution Oplossing	Marks Punte
4.1		(1)
4.2.1		(3)
4.2.2		(1)

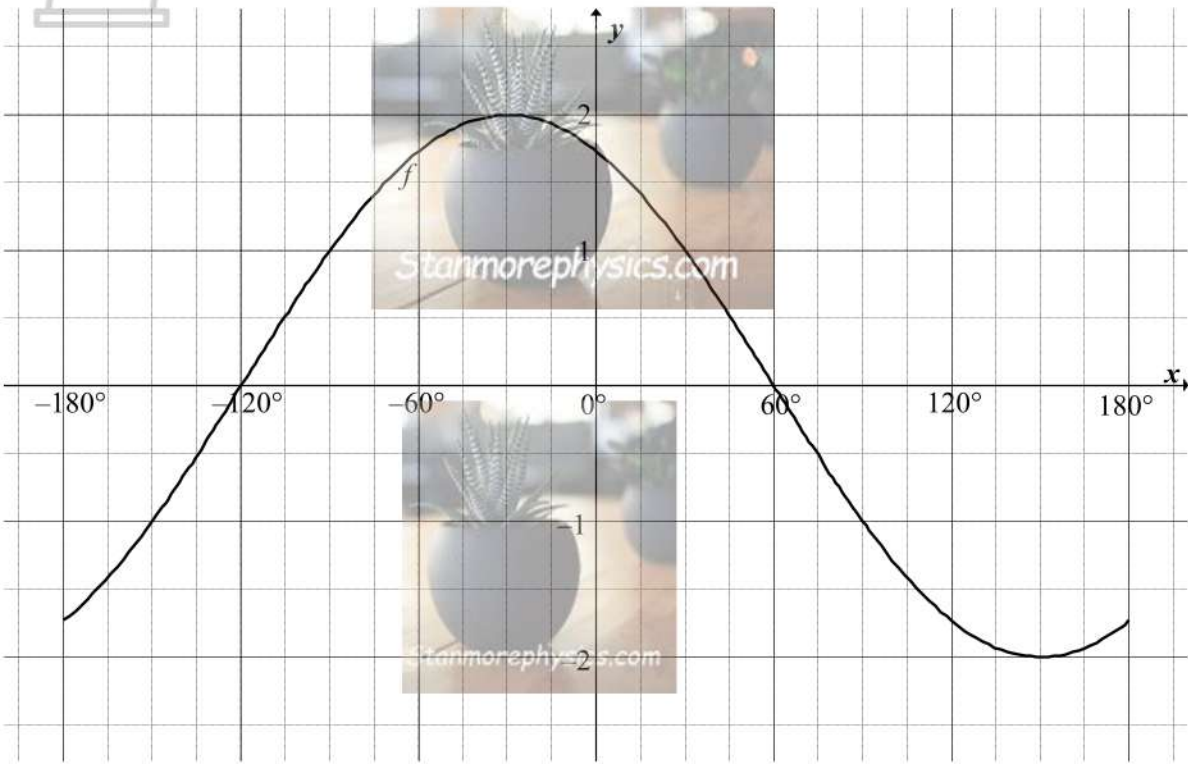
	Solution Oplossing	Marks Punte
4.3		(4)
4.4.1		(2)
4.4.2		(5)
		[16]

QUESTION/VRAAG 5

	Solution Oplossing	Marks Punte
5.1.1		(4)
5.1.2		(2)
5.2		(5)

	Solution Oplossing	Marks Punte
5.3		(6)
5.4.1		(2)
5.4.2		(5)
		[24]

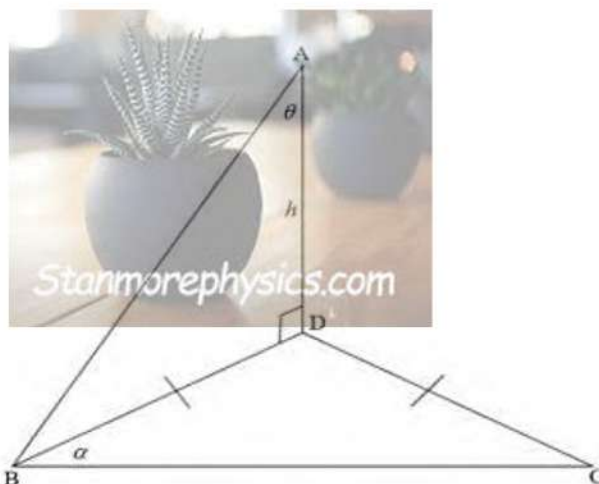
QUESTION/VRAAG 6

Solution Oplossing		Marks Punte
6.1		(2)
6.2		
6.3		(1)
6.4		
6.5.1		(2)

	Solution Oplossing	Marks Punte
6.5.2		(2)
6.6		(2)
		[15]



QUESTION/VRAAG 7

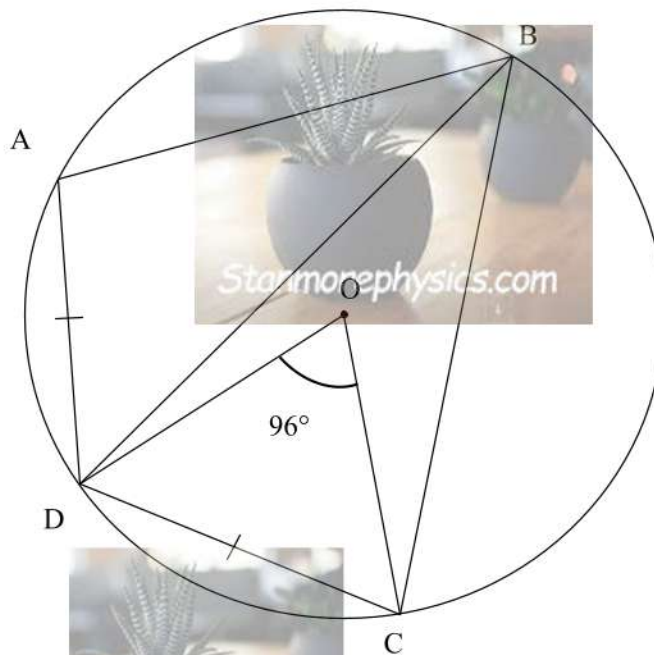


	Solution Oplossing	Marks Punte
7.1		(2)
7.2		(4)
7.3		(3)
		[9]

Provide reasons for your statements in QUESTIONS 8, 9 and 10.
Verskaf redes vir jou bewerings in VRAAG 8, 9 en 10.

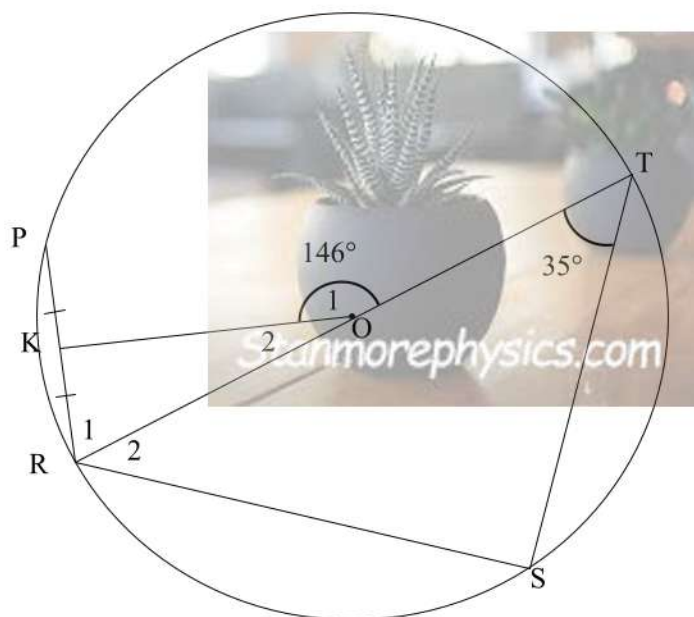
QUESTION/VRAAG 8

8.1



	Solution Oplossing	Marks Punte
8.1.1		(2)
8.1.2		(2)
8.1.3		(5)

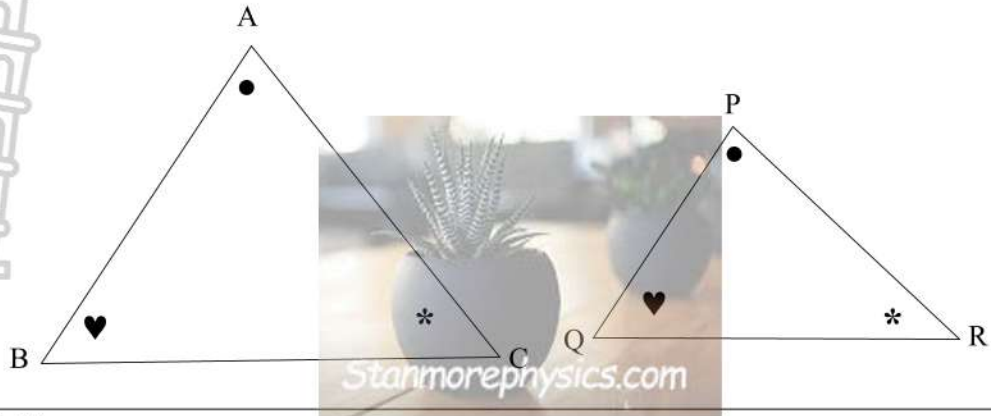
8.2



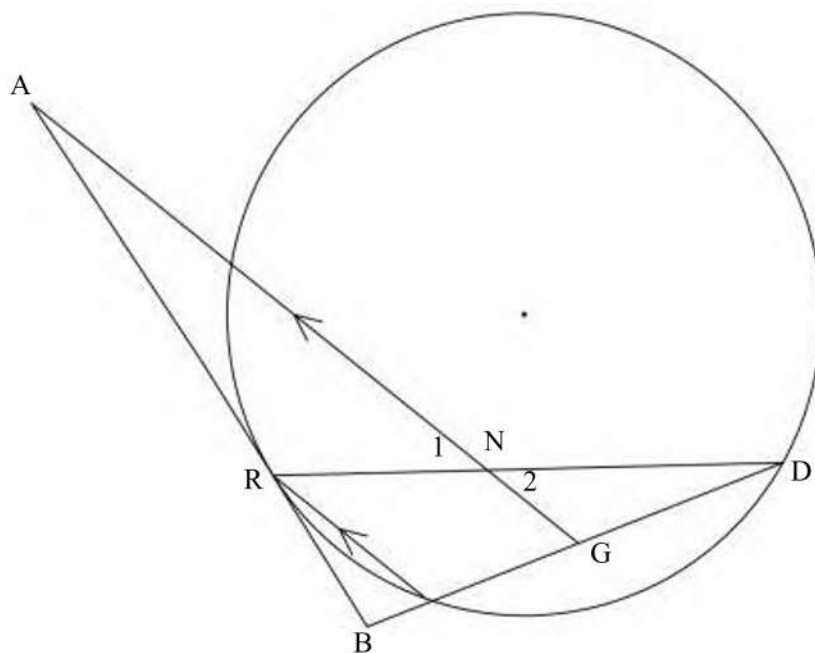
	Solution Oplossing	Marks Punte
		(7)
		[16]

QUESTION/VRAAG 9

9.1

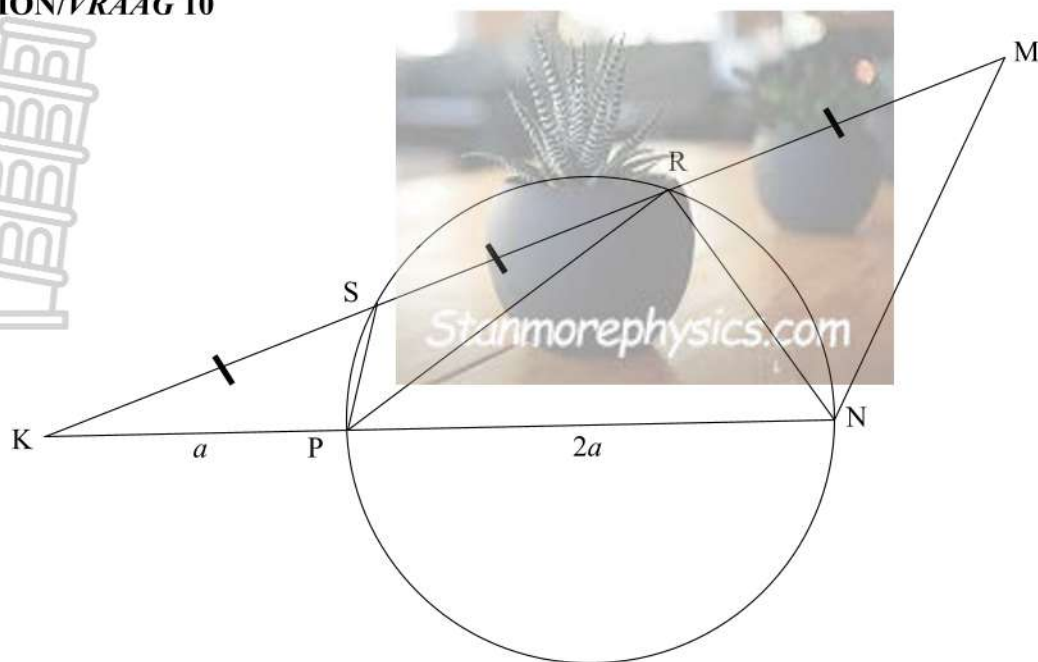
[illegible]

9.2

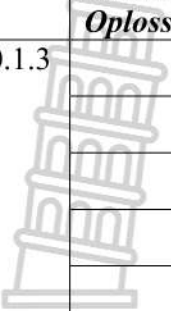


	Solution Oplossing		Marks Punte
9.2.1			(5)
9.2.2			(2)
			[13]

QUESTION/VRAAG 10



	Solution Oplossing	Marks Punte
10.1.1		(2)
10.1.2		(4)

	Solution Oplossing	Marks Punte
10.1.3		(4)
10.2		(3)
		[16]

TOTAL/TOTAAL: 150

Bykom



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**PROVINCIAL PREPARATORY EXAMINATION/
PROVINSIALE VOORBEREIDENDE EKSAMEN**

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GRADE/GRAAD 12

MATHEMATICS/WISKUNDE

PAPER/VRAESTEL 2

SEPTEMBER 2025

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 18 pages./
Hierdie nasienriglyne bestaan uit 18 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

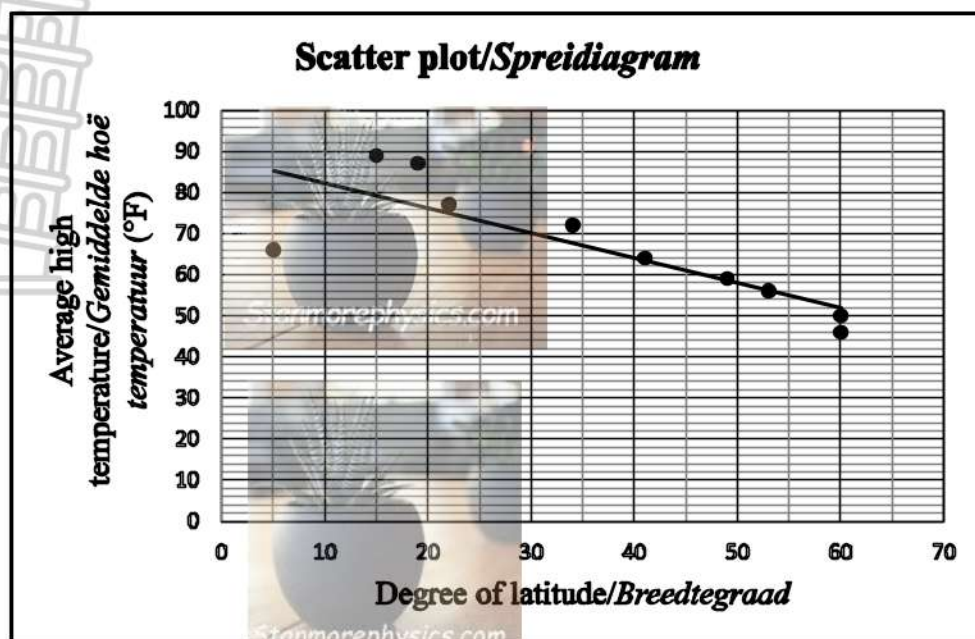
NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien ná die tweede berekeningsfout.
- Om antwoorde/waardes aan te neem om 'n probleem op te los, word NIE toegelaat NIE.

GEOMETRY/MEETKUNDE	
S	A mark for a correct statement (A statement is independent of a reason)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct
	Ken 'n punt toe as die bewering EN rede beide korrek is



QUESTION/VRAAG 1

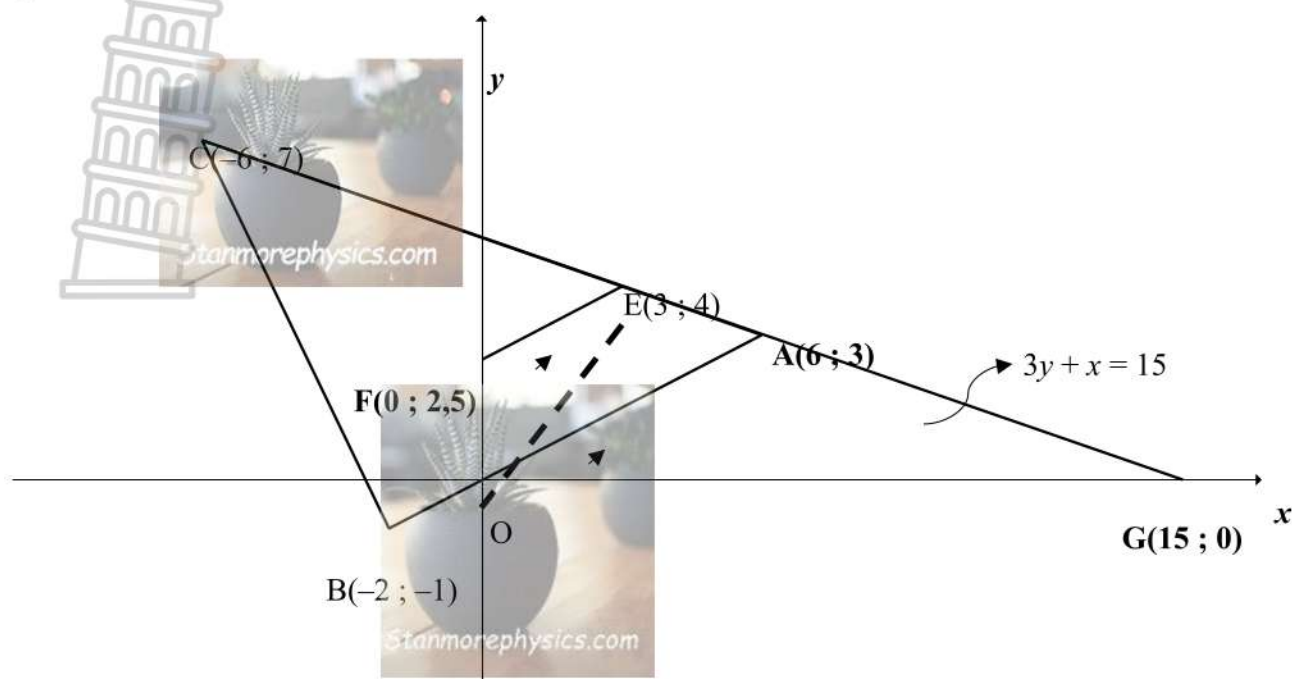


1.1	Strong negative correlation/ <i>Sterk negatiewe korrelasie</i> Data points are close to the regression line/ <i>Data punte lê naby regressielyn.</i>	✓ strong/ <i>sterk</i> ✓ reason/ <i>rede</i>	(2)																						
	<table><tr><td>Latitude in degrees/ <i>Beedtegraad</i></td><td>5</td><td>19</td><td>34</td><td>53</td><td>22</td><td>41</td><td>60</td><td>15</td><td>60</td><td>49</td></tr><tr><td>Average high temperature/<i>Gemiddelde hoë temperatuur (in °F)</i></td><td>66</td><td>87</td><td>72</td><td>56</td><td>77</td><td>64</td><td>46</td><td>89</td><td>50</td><td>59</td></tr></table>	Latitude in degrees/ <i>Beedtegraad</i>	5	19	34	53	22	41	60	15	60	49	Average high temperature/ <i>Gemiddelde hoë temperatuur (in °F)</i>	66	87	72	56	77	64	46	89	50	59		
Latitude in degrees/ <i>Beedtegraad</i>	5	19	34	53	22	41	60	15	60	49															
Average high temperature/ <i>Gemiddelde hoë temperatuur (in °F)</i>	66	87	72	56	77	64	46	89	50	59															
1.2	$a = 88,33$ $b = -0,61$ $\hat{y} = 88,33 - 0,61x$ Answer only: Full Marks/ <i>Slegs antwoord: Volpunte</i>	✓ a ✓ b ✓ eq/vgl	(3)																						
1.3	$y = 88,33 - 0,61(28)$ $= 71,25^{\circ}\text{F}$ OR/OF $y = 71,33^{\circ}\text{F}$ (calc/sakr) OR/OF From Diagram: 71-72	✓ subst ✓ answ/antw ✓✓ answ/antw	(2) (2)																						
1.4	$\sigma_y = 13,9^{\circ}\text{F}$	✓ answ/antw	(1)																						
1.5	$\bar{y} = 66,6$ $66,6 + 13,9 = 80,5$ $\therefore$ 2 Cities / <i>Stede</i> CA: From 1.4 if learner used $\sigma_x = 18,77$ $35,8 + 18,77 = 54,58$ 2 cities (3/3)	✓ mean ✓ mean + σ_y ✓ answ/antw	(3)																						
			[11]																						

QUESTION/VRAAG 2

2.1	$k = 44$ $f = 6$	✓ k ✓ f (2)
2.2	OGIVE / OGIEF	✓ (60 ; 0) ✓ (90 ; 50) ✓ S-curve/- kurwe (3)
2.3	$M = 77 \text{ kg}$ (accept: 76 – 78 kg) CA from OGIVE/ van OGIEF	✓ $CF = 25$ ✓ answ/antw (2)
2.4	$(83 ; \pm 40)$ (accept: 39 – 41) $\therefore \frac{10}{50} \times 100 = 20\%$	✓ CF ✓ % (2)
		[9]

QUESTION/VRAAG 3

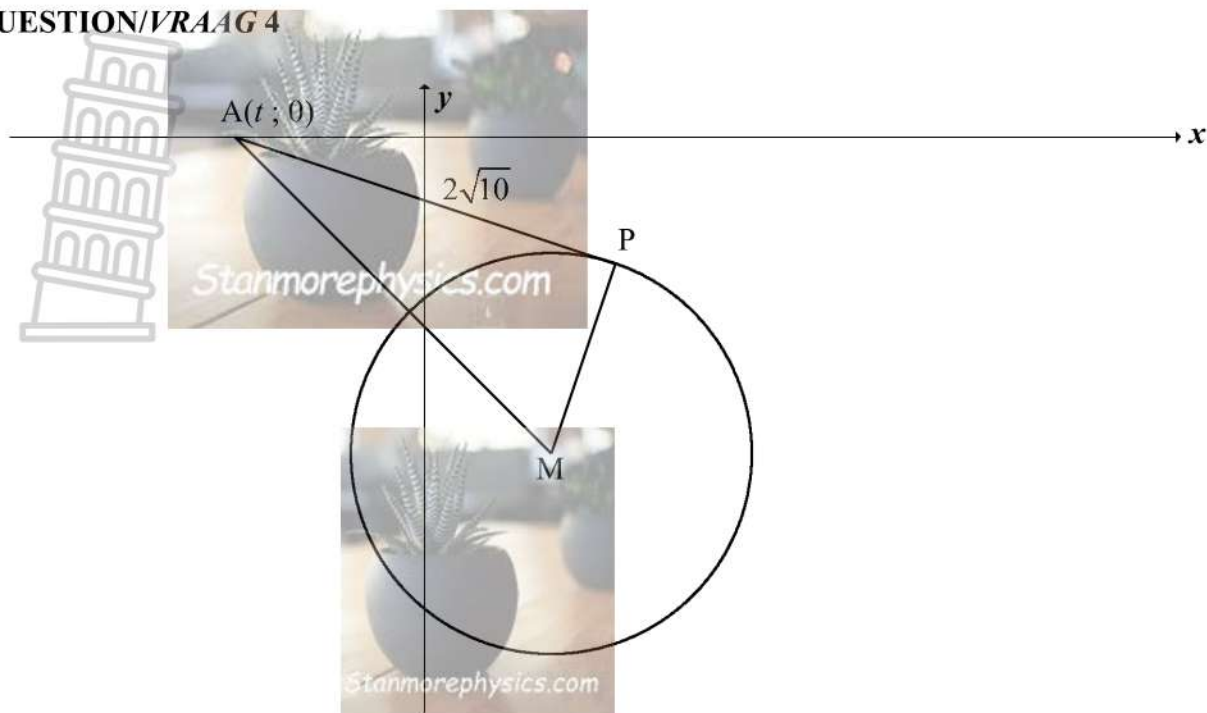


3.1	$m_{AB} = \frac{-1-0}{-2-0} = \frac{1}{2}$	✓ subst B and O ✓ answ/antw (2)
3.2	$m_{CB} = \frac{7+1}{-6+2} = -2$ $m_{AB} \times m_{CB} = \frac{1}{2} \times -2 = -1$ $\therefore CB \perp BA$	✓ subst B and C ✓ m_{CB} ✓ $m_1 \times m_2 = -1$ (3)
3.3	$y = mx + c$ OR / OF $y - y_1 = m(x - x_1)$ $4 = \frac{1}{2}(3) + c$ $y - 4 = \frac{1}{2}(x - 3)$ $c = \frac{5}{2}$ $y = \frac{1}{2}x + \frac{5}{2}$ $y = \frac{1}{2}x + \frac{5}{2}$	✓ $m_{EF} = m_{AB}$ ✓ subst E into equation ✓ eq/vgl (3)
3.4.1	$\tan \hat{AOG} = \frac{1}{2}$ $\hat{AOG} = 26,57^\circ$ $\hat{OFE} = 26,57^\circ + 90^\circ$ (Corr $\angle$'s; $FE \parallel BA$) $= 116,57^\circ$	✓ $\tan \hat{AOG} = \frac{1}{2}$ ✓ size of $\hat{AOG}$ ✓ $\hat{OFE}$ (3)

3.4.2	F(0 ; 2,5) G(15 ; 0) <u>Draw EO</u> area OFEG = area $\triangle OFE$ + area $\triangle EOG$ $= \frac{1}{2} \left(\frac{5}{2} \right) (3) + \frac{1}{2} (15)(4)$ $= 3\frac{3}{4} + 30$ $= 33\frac{3}{4} \text{ units}^2$	✓ coordinates of F ✓ coordinates of G ✓ method ✓ area of $\triangle OFE$ ✓ area of $\triangle EOG$ ✓ area of OFEG (6)
3.5.1	Use midpoint of AC: D(2 ; 11)	✓ x ✓ y (2)
3.5.2 (a)	AC = diameter (converse $\angle$ in semi circle) Centre $\left(\frac{-6+6}{2} ; \frac{7+3}{2} \right) = (0 ; 5)$	✓ x ✓ y (2)
3.5.2 (b)	$r = \sqrt{(0-6)^2 + (5-3)^2}$ $= \sqrt{40}$ $x^2 + (y-5)^2 = 40$	✓ subst into distance formula ✓ LHS ✓ RHS (3)
		[24]



QUESTION/VRAAG 4



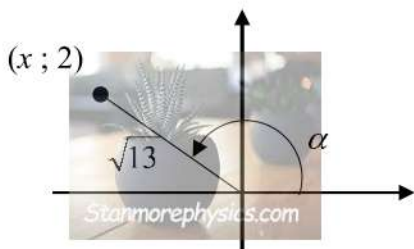
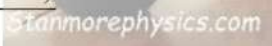
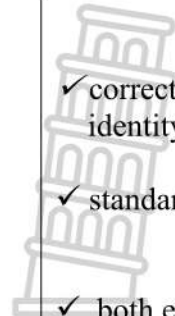
4.1	Radius $\perp$ Tangent	✓ reason (1)
4.2.1	$x^2 - 4x + 4 + y^2 + 10y + 25 = -19 + 4 + 25$ $(x - 2)^2 + (y + 5)^2 = 10$ $\therefore M(2; -5)$ OR/OF $M\left(-\frac{1}{2}(-4); -\frac{1}{2}(10)\right)$ $\therefore M(2; -5)$ Need to show some calculation	✓ LHS & RHS ✓ x ✓ y (3) ✓ using formula ✓ x ✓ y (3)
4.2.2	$r = \sqrt{2^2 + (-5)^2 - 19}$ $= \sqrt{10}$ Answer only: Full Marks/ Slegs antwoord: Volpunte	✓ answer/antwoord (1)
4.3	$AM^2 = AP^2 + MP^2 \text{ (Pyth th)}$ $(t - 2)^2 + (0 + 5)^2 = (2\sqrt{10})^2 + (\sqrt{10})^2$ $t^2 - 4t + 4 + 25 = 40 + 10$ $t^2 - 4t - 21 = 0$ $(t - 7)(t + 3) = 0$ $\therefore t = -3 \text{ (given)}$	✓ subst into distance formula ✓ correct use of Pyth th ✓ standard form ✓ factors (4)

4.4.1	$y = -5 - 3$ $= -8$ $\therefore N(5; -8)$	✓ substitution ✓ y-value of N (2)
4.4.2	$MN = \sqrt{(-5 - (-8))^2 + (2 - 5)^2}$ $= 3\sqrt{2} = 4,24 \text{ units}$ Radius of circle N = $\sqrt{40} = 2\sqrt{10} = 6,32 \text{ units}$ Sum of two radii = $\sqrt{10} + \sqrt{40} = 3\sqrt{10} = 9,49 \text{ units}$ $\therefore MN < \text{sum of 2 radii}$ $\therefore 2 \text{ circles intersect}$	✓ subst into distance formula ✓ MN ✓ radius of N ✓ sum of radii ✓ answer (5)
		[16]

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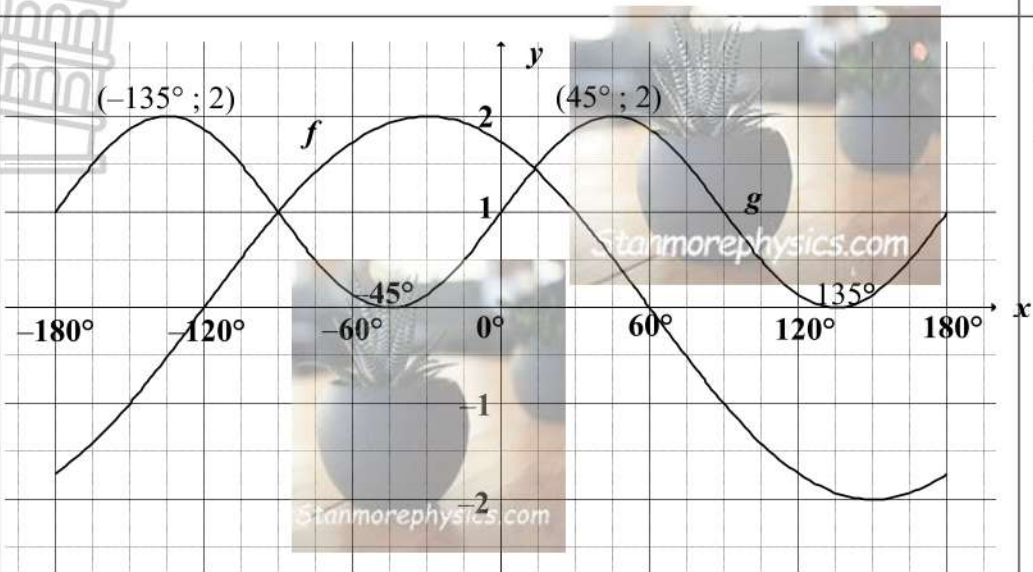


QUESTION/VRAAG 5

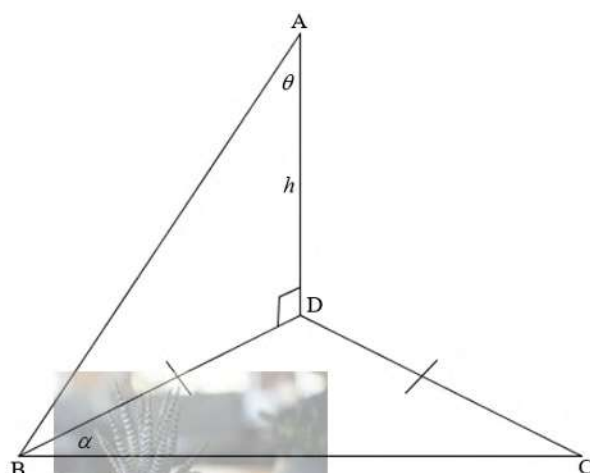
5.1.1	$\sin \alpha = \frac{2}{\sqrt{13}}$ $x^2 + y^2 = r^2$ $x^2 + 4 = 13$ $x^2 = 9$ $x = -3$ $\tan \alpha = -\frac{2}{3}$ 	✓ 2nd quadrant ✓ subst into Pyth ✓ value of x ✓ answer/antw (4)
5.1.2	$\sin(90^\circ - \alpha) = \cos \alpha$ $= \frac{-3}{\sqrt{13}}$ 	✓ co-function ✓ answer/antwoord (2)
5.2	$\frac{\sin(180^\circ + \theta) \cdot \cos(90^\circ + \theta)}{\tan \theta \cdot \cos(-\theta)}$ $\frac{-\sin \theta \cdot -\sin \theta}{\frac{\sin \theta}{\cos \theta} \cdot \cos \theta}$ $= \sin \theta$ OR $= \frac{-\sin \theta \cdot -\sin \theta}{\tan \theta \cdot \cos \theta}$ $= \frac{\tan \theta \sin \theta}{\tan \theta}$ $= \sin \theta$ 	✓ $-\sin \theta$ ✓ $-\sin \theta$ ✓ $\cos \theta$ ✓ $\frac{\sin \theta}{\cos \theta}$ ✓ $\sin \theta$ (5) ✓ $-\sin \theta$ ✓ $-\sin \theta$ ✓ $\cos \theta$ ✓ $\tan \theta$ ✓ $\sin \theta$ (5)
5.3	$\sin x = 1 - \cos 2x$ $\sin x = 1 - (1 - 2\sin^2 x)$ $\sin x = 1 - 1 + 2\sin^2 x$ $\sin x - 2\sin^2 x = 0$ $\sin x(1 - 2\sin x) = 0$ $\sin x = 0 \text{ or / of } \sin x = \frac{1}{2}$ $\sin x = 0:$ $x = 0^\circ + k \cdot 360^\circ \text{ or } 180^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ $\sin x = \frac{1}{2}:$ $x = 30^\circ + k \cdot 360^\circ \text{ or / of } x = 150^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$	 ✓ correct double identity ✓ standard form ✓ both equations ✓ 0° & 180° ✓ 30° & 150° ✓ $+ k \cdot 360^\circ; k \in \mathbb{Z}$ (6)

5.4.1	$\begin{aligned} \text{LHS} &= \sin A \cos B + \cos A \sin B + \sin A \cos B - \cos A \sin B \\ &= 2 \sin A \cos B \\ &= \text{RHS} \end{aligned}$	✓ expansion of $\sin(A+B)$ ✓ expansion of $\sin(A-B)$ (2)
5.4.2	$\begin{aligned} \text{LHS} &= \frac{\sin(5x+2x) + \sin(5x-2x)}{\cos(5x+2x) + \cos(5x-2x)} \\ &= \frac{2 \sin 5x \cos 2x}{\cos 5x \cos 2x - \sin 5x \sin 2x + \cos 5x \cos 2x + \sin 5x \sin 2x} \\ &= \frac{2 \sin 5x \cos 2x}{2 \cos 5x \cos 2x} \\ &= \frac{\sin 5x}{\cos 5x} \\ &= \tan 5x = \text{RHS} \end{aligned}$ OR/OF $\begin{aligned} \text{LHS} &= \frac{\sin(5x+2x) + \sin(5x-2x)}{\cos(5x+2x) + \cos(5x-2x)} \\ &= \frac{2 \sin 5x \cos 2x}{\sin[90^\circ - (5x+2x)] + \sin[90^\circ - (5x-2x)]} \\ &= \frac{2 \sin 5x \cos 2x}{\sin[(90^\circ - 5x) - 2x] + \sin[(90^\circ - 5x) + 2x]} \\ &= \frac{2 \sin 5x \cos 2x}{2 \sin(90^\circ - 5x) \cos(2x)} \\ &= \frac{2 \sin 5x \cos 2x}{2 \cos 5x \cos 2x} \\ &= \tan 5x \end{aligned}$	✓ $5x+2x$ & $5x-2x$ ✓ simplify numerator ✓ expand denominator ✓ simplify denominator ✓ simplify fraction (5) ✓ $5x+2x$ & $5x-2x$ ✓ simplify numerator ✓ expand denominator ✓ simplify denominator ✓ simplify fraction (5)
		[24]

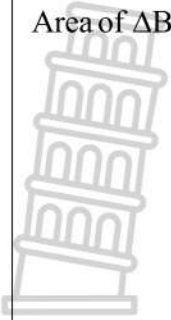
QUESTION/VRAAG 6

6.1	$a = 2$ $b = 30^\circ$	✓ a ✓ b (2)
6.2		✓ y-intercept ✓ max tps ✓ min tps (3)
6.3	180°	✓ answer/antw (1)
6.4	$2g(x) = 2 \sin 2x + 2$ $\therefore 0 \leq y \leq 4$ OR $y \in [0; 4]$	✓ $2g(x)$ ✓ critical values ✓ inequalities (3)
6.5.1	$x \in [-180^\circ; -90^\circ)$ OR $-180^\circ \leq x < -90^\circ$	✓ critical values ✓ inequalities (2)
6.5.2	where $\cos(x + b) = 0$ $x = -120^\circ$ or 60°	✓ -120° ✓ 60° (2)
6.6	$p(x) = \sin[2(x + 45^\circ)] + 1$ $= \sin(2x + 90^\circ) + 1$ $p(x) = \cos 2x + 1$	Answer only: Full Marks/ Slegs antwoord: Volpunte ✓ correct subst of $x + 45^\circ$ ✓ answer/antwoord (2)
		[15]

QUESTION/VRAAG 7

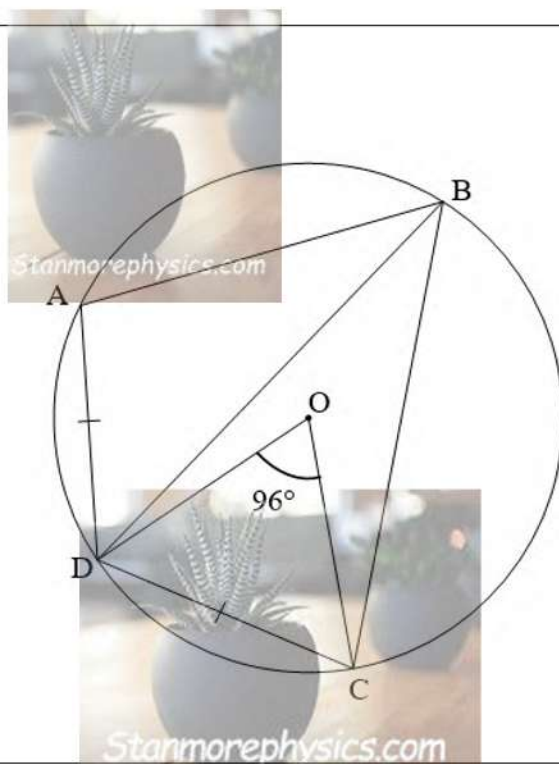


7.1	$\tan \theta = \frac{BD}{h}$ $BD = h \tan \theta$	✓ tan ratio ✓ answer/ antwoord (2)
7.2	$\hat{DCB} = \alpha \quad (\angle s \text{ opp} = \text{sides}) / (\angle e \text{ to} = \text{sye})$ $\hat{BDC} = 180^\circ - 2\alpha \quad (\angle^{s/e} \text{ of/van } \Delta)$ $\frac{BC}{\sin(180^\circ - 2\alpha)} = \frac{BD}{\sin \alpha}$ $\frac{BC}{\sin 2\alpha} = \frac{h \tan \theta}{\sin \alpha}$ $BC = \frac{h \tan \theta \sin 2\alpha}{\sin \alpha}$ $= \frac{h \tan \theta \cdot 2 \sin \alpha \cos \alpha}{\sin \alpha}$ $= 2h \tan \theta \cos \alpha$ OR/OF $BC^2 = (h \tan \theta)^2 + (h \tan \theta)^2 - 2(h \tan \theta) \cos(180^\circ - 2\alpha)$ $= h^2 \tan^2 \theta + h^2 \tan^2 \theta - 2h^2 \tan^2 \theta (-\cos 2\alpha)$ $= 2h^2 \tan^2 \theta (1 + \cos 2\alpha)$ $= 2h^2 \tan^2 \theta (2 \cos^2 \alpha)$ $BC = \sqrt{4h^2 \tan^2 \theta \cos^2 \alpha}$ $BC = 2h \tan \theta \cos \alpha$	✓ $180^\circ - 2\alpha$ ✓ subst into sin rule ✓ reduction ✓ sin double angle identity (4) ✓ $180^\circ - 2\alpha$ ✓ subst into cos rule ✓ reduction ✓ cos double angle identity (4)

7.3	 $\begin{aligned}\text{Area of } \triangle BDC &= \frac{1}{2} BD \cdot DC \cdot \sin \hat{BDC} \\ &= \frac{1}{2} (h \tan \theta) (h \tan \theta) \sin(180^\circ - 2\alpha) \\ &= \frac{1}{2} (5 \tan 40^\circ) (5 \tan 40^\circ) \sin 130^\circ \\ &= 6,74 \text{ units}^2\end{aligned}$	✓ subst into area rule ✓ subst values ✓ answer/antwoord (3)
		[9]

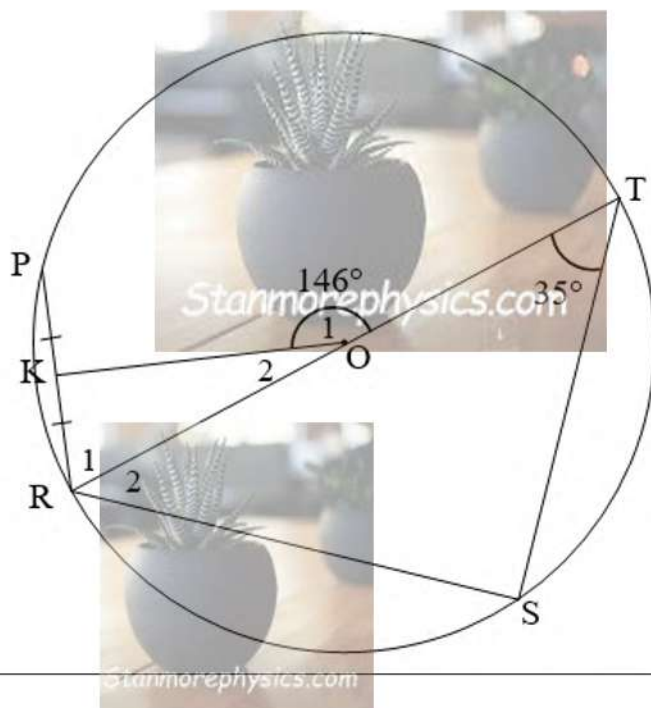
QUESTION/VRAAG 8

8.1



8.1.1	$\hat{DBC} = 48^\circ$ ($\angle$ at centre = $2 \angle$ at circle / midpts $\angle = 2 \text{ omtreks } \angle$)	✓ S ✓ R (2)
8.1.2	$\hat{ABD} = 48^\circ$ (= chords; = $\angle$ s / = koorde; = $\angle$ e)	✓ S ✓ R (2)
8.1.3	$\hat{ODC} = 42^\circ$ ($\angle^s$ opp = sides / $\angle^e$ to = sye) $\hat{ADC} = 180^\circ - 96^\circ$ (opp $\angle^s$ cyc quad / to $\angle^e$ koordevh) $= 84^\circ$ $\hat{ADO} = 84^\circ - 42^\circ$ $= 42^\circ$	✓ S ✓ R ✓ S ✓ R ✓ answer/antwoord (5)

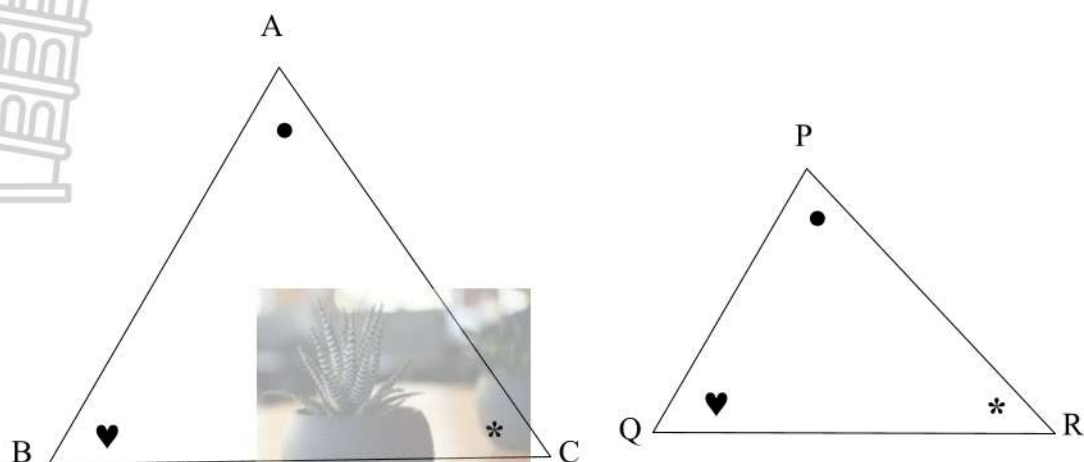
8.2



$\hat{S} = 90^\circ$ (ϕ in half circle) $\hat{R}_2 = 55^\circ$ ($\angle^s$ of Δ) $O_2 = 34^\circ$ ($\angle^s$ on a straight line) $\hat{RKO} = 90^\circ$ (line from centre to midpt of chord) $\hat{R}_1 = 56^\circ$ (ext $\angle$ of Δ) $\hat{PRS} = 56^\circ + 55^\circ = 111^\circ$ OR/OF $\hat{S} = 90^\circ$ (ϕ in half circle) $\hat{R}_2 = 55^\circ$ ($\angle^s$ of Δ) $\hat{RKO} = 90^\circ$ (line from centre to midpt of chord) $\hat{R}_1 = 56^\circ$ (ext $\angle$ of Δ) $\hat{PRS} = 56^\circ + 55^\circ = 111^\circ$	$\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark S \quad \checkmark R$ $\checkmark S/R$ $\checkmark \text{ answer/}$ antwoord (7) $\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark S \quad \checkmark R$ $\checkmark S/R$ $\checkmark \text{ answer/}$ antwoord (7)
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QUESTION/VRAAG 9

9.1



Constr:

Let G and H lie on AB and AC respectively such that AG = PQ and AH = PR. Draw GH.

Proof:

In $\triangle AGH$ and $\triangle PQR$:

AG = PQ [Constr]

AH = PR [Constr]

$\hat{A} = \hat{P}$ [Given]

$\therefore \triangle AGH \equiv \triangle PQR$ (SAS)

$\therefore \hat{AGH} = \hat{PQR}$

GH $\parallel$ BC [corresp $\angle$'s are equal]

$\frac{AB}{AG} = \frac{AC}{AH}$ [line $\parallel$ one side of $\triangle$ OR prop theorem; MN $\parallel$ BC]

$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$ [AM = DE and AN = DF]

✓ constr

✓ S ✓ R

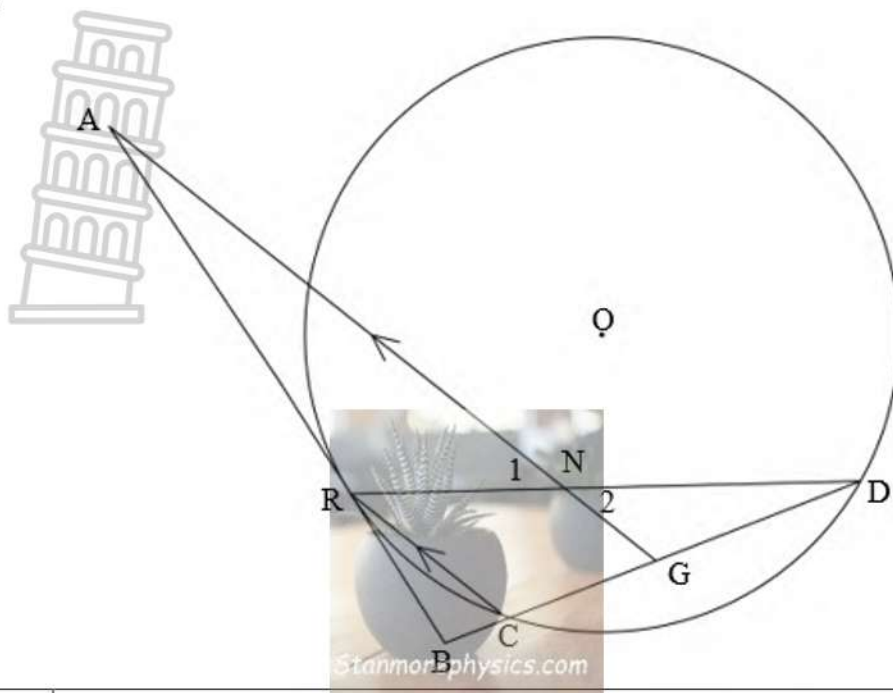
✓ S/R

✓ S ✓ R

(6)



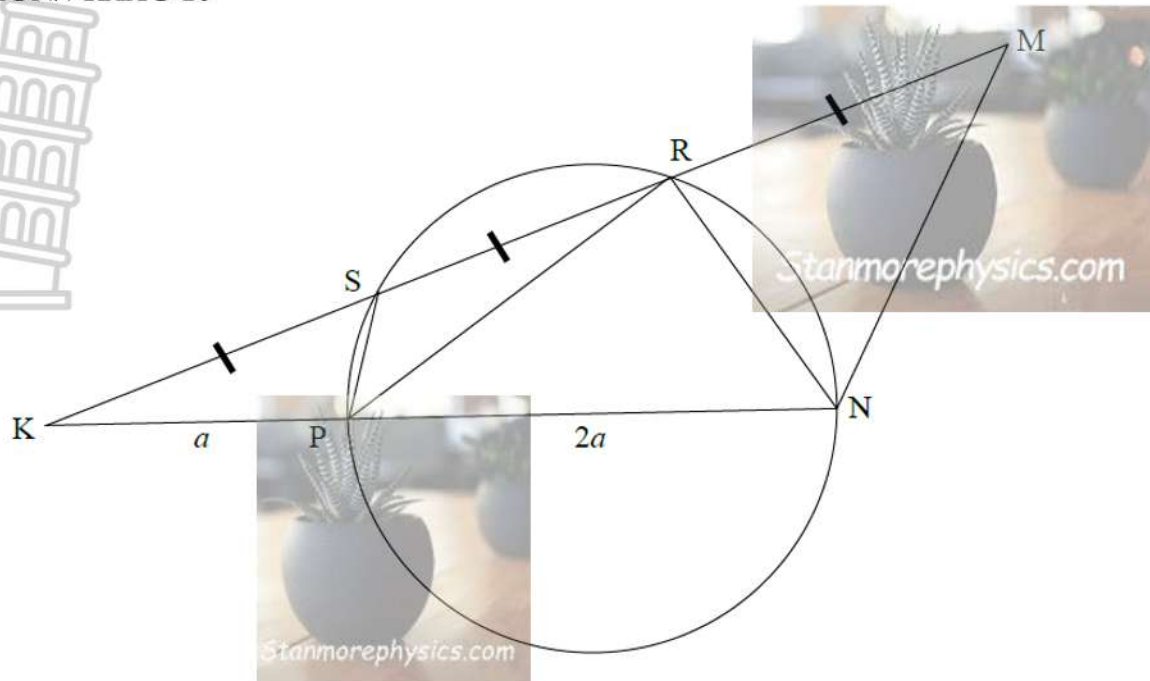
9.2



9.2.1	In $\Delta DNG \parallel \Delta ANR$: $\hat{N}_1 = \hat{N}_2$ (vert opp $\angle s / regootst \angle e$) $\hat{A}R\hat{N} = R\hat{C}\hat{D}$ (tan chord th/rkl koord st) $R\hat{C}\hat{D} = N\hat{G}\hat{D}$ (corresp $\angle s / ooreenk \angle e$; $RC \parallel AG$) $\therefore \Delta DNG \parallel \Delta ANR$ (AAA) OR/OF In $\Delta DNG \parallel \Delta ANR$: $\hat{N}_1 = \hat{N}_2$ (vert opp $\angle s / regootst \angle e$) $\hat{A}R\hat{N} = R\hat{C}\hat{D}$ (tan chord th/rkl koord st) $R\hat{C}\hat{D} = N\hat{G}\hat{D}$ (corresp $\angle s / ooreenk \angle e$; $RC \parallel AG$) $\hat{A} = \hat{D}$ ($\angle s$ of $\Delta / \angle e$ v Δ) $\therefore \Delta DNG \parallel \Delta ANR$	✓ S/R ✓ S ✓ R ✓ S/R ✓ R (5) ✓ S/R ✓ S ✓ R ✓ S/R ✓ R (5)
9.2.2	$B\hat{R}C = \hat{D}$ (tan chord th/rkl koord st) $\hat{A} = \hat{D}$ ($\parallel \Delta s / e$) $\therefore B\hat{R}C = \hat{A}$	✓ S/R ✓ S (2)

[13]

QUESTION/VRAAG 10



10.1.1	$\frac{KP}{PN} = \frac{KS}{SM} = \frac{1}{2}$ PS NM (side in proportion/sye in dieselfde verh) OR/OF $\frac{KP}{KN} = \frac{KS}{KM} = \frac{1}{3}$ ∴ PS NM (side in proportion/sye in dieselfde verh)	✓ S ✓ R	(2)
10.1.2	$\hat{KSP} = \hat{M}$ (Corr $\angle^s$ / ooreenk $\angle^e$; PS NM) $\hat{KSP} = \hat{PNR}$ (ext $\angle$ of cyc quad/buite $\angle$ v koordevh) $\hat{PNR} = \hat{M}$ ∴ PN = tangent/raaklyn (conv tan chord th/omg rkl koordst)	✓ S/R ✓ S ✓ R ✓ R	(4)
10.1.3	In $\triangle KRN$ and $\triangle KMN$: $\hat{KNR} = \hat{M}$ (proven/bewys) $\hat{K} = \hat{K}$ (common/gemeen) ∴ $\triangle KRN \parallel \triangle KMN$ (AAA) $\frac{NM}{RN} = \frac{KN}{KR} = \frac{KM}{KN}$ $\frac{NM}{RN} = \frac{3a}{12} = \frac{a}{4}$	✓ $2 \angle s = + R$ OR $3 \angle s =$ ✓ correct $\Delta s \parallel$ ✓ proportion ✓ $\frac{3a}{12}$	(4)

10.2	$\frac{PN}{NM} = \frac{NR}{MR} \quad (\text{Similar } \Delta\text{'s})$ $\frac{2a}{NM} = \frac{NR}{6}$ $\text{but } NR = \frac{4NM}{a}$ $\frac{2a}{NM} = \frac{4NM}{6a}$ $4NM^2 = 12a^2$ $NM = \sqrt{3}a$	✓ ratio ito NM and NR ✓ subst NR ✓ answer/antwoord (3)
		[16]
		TOTAL: 150



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