



education

**MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA**

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

SEPTEMBER 2025

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages and 1 information sheet, and an answer book is provided.



INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

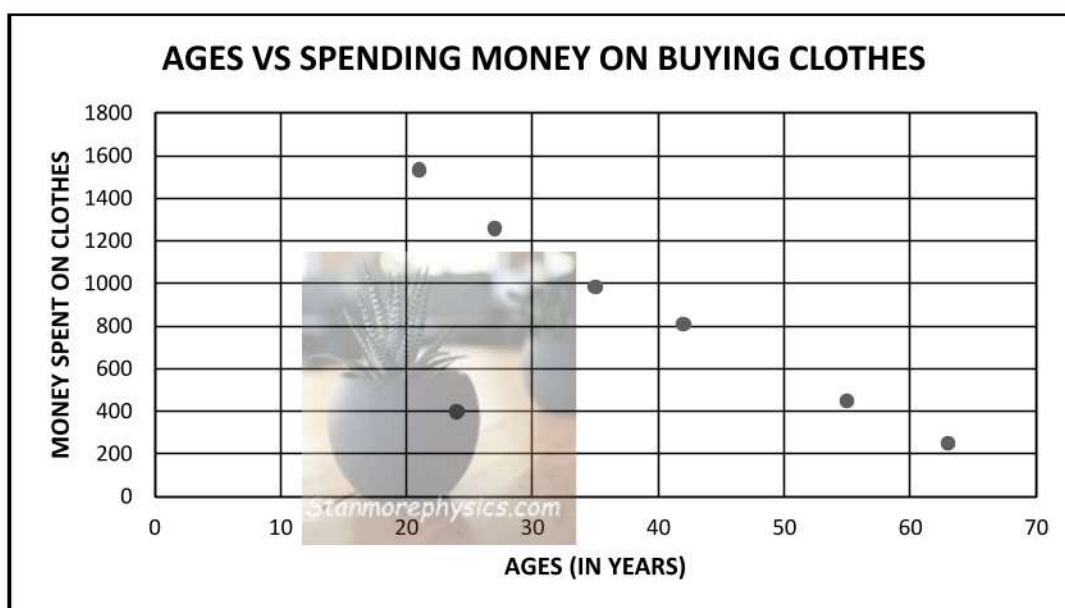
1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.



QUESTION 1

The relationship between age and spending money on buying clothes per month has been studied for years. Research has shown the following results:

Ages (in years)	21	24	27	35	42	55	63
Spending money on buying clothes (in rands)	1534	400	1260	986	810	450	250



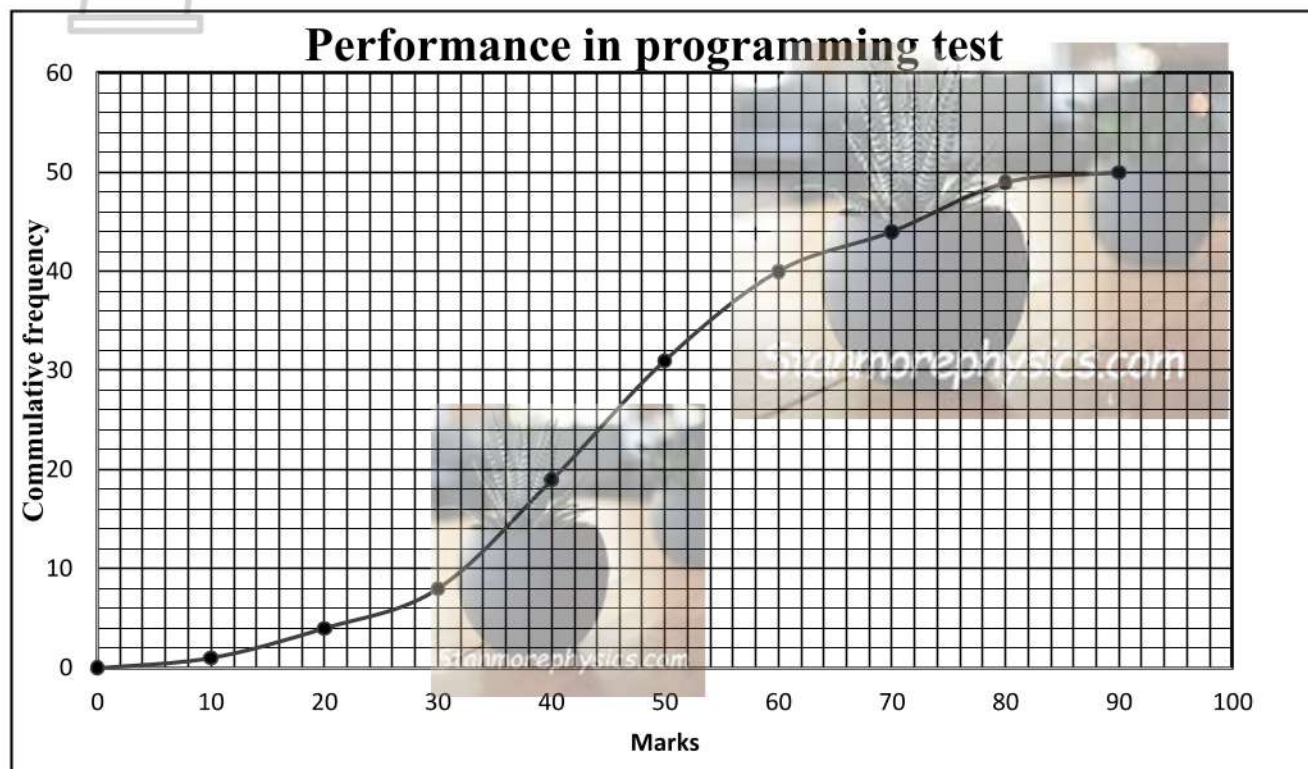
- 1.1 Determine the equation of the least squares regression line. (3)
- 1.2 Draw the least squares regression line on the answer book. (2)
- 1.3 Determine the correlation coefficient. (1)
- 1.4 Predict the money spend by a 50-year-old person on buying clothes. (2)
- 1.5 Is the answer calculated in QUESTION 1.4 reliable? Justify your answer. (2)
- 1.6 Explain the effect on the gradient of the regression line if the point (24; 400) is removed. (1)

[11]



QUESTION 2

- 2.1 The ogive (cumulative frequency graph) shows the performance of the students who took a test in basics programming skills. The test had a maximum of 100 marks.



- 2.1.1 How many students took the test? (1)
- 2.1.2 Only the top 25% of the students are allowed to do an advanced course in programming. Determine the cut-off mark for the top 25%. (2)
- 2.1.3 Complete a frequency table on the ANSWER BOOK provided. (2)
- 2.1.4 Calculate the mean estimate of the data. (3)



NSC

- 2.2 The average time to deliver a pizza ordered telephonically is 25 minutes. The standard deviation of the delivery time allowed, is 6 minutes. The delivery times in a day is summarised below.



Delivery times (minutes)
32
28
20
38
15
24

If the policy of the company is that you will receive the pizza free of charge when the delivery time is more than one standard deviation from the mean, determine the number of free pizzas for the day. Show all supporting calculations.

(3)

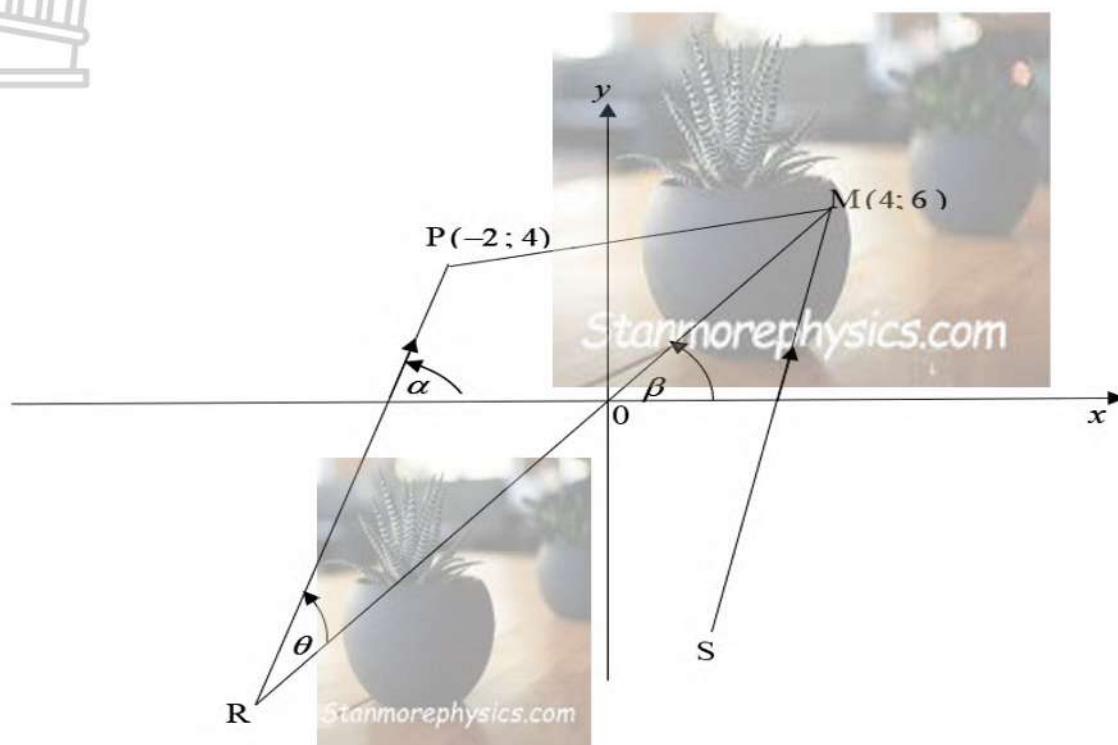


[11]



QUESTION 3

In the diagram below, $\triangle PMR$ is drawn with vertices $P(-2; 4)$, $M(4; 6)$ and R in the Cartesian plane. Line MR passes through the origin at O . $\widehat{PRM} = \theta$ and $PR \parallel MS$. The equation of MS is given by $y - 5x + 14 = 0$.



- 3.1 Calculate the length of MR , in simplified surd form. (2)
- 3.2 Determine the equation of MR . (3)
- 3.3 Determine the equation of PR . (3)
- 3.4 Determine the coordinates of R . (3)
- 3.5 Calculate the size of θ , rounded off to TWO decimal places. (4)
- 3.6 Calculate the area of $PMSR$ if it is further given that $PMSR$ is a parallelogram. (4)

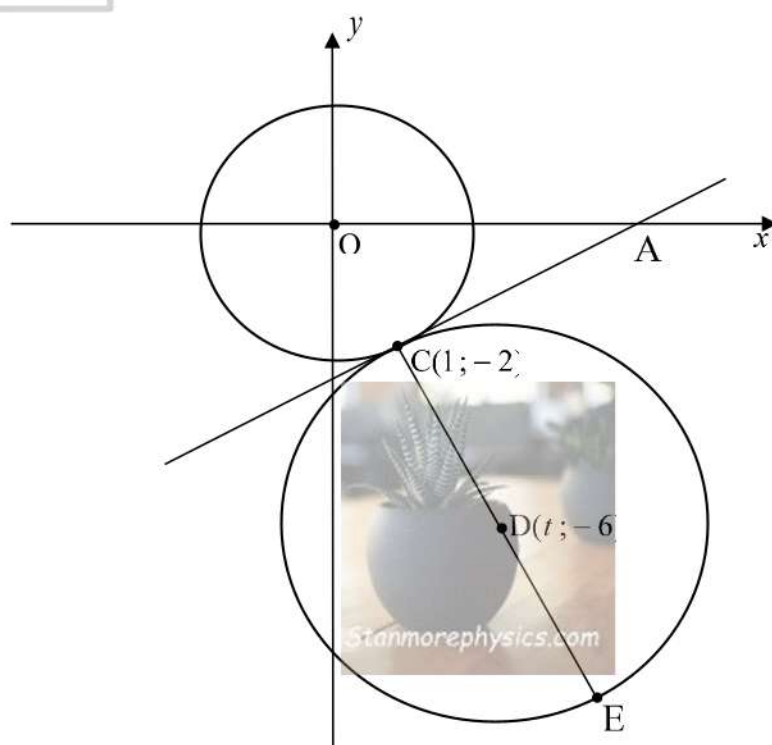
[19]



QUESTION 4

The diagram below consists of two circles, which touch each other externally at $C(1; -2)$.

The smaller circle has its centre O at the origin. The other circle has centre $D(t; -6)$. CA is a common tangent which intersects the x -axis at A . CDE is the diameter of the larger circle.



- 4.1 Give a reason why the points O , C and D lie on a straight line. . (2)
- 4.2 Calculate the gradient of OC (2)
- 4.3 Hence, show that the value of $t = 3$. (2)
- 4.4 Determine the equation of the tangent AC in the form $y = mx + c$. (3)
- 4.5 Calculate the coordinates of E . (2)
- 4.6 Determine the equation of a circle passing through the points $A(5; 0)$, C and E in the form $(x - a)^2 + (y - b)^2 = r^2$. (5)
- 4.7 If a circle with centre D and equation $(x - 3)^2 + (y + 6)^2 = r^2$ has to cut the circle with centre O twice, give all possible values of r . (4)

[20]



QUESTION 5

- 5.1 If $\tan A = \frac{5}{12}$ and $A \in (180^\circ; 360^\circ)$, determine without using a calculator and with an aid of a diagram the value of:



5.5.1 $\cos A$

(3)

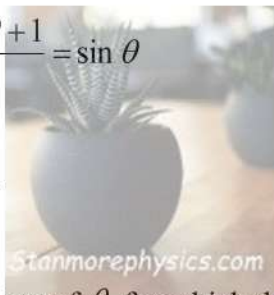
5.5.2 $\sin 2A$

(3)

- 5.2 Determine the value of: $\frac{1}{\cos(360^\circ - \theta) \cdot \sin(90^\circ - \theta)} - \tan^2(180^\circ + \theta)$

(6)

- 5.3 Given: $\frac{\sin(-2\theta) - \cos^2\theta + 1}{\sin\theta - 2\cos\theta} = \sin\theta$



- 5.3.1 Prove the identity

(5)

- 5.3.2 Determine the values of θ for which the identity will be undefined if $\theta \in [-180^\circ; 180^\circ]$

(4)

- 5.4 Determine the general solution of the following equation:

$$\sin(2x + 40^\circ)\cos(x + 30^\circ) - \cos(2x + 40^\circ)\sin(x + 30^\circ) = \cos(2x - 20^\circ)$$

(6)

- 5.5 Given: $f(x) = 2\cos^2 x - 2$

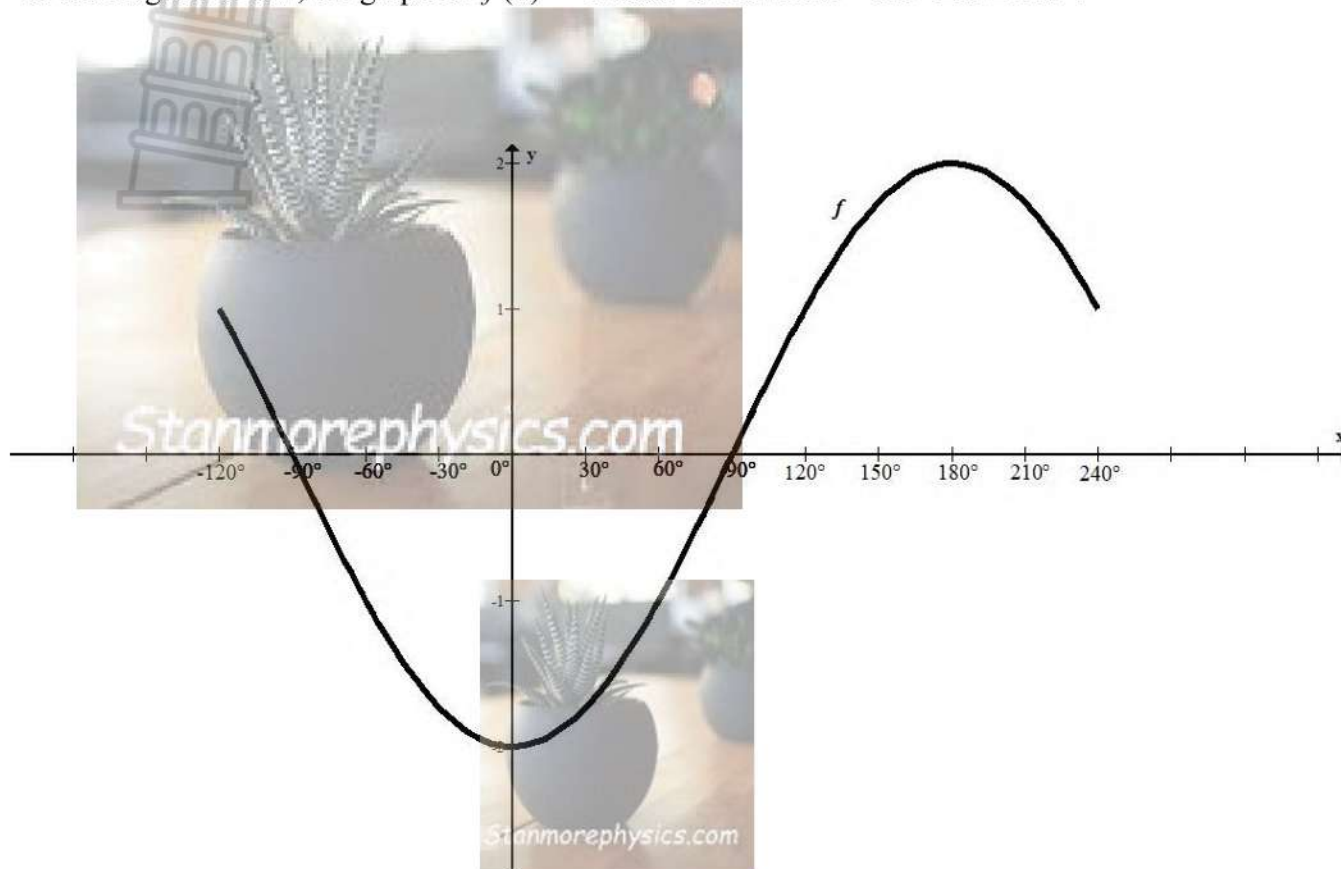
Determine the maximum value of $f(x)$

(3)

[30]

QUESTION 6

In the diagram below, the graph of $f(x) = -2 \cos x$ is drawn for $-120^\circ \leq x \leq 240^\circ$.

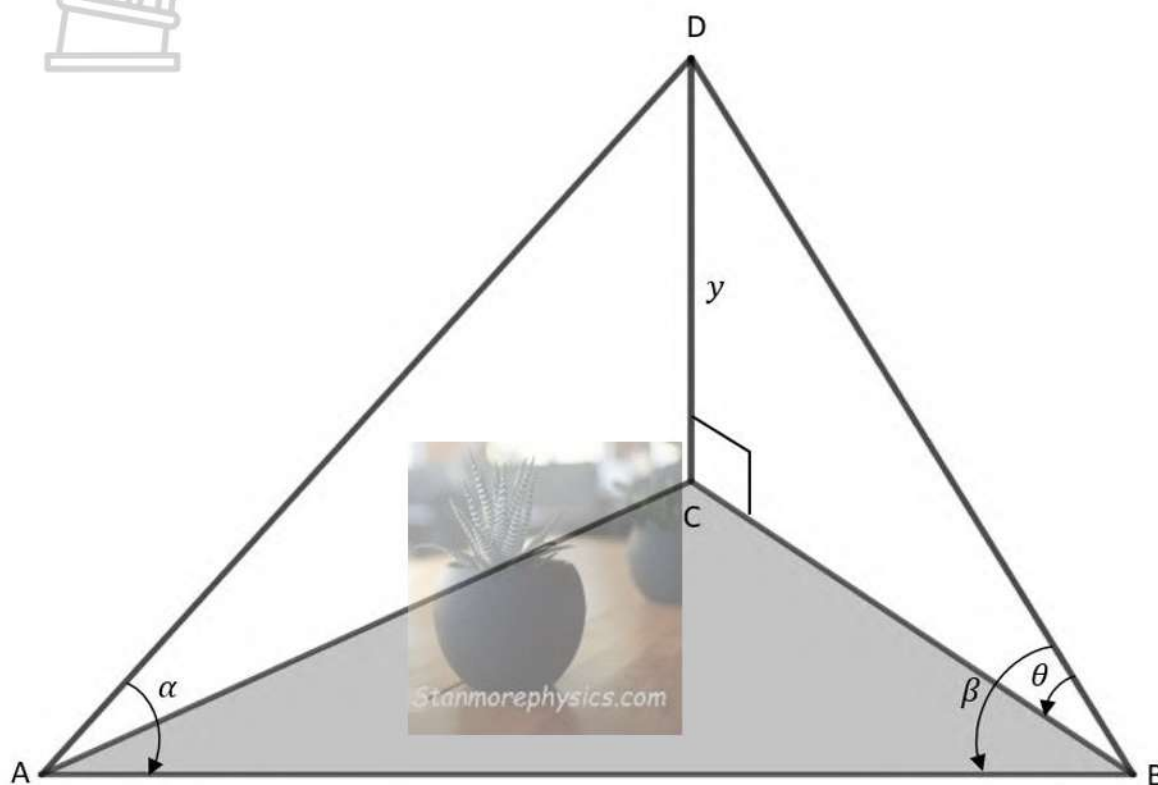


- 6.1 Write down the amplitude of f . (1)
- 6.2 Write down the range of $f(x) + 3$. (2)
- 6.3 Draw the graph of $g(x) = \sin(x + 60^\circ)$ for $-120^\circ \leq x \leq 240^\circ$ on the grid provided in the ANSWER BOOK. (3)
- 6.4 For which values of k will $f(x) = k$ have no real roots? (2)
- 6.5 Determine the values of x in the interval $-120^\circ \leq x \leq 240^\circ$ for which $x \cdot g'(x) < 0$. (3)

[11]

QUESTION 7

In the diagram alongside, D is a point vertically above C. DC is y metres in length. The angle of elevation of D from B is θ . $\hat{DAB} = \alpha$ and $\hat{DBA} = \beta$.



7.1 Determine the length of DB in terms of y and θ . (3)

7.2 Show that $AB = \frac{y \sin (\alpha + \beta)}{\sin \theta \sin \alpha}$ (4)

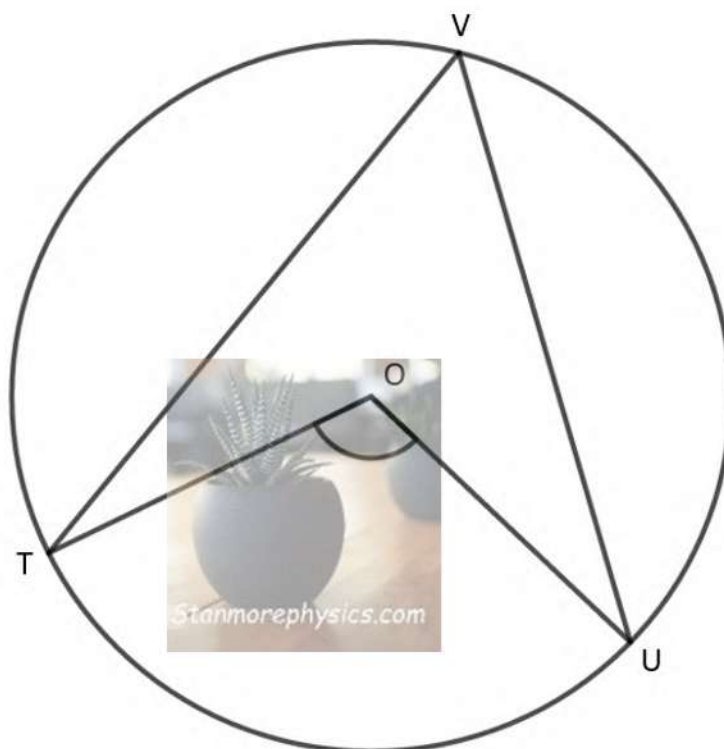
[7]



Give reasons for your statements and calculations in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, O is the centre of circle TUV.

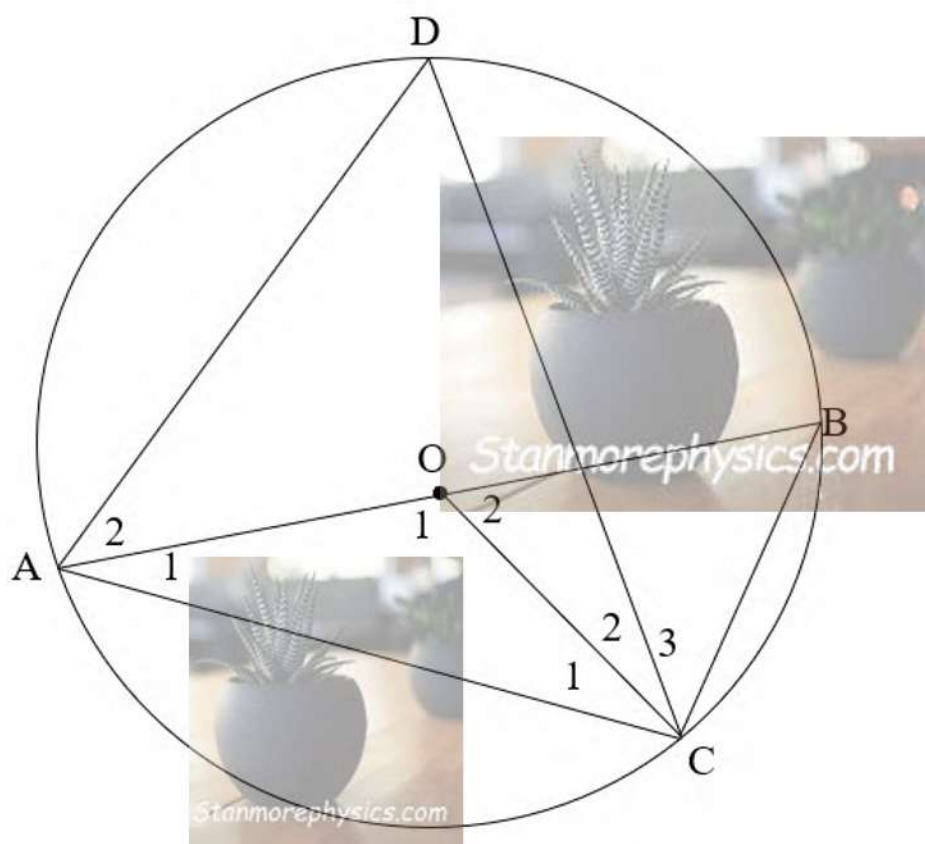


Use the diagram on the ANSWER BOOK to prove the theorem which states that $\angle TOU = 2\angle TVU$ (6)



NSC

8.2 O is the centre of the circle. AB is a diameter. $\hat{AOC} = 104^\circ$ and $\hat{DAB} = 32^\circ$.



Calculate the size of:

8.2.1 $\hat{D}$ (2)

8.2.2 $\hat{C}_3$ (1)

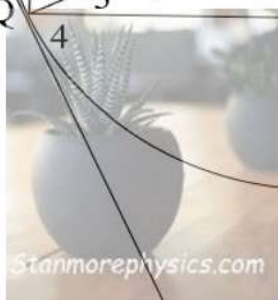
8.2.3 $\hat{A}_1$ (3)

8.2.4 $\hat{C}_2$ (3)



NSC

In the figure, a circle with center O is tangent to the line AB at point A . The line AB is produced at point B to point C such that $BC = OA$. The line OC is drawn. Find $\angle C$.



- 8.3.1 Prove that RBOQ is a cyclic quadrilateral. (4)

- 8.3.2 Prove that RB is a tangent to the circle passing through T, O and B. (4)

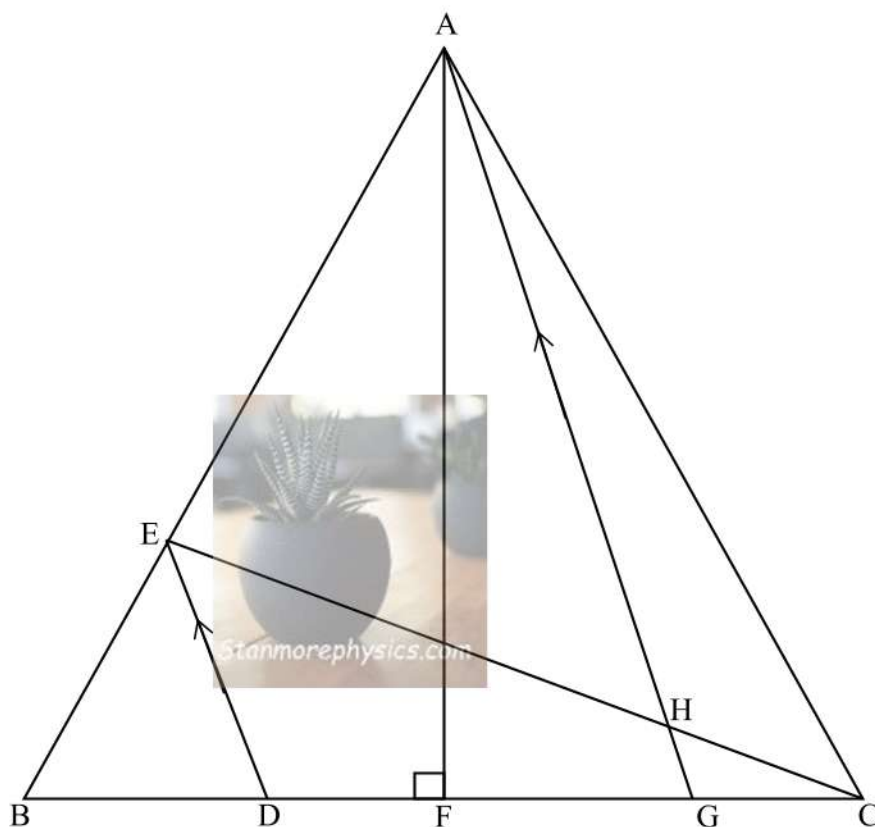
- 8.3.3 Express $\hat{P}$ in terms of $\hat{B}_1$. (3)

[26]



QUESTION 9

In the diagram below $\frac{DG}{GC} = \frac{3}{2}$; $\frac{BE}{EA} = \frac{1}{2}$ and $AG \parallel DE$.



Determine, with reasons, the value of:

9.1 $\frac{EH}{HC}$. (2)

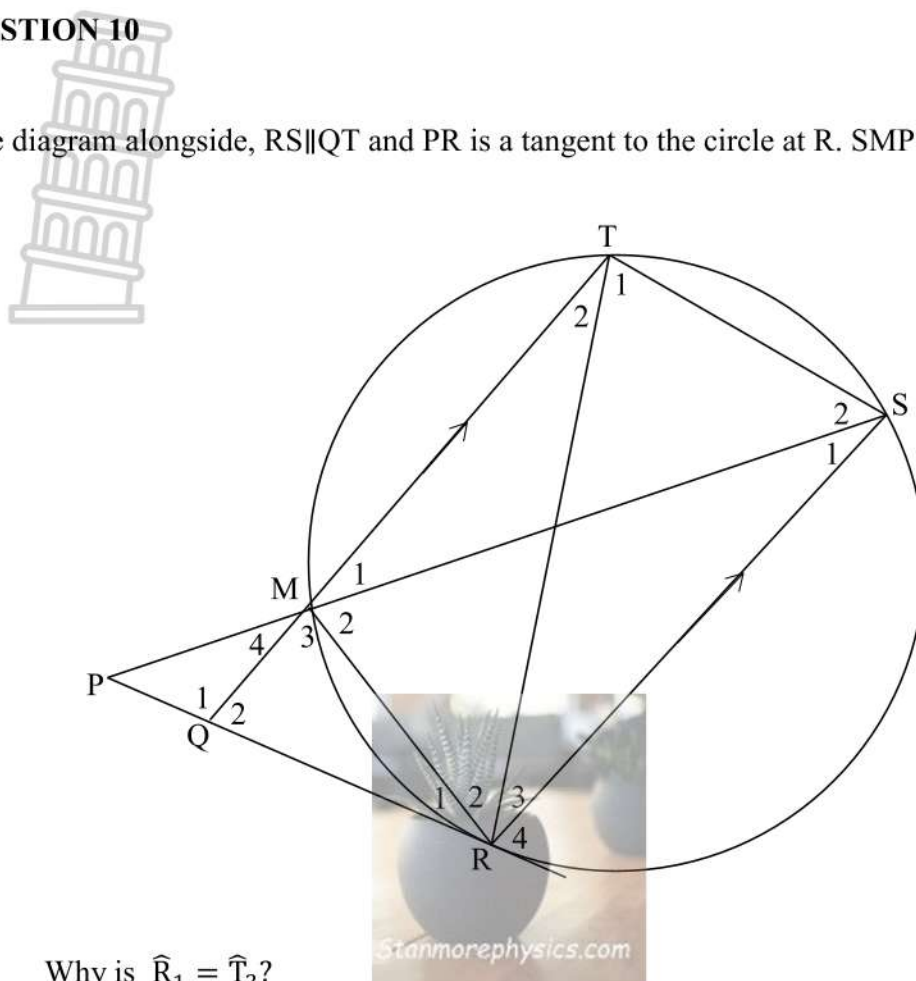
9.2 $\frac{\text{Area } \triangle BAG}{\text{Area } \triangle CAG}$. (3)

[5]



QUESTION 10

In the diagram alongside, $RS \parallel QT$ and PR is a tangent to the circle at R . SMP is a straight line.



10.1 Why is $\widehat{R}_1 = \widehat{T}_2$? (1)

10.2 Prove that $\triangle RTS \parallel \triangle RQM$ (4)

10.3 Prove that $RQ \times TS = QM \times RT$ (1)

10.4 Calculate the numerical value of $\frac{PQ}{RT}$ if it is given that $\frac{PM}{ST} = \frac{2}{3}$ and $\frac{QM}{SM} = \frac{1}{2}$. (4)

[10]

TOTAL: 150

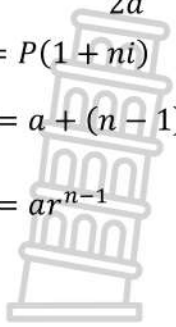


$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \quad A = P(1 - ni)$$

$$T_n = a + (n - 1)d$$

$$T_n = ar^{n-1}$$



$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c \quad y - y_1 = m(x - x_1)$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$



$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= 1 - 2\sin^2 \alpha$$

$$= 2\cos^2 \alpha - 1$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\hat{y} = a + bx$$

$$A = P(1 - i)^n \quad A = P(1 + i)^n$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

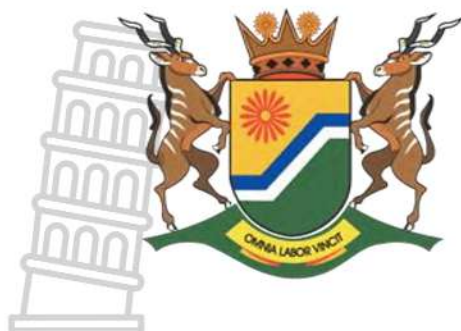
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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SERTIFIKAAT**



GRADE 12 / GRAAD 12

MATHEMATICS PAPER 2/ WISKUNDE VRAESTEL 2

SEPTEMBER 2025

MARKING GUIDELINE / NASIENRIGLYN

MARKS / PUNTE: 150

These marking guidelines consist of 17 pages.

Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

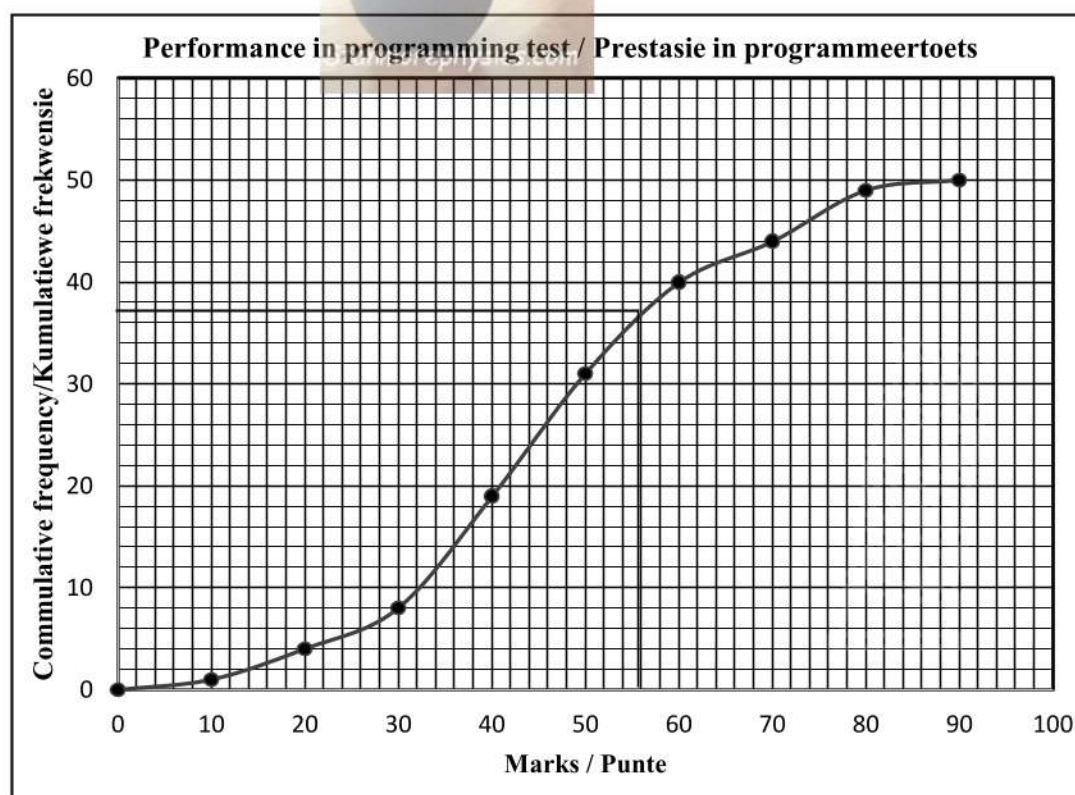
LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyn van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

QUESTION 1		
1.1	$A = 1\,607,113187$ $B = -20,82319216$ $\hat{y} = 1607,11 - 20,82x$	✓ 1578,539116 ✓ -22,32125028 ✓ equation / vergelyking (3)
1.2	AGES VS SPENDING MONEY ON BUYING CLOTHES OUDERDOM TEENoor GELD SPANDEER OP KLERE MONEY SPENT ON CLOTHES / GELD SPANDEER OP KLERE AGES (IN YEARS) / OUDERDOM (IN JARE) Mean point: (38,14; 812,86)	✓ shape/vorm ✓ passes through the mean point (38,14; 812,86) / deur die gemiddelde punt (2)
1.3	$r = -0,68 \quad -0,69995 \dots = -0,70$	✓ answer antwoord (1)
1.4	$\hat{y} = 1578,54 - 22,32(50) \quad 1607,11 - 20,82(50)$ $= 462,54 \quad 566,11$ OR / OF	✓ substitution / substitusie ✓ answer antwoord (2)

	$\hat{y} = 462,48$ 565,95 (from calculator)(sakrekenaar)	OR / OF ✓ ✓ Answer
1.5	It is NOT reliable since there is NO STRONG correlation between the variables / It is FAIRLY reliable since there is an AVERAGE correlation between the variables. Dit is NIE betroubaar nie want daar is nie 'n STERK korrelasie tussen die veranderlikes nie / Dit is REDELIK betroubaar, want daar is 'n GEMIDDELDE korrelasie.	✓ Not reliable Nie betroubaar ✓ Not strong / nie sterk OR / OF ✓ Fairly reliable / redelik betroubaar ✓ average correlation / gemiddelde korrelasie (2)
1.6	The gradient will DECREASE (become more negative) when the point is removed. Die gradiënt sal VERMINDER (meer negatief word) as die punt verwyder word.	✓ increase (more negative) / vermeerder (meer negatief) (1)
		[11]

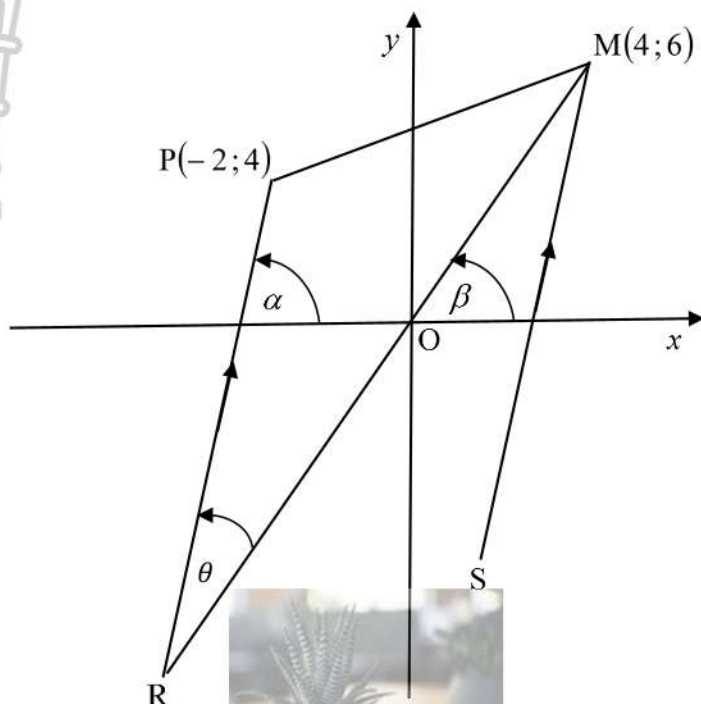
QUESTION 2 / VRAAG 2



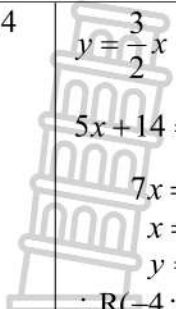
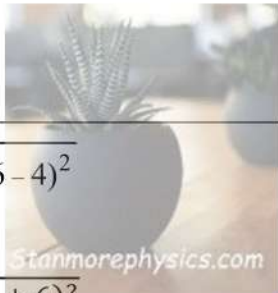
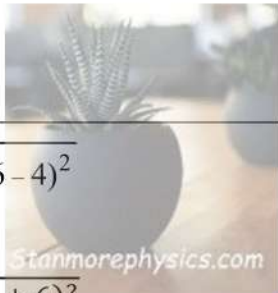
2.1.1	50 students/studente	✓ answer / antwoord (1)
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2.1.2	$\frac{75}{100} \times 50 = 37,5$ 65 marks/punte or/of 65%	✓ 37,5 ✓ 65 (2)																														
2.1.3	<table border="1"> <thead> <tr> <th>Marks(out of 100) Punte (uit 100)</th><th>Frequency(f) Frekwensie(f)</th><th>Cumulative frequency Kumulatiewe frekwensie</th></tr> </thead> <tbody> <tr> <td>$0 \leq x < 10$</td><td>1</td><td>1</td></tr> <tr> <td>$10 \leq x < 20$</td><td>3</td><td>4</td></tr> <tr> <td>$20 \leq x < 30$</td><td>4</td><td>8</td></tr> <tr> <td>$30 \leq x < 40$</td><td>11</td><td>19</td></tr> <tr> <td>$40 \leq x < 50$</td><td>12</td><td>31</td></tr> <tr> <td>$50 \leq x < 60$</td><td>9</td><td>40</td></tr> <tr> <td>$60 \leq x < 70$</td><td>4</td><td>44</td></tr> <tr> <td>$70 \leq x < 80$</td><td>5</td><td>49</td></tr> <tr> <td>$80 \leq x < 90$</td><td>1</td><td>50</td></tr> </tbody> </table>	Marks(out of 100) Punte (uit 100)	Frequency(f) Frekwensie(f)	Cumulative frequency Kumulatiewe frekwensie	$0 \leq x < 10$	1	1	$10 \leq x < 20$	3	4	$20 \leq x < 30$	4	8	$30 \leq x < 40$	11	19	$40 \leq x < 50$	12	31	$50 \leq x < 60$	9	40	$60 \leq x < 70$	4	44	$70 \leq x < 80$	5	49	$80 \leq x < 90$	1	50	✓ 1st 3 values / 1e 3 waardes ✓ all values / al die waardes (2)
Marks(out of 100) Punte (uit 100)	Frequency(f) Frekwensie(f)	Cumulative frequency Kumulatiewe frekwensie																														
$0 \leq x < 10$	1	1																														
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$60 \leq x < 70$	4	44																														
$70 \leq x < 80$	5	49																														
$80 \leq x < 90$	1	50																														
2.1.4	$\bar{x} = \frac{(1 \times 5) + (3 \times 15) + (4 \times 25) + (11 \times 35) + \dots + (1 \times 85)}{50}$ $= \frac{2290}{50} = 45,8$ OR / OF 45,8 (from calculator)(van sakrekenaar)	✓ mid pts / middelwaardes ✓ divide by 50 / deel deur 50 ✓ answer/ antwoord OR / OF ✓✓✓ answer / antwoord (3)																														
2.2	$\bar{x} = 25 \text{ min}$ $\sigma = 6 \text{ min}$ $= 25 + 6 = 31$ $\therefore 2 \text{ vouchers will be free} / 2 \text{ bewyse sal gratis wees}$	✓ 25 ✓ 6 ✓ 2 (3)																														
		[11]																														

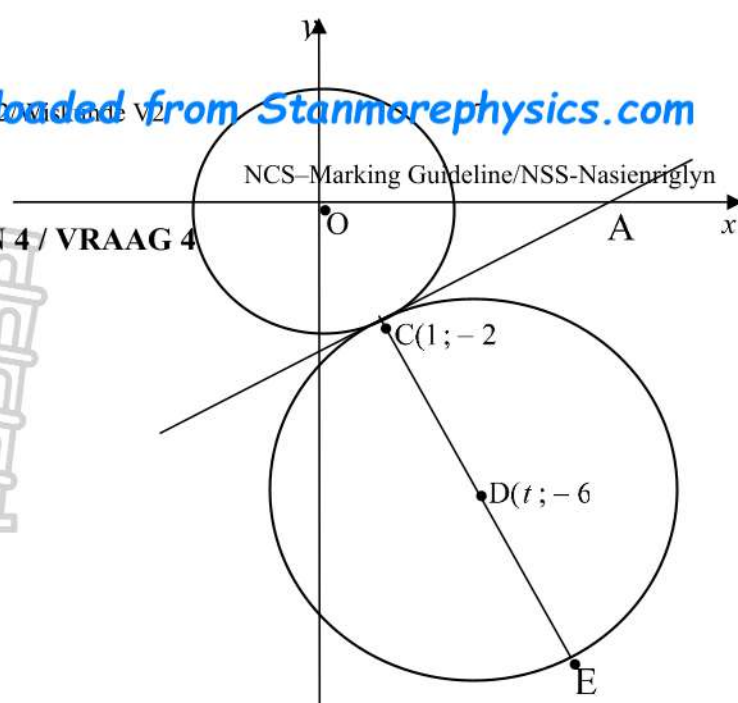
QUESTION 3 / VRAAG 3



3.1	$m_{MP} = \frac{0-6}{0-4}$ $= \frac{3}{2}$	✓ Substitution into the correct formula vervang in korrekte formule $\checkmark \frac{3}{2}$ (2)
3.2	$m_{MR} = \frac{0-6}{0-4}$ $= \frac{3}{2}$ Equation of MR is $y = \frac{3}{2}x$ (Vergelyking van MR)	✓ substitution vervang $\checkmark \frac{3}{2}$ ✓ equation / vergelyking (3)
3.2	$y = 5x - 14$ $m_{MS} = m_{PR} = 5$ PR MS Equation of PR $y - y_1 = m(x - x_1)$ $y - 4 = 5(x + 2)$ $\therefore y = 5x + 14$ OR / OF $4 = 5(-2) + c$ $4 + 10 = c$ $c = 14$	$\checkmark m_{PR} = 5$ ✓ sub of/vervang (-2; 4) ✓ answer/antwoord OR / OF $\checkmark m_{PR} = 5$ ✓ sub of /vervang (-2; 4) ✓ answer/antwoord (3)

3.4	 $y = \frac{3}{2}x \text{ and/en } y = 5x + 14$ $5x + 14 = \frac{3}{2}x$ $7x = -28$ $x = -4$ $y = -6$ $\therefore R(-4; -6)$	✓ equating /vergelykings gelyk ✓ $x = -4$ ✓ $y = -6$ (3)
3.5	$m_{PR} = 5$ $\tan \alpha = 5$ $\alpha = 78,69^\circ$ $m_{MR} = \frac{3}{2}$ $\tan \beta = \left(\frac{3}{2}\right)$ $\beta = 56,31^\circ$ $\therefore \theta = \alpha - \beta$ $= 22,38^\circ$ 	✓ $\tan \alpha = 5$ ✓ $78,69^\circ$ ✓ $56,31^\circ$ ✓ $22,38^\circ$ (4)
3.6	 $PR = \sqrt{(-4 + 2)^2 + (-6 - 4)^2}$ $= \sqrt{104} \text{ or } 2\sqrt{26}$ $MR = \sqrt{(4 + 4)^2 + (6 + 6)^2}$ $= \sqrt{208} \text{ or/of } 4\sqrt{13}$ Area of $\triangle PMR$ / Oppervlakte van $\triangle PMR$ $= \frac{1}{2} PR.MR.\sin \theta$ $= \frac{1}{2} 2\sqrt{26}.4\sqrt{13}.\sin 22,38^\circ \text{ or } \frac{1}{2} \sqrt{104} .4\sqrt{13}.\sin 22,38^\circ$ $= 28$ Area PMSR = 56 units²/eenhede²	✓ PR ✓ MR ✓ Sub into the area rule/vervang in opp reël ✓ Answer /antwoord (4)
		[19]

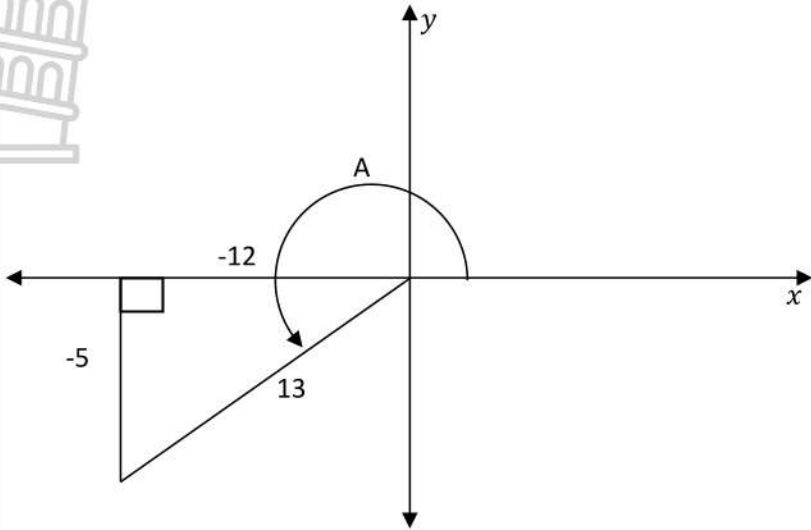
QUESTION 4 / VRAAG 4



4.1	$OC \perp \text{tangent}$ [radius $\perp$ tangent] [radius $\perp$ raaklyn] $CE \perp \text{tangent}$ [radius $\perp$ tangent] [radius $\perp$ raaklyn] $\therefore O, C$ and E lies on a straight line [adjacent $\angle s = 180^\circ$] $\therefore O, C$ en E lê op 'n reguit lyn. [Aangrensende $\angle e = 180^\circ$]	$\checkmark$ both S / beide bewerings $\checkmark$ both reasons/ beide redes (2)
4.2	$m_{OC} = \frac{-2-0}{1-0} = -2$	$\checkmark$ subst into formula vervang in formule $\checkmark$ answer/antw (2)
4.3	$m_{CD} = -2$ $\frac{y_2 - y_1}{x_2 - x_1} = -2$ $\frac{-6+2}{t-1} = -2$ $t = 3$	$\checkmark m_{CD}$ $\checkmark$ subst / vervanging (2)
4.4	$m_{\text{tangent}} = \frac{1}{2}$ $AC: y - y_1 = m(x - x_1)$ $y + 2 = \frac{1}{2}(x - 1)$ $y = \frac{1}{2}x - \frac{5}{2}$	$\checkmark m$ $\checkmark$ subst/vervang (1;-2) $\checkmark$ equation vergelyking (3)

4.5	D is the midpoint of circle / D is die middelpunt van die sirkel $\frac{1+x}{2} = 3$ $x = 5$ $\frac{y-2}{2} = -6$ $y = -10$ $E(5; -10)$	✓ $x = 5$ ✓ $y = -10$ OR/OF answer only / slegs antw (2)
4.6	$\hat{ACD} = 90^\circ$ [radius $\perp$ tangent] [radius $\perp$ raaklyn] $\therefore AE$ is the diameter of circle ACE [converse $\angle$ in semi-circle] AE is die middellyn van die sirkel [hoek in halwe sirkel omgekeerde] $AE = \sqrt{(5-5)^2 + (0+10)^2}$ $AE = \sqrt{0+100}$ $AE = 10$ $\therefore \text{radius} = 5$ $\text{midpt}_{AE} = \left(\frac{5+5}{2}; \frac{-10}{2} \right)$ $= (5; -5) = \text{centre of circle / mdpt van sirkel}$ Vgl van sirkel/ Equation of circle ACE: $(x-5)^2 + (y+5)^2 = 25$	✓ S / R ✓ R ✓ Radius = 5 ✓ midpt/middelpunt ✓ LHS & RHS LK en RK (5)
4.7	Circle centre O: $x^2 + y^2 = 5$ Sirkel met middelpunt O Diameter = $2\sqrt{5}$ middellyn $\sqrt{20} < r < \sqrt{80}$ $2\sqrt{5} < r < 4\sqrt{5}$	✓ $r^2 = 5$ ✓ diameter middellyn ✓ $\sqrt{80} = 4\sqrt{5}$ ✓ endpoints eindpunte (4)
		[20]

QUESTION 5 / VRAAG 5

5.1		
5.1.1	$\cos A = -\frac{12}{13}$	✓ Diagram in quadrant 3 ✓ skets in kwadrant 3 ✓ $r = 13$ ✓ $-\frac{12}{13}$ (3)
5.1.2	$\sin 2A = 2 \sin A \cos A$ $= 2 \left(-\frac{5}{13}\right) \left(-\frac{12}{13}\right)$ $= \frac{120}{169}$	✓ $2 \sin A \cos A$ ✓ substitution / vervanging ✓ answer / antwoord (3)
5.2	$= \frac{1}{\cos \theta \cdot \cos \theta} - (-\tan \theta)^2$ $= \frac{1}{\cos^2 \theta} - \tan^2 \theta$ $= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$ $= \frac{1 - \sin^2 \theta}{\cos^2 \theta}$ $= \frac{\cos^2 \theta}{\cos^2 \theta}$ $= 1$	✓ $\cos \theta$. ✓ $\cos \theta$ ✓ $-\tan^2 \theta$ ✓ $\frac{\sin^2 \theta}{\cos^2 \theta}$ ✓ $\cos^2 \theta$ ✓ answer/antwoord (6)
		[12]

QUESTION 6 / VRAAG 6

6.1.1

$$\begin{aligned} LHS &= \frac{-\sin 2\theta - \cos^2 \theta + 1}{\sin \theta - 2\cos \theta} \\ &= \frac{-2\sin \theta \cdot \cos \theta - \cos^2 \theta + \sin^2 \theta + \cos^2 \theta}{\sin \theta - 2\cos \theta} \\ &= \frac{\sin^2 \theta - 2\sin \theta \cdot \cos \theta}{\sin \theta - 2\cos \theta} \\ &= \frac{\sin \theta (\sin \theta - 2\cos \theta)}{\sin \theta - 2\cos \theta} \\ &= \sin \theta \end{aligned}$$

$\therefore LHS = RHS$

OR/OF

$$\begin{aligned} LHS &= \frac{-\sin 2\theta - \cos^2 \theta + 1}{\sin \theta - 2\cos \theta} \\ &= \frac{-2\sin \theta \cdot \cos \theta + \sin^2 \theta}{\sin \theta - 2\cos \theta} \\ &= \frac{\sin^2 \theta - 2\sin \theta \cdot \cos \theta}{\sin \theta - 2\cos \theta} \\ &= \frac{\sin \theta (\sin \theta - 2\cos \theta)}{\sin \theta - 2\cos \theta} \\ &= \sin \theta \end{aligned}$$

$\therefore LHS = RHS$

✓ $-\sin 2\theta$
✓ $2\sin \theta \cdot \cos \theta$

✓ $\sin^2 \theta$
✓ common factor /
gemeensk faktor
✓ simplification
vereenvoudiging

OR /OF

✓ $-\sin 2\theta$
✓ $2\sin \theta \cdot \cos \theta$

✓ $\sin^2 \theta + \cos^2 \theta$
✓ common factor/gemeensk
faktor
✓ simplification /
vereenvoudig

(5)

6.1.2

$$\begin{aligned} \sin \theta - 2\cos \theta &= 0 \\ \frac{\sin \theta}{\cos \theta} &= \frac{2\cos \theta}{\cos \theta} \\ \tan \theta &= 2 \\ \theta &= 63,43^\circ \\ \theta &= \{-116,57^\circ; 63,43^\circ\} \end{aligned}$$

✓ $\sin \theta - 2\cos \theta = 0$

✓ $\tan \theta = 2$

✓ $-116,57^\circ$

✓ $63,43^\circ$

(4)

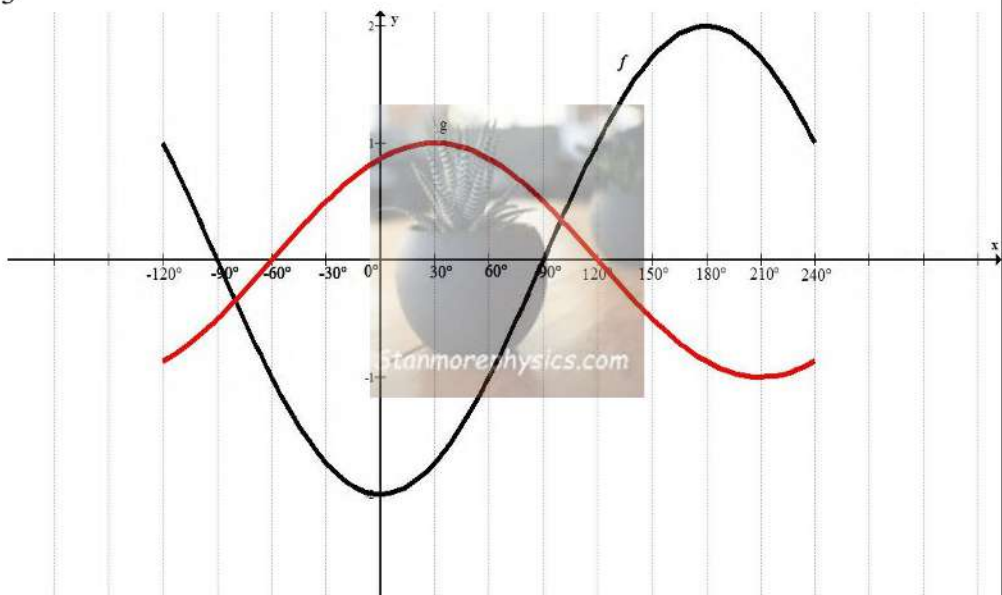
6.2

$$\begin{aligned} \sin[(2x + 40^\circ) - (x + 30^\circ)] &= \cos(2x - 20^\circ) \\ \sin(x + 10^\circ) &= \cos(2x - 20^\circ) \\ \cos[90^\circ - (x + 10^\circ)] &= \cos(2x - 20^\circ) \\ 90^\circ - (x + 10^\circ) &= \pm(2x - 20^\circ) + k360^\circ \quad k \in \mathbb{Z} \\ x = -60^\circ + k.360^\circ \quad k \in \mathbb{Z} \quad \text{or} \quad x &= 33,33^\circ - k.120^\circ \quad k \in \mathbb{Z} \end{aligned}$$

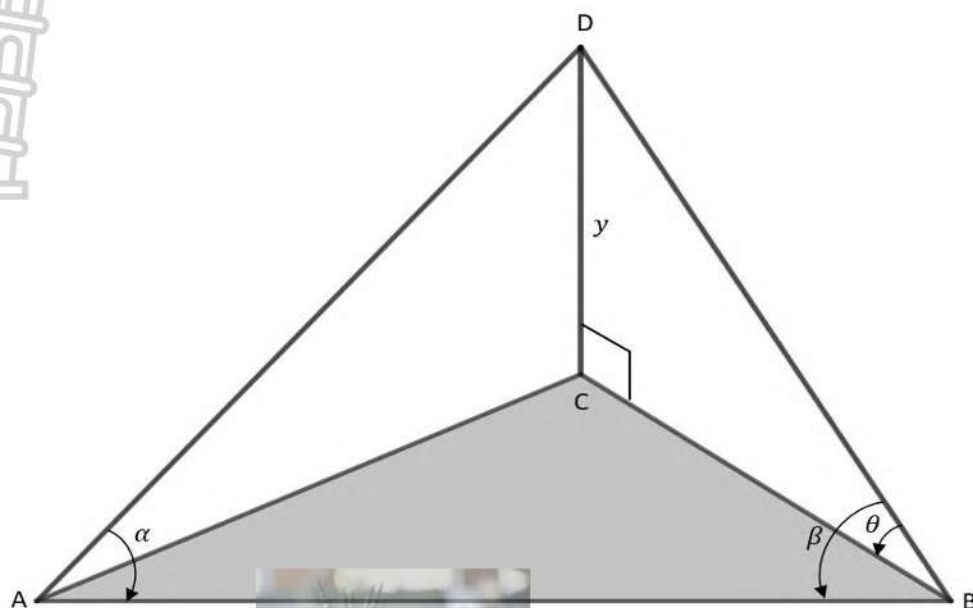
✓ Compound angle /
saamgestelde hoek
✓ $\sin(x + 10^\circ)$
✓ $\cos[90^\circ - (x + 10^\circ)]$
✓ equating $\angle$ s/ stel $\angle$ e gelyk
✓ $x = -60^\circ + k360^\circ$
✓ $x = 33,33^\circ + k120^\circ$ (6)

6.3	$f(x) = 2\cos^2 x - 2$ $= 2\cos^2 x - 1 - 1$ $= \cos 2x - 1$ Max value/Maks waarde = 0	✓ $2\cos^2 x - 1 - 1$ ✓ identity/identiteit ✓ answer/antwoord (3)
		[18]

QUESTION 7 / VRAAG 7

7.1	Amplitude = 2	✓ answer/antwoord (1)
7.2	$y \in [-2 + 3; 2 + 3]$ $y \in [1; 5]$	✓✓ $y \in [1; 5]$ (2)
7.3		✓ y – intercept y – afsnit ✓ x – intercepts x – afsnitte ✓ Min / Max points Min/maks punte (3)
7.4	$k < -2$ or/of $k > 2$	✓ $k < -2$ ✓ $k > 2$ (2)
7.5	$-120^\circ \leq x < 0^\circ$ $30^\circ < x < 210^\circ$	✓✓ $-120^\circ \leq x < 0^\circ$ ✓ $30^\circ < x < 210^\circ$ (3)
		[11]

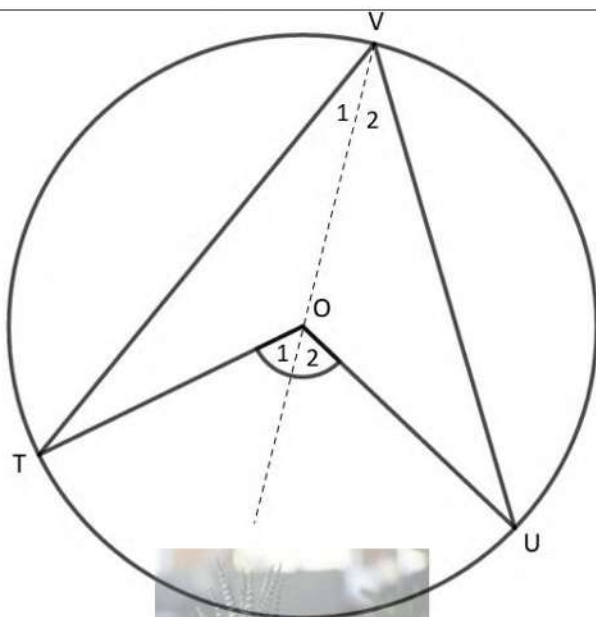
QUESTION 8 / VRAAG 8



8.1	In $\triangle CDB$ $\sin \hat{B} = \frac{DC}{DB}$ $\sin \theta = \frac{y}{DB}$ $DB = \frac{y}{\sin \theta}$	✓ trig ratio/verhouding ✓ subst / vervang ✓ answer / antwoord (3)
8.2	In $\triangle ADB$: $\hat{A}DB = 180^\circ - (\alpha + \beta) \quad [\text{sum of } \angle\text{s in } \triangle][\text{som binne } \angle\text{e van } \triangle]$ $\frac{\sin \hat{A}}{DB} = \frac{\sin \hat{D}}{AB}$ $\frac{\sin \alpha}{y} = \frac{\sin[180^\circ - (\alpha + \beta)]}{AB}$ $\sin \theta$ $AB \cdot \sin \alpha \cdot \sin \theta = y \cdot \sin(\alpha + \beta)$ $AB = \frac{y \sin(\alpha + \beta)}{\sin \alpha \cdot \sin \theta}$	✓ $180^\circ - (\alpha + \beta)$ ✓ subst into sine rule/ vervang in sin-reël ✓ simplify/vereenvoudig ✓ $\sin(\alpha + \beta)$ (4)
		[7]

QUESTION 9 / VRAAG 9

9.1



Construction/Konstruksie: Join/verbind VO and extend to Z / en verleng na Z

In $\triangle TOV$:

$$TO = OV \quad [\text{radii}]$$

$$\hat{T} = \hat{V}_1 \quad [\angle\text{'s opp = sides}] [\text{hoeke teenoor gelyke sye}]$$

$$\hat{O}_1 = \hat{T} + \hat{V}_1 \quad [\text{ext } \angle \text{ of } \triangle] [\text{buitehoek van } \triangle]$$

$$\hat{O}_1 = 2\hat{V}_1$$

Similarly in $\triangle OUV$ / soortgelyk in $\triangle OUV$:

$$\hat{O}_2 = 2\hat{V}_2$$

$$\hat{O}_1 + \hat{O}_2 = 2\hat{V}_1 + 2\hat{V}_2$$

$$\hat{TOU} = 2(\hat{V}_1 + \hat{V}_2)$$

$$\therefore \hat{TOU} = 2\hat{T\hat{V}U}$$

✓
construction/konstruksie

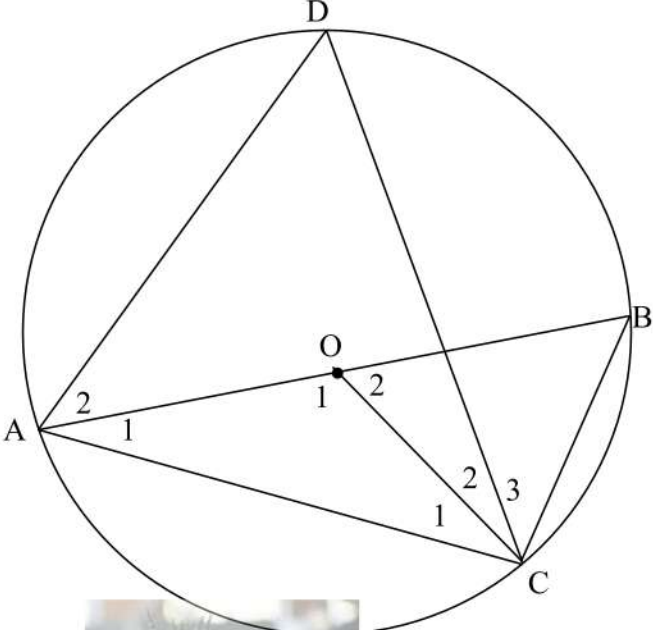
✓ S ✓ R

✓ S/R

✓ S

✓ S

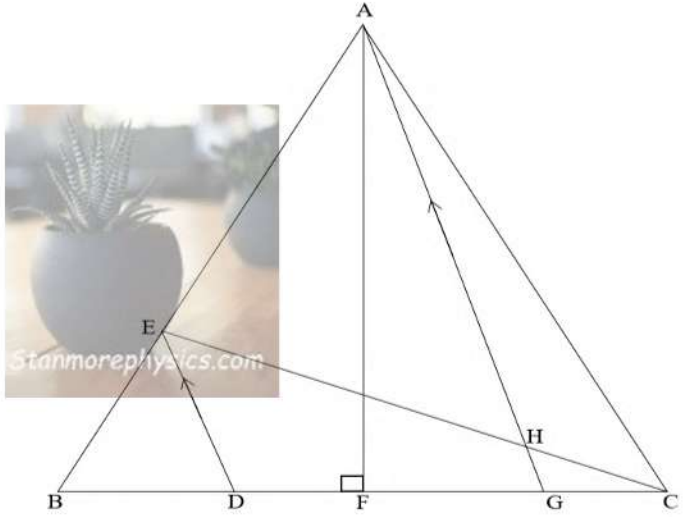
(6)

9.2		
9.2.1	$\hat{AOC} = 2\hat{D}$ $104^\circ = 2\hat{D}$ $\therefore \hat{D} = 52^\circ$	[$\angle$ at the centre = $2 \times \angle$ at circumference.] [mdpts $\angle = 2 \times$ omtreks $\angle$] ✓ S ✓ R (2)
9.2.2	$\hat{DAB} = \hat{C}_3 = 32^\circ$	[$\angle$s in the same segment] [$\angle$e in selfde sirkelsegment] ✓ S/R (1)
9.2.3	$\hat{A}_1 = \hat{C}_1$ $\hat{A}_1 + \hat{C}_1 + \hat{O} = 180^\circ$ $2\hat{A}_1 + 104^\circ = 180^\circ$ $\hat{A}_1 = 38^\circ$	[$\angle$'s opp = radii] [$\angle$e teenoor gelyke radiusse] [sum of $\angle$'s in Δ] [som binne $\angle$e van Δ] ✓ S/R ✓ S/R ✓ answer/antwoord (3)
9.2.4	$\hat{ACB} = 90^\circ$ $\hat{C}_3 = 32^\circ$ Maar/ But $\hat{ACB} = \hat{C}_1 + \hat{C}_2 + \hat{C}_3$ $90^\circ = 38^\circ + \hat{C}_2 + 32^\circ$ $\hat{C}_2 = 20^\circ$	[$\angle$'s in the semi-circle] [$\angle$e in halwe sirkel] [$\angle$'s in same circle segment] [$\angle$e in selfde sirkelsegment] ✓ S / R ✓ S/R ✓ answer (3)

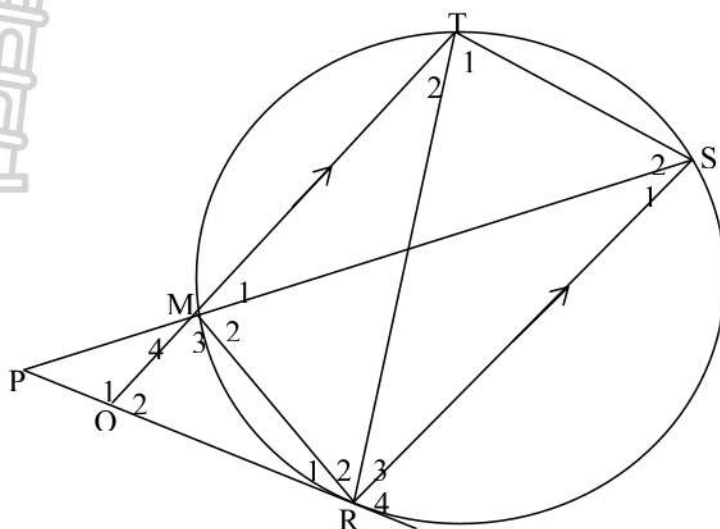
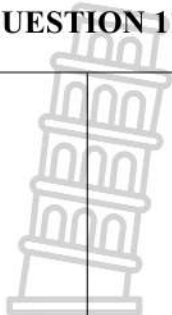
9.3		
9.3.1	$\hat{O}QR = 90^\circ$ [tan $\perp$ radius][radius $\perp$ raaklyn] $\hat{O}BR = 90^\circ$ [tan $\perp$ radius][radius $\perp$ raaklyn] $\therefore \hat{O}QR + \hat{O}BR = 180^\circ$ <i>RBOQ</i> is a cyclic quad [opp $\angle$s of quad supp] $\therefore$ <i>RBOQ</i> is 'n KVH [teenoorst $\angle$e van KVH omgekeerde] OR/OF $\hat{P}QO = 90^\circ$ [tan $\perp$ radius][radius $\perp$ raaklyn] $\hat{O}BR = 90^\circ$ [tan $\perp$ radius][radius $\perp$ raaklyn] $\therefore$ <i>RBOQ</i> is a cyclic quad [converse ext $\angle$ of a cyclic quad] $\therefore$ <i>RBOQ</i> is 'n KVH [buite $\angle$ van KVH omgekeerde]	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ R OR/OF $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ R (4)
9.3.2	$\hat{O}TB = 90^\circ$ [line from centre circle to midpt of chord] [lyn vanaf midpt sirkel na mdpt koord] $\therefore \hat{O}_3 = 90^\circ - \hat{B}_1$ [sum of $\angle$'s in Δ][som binne $\angle$e van Δ] maar/ but $\hat{B}_2 = 90^\circ - \hat{B}_1$ $\hat{O}_3 = \hat{B}_2$ [both/beide = $90^\circ - \hat{B}_1$] $\therefore$ <i>RB</i> is a tangent passing through T, O and B [converse tan chord theorem] $\therefore$ <i>RB</i> is 'n raaklyn deur T, O en B [raaklyn koord stelling omgekeerde]	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
9.3.3	$\hat{Q}_2 = 90^\circ - \hat{Q}_1$ [tan $\perp$ radius][radius $\perp$ raaklyn]	$\checkmark$ S/R $\checkmark$ S/R

maar/ but $\hat{Q}_1 = \hat{B}_1$ [tan-chord theorem][raaklyn koord stelling] en/ and $\hat{Q}_3 = \hat{B}_1$ [$\angle$'s opp = sides][$\angle$e teenoor gelyke sye] In ΔPQB $\hat{P} + \hat{B}_1 + \hat{Q}_1 + \hat{Q}_2 = 180^\circ$ [sum of $\angle$'s in Δ][som binne $\angle$e van Δ] $\hat{P} + \hat{B}_1 + \hat{B}_1 + 90^\circ - \hat{B}_1 + \hat{B}_1 = 180^\circ$ $\hat{P} = 90^\circ - 2\hat{B}_1$	✓ answer/antwoord (3)
	[26]

QUESTION 10 / VRAAG 10

		
10.1	$\frac{DG}{GC} = \frac{EH}{HC}$ [prop theorem , ED $\parallel$ HG] [eweredigheidstelling]	✓ S ✓ R (2)
10.2	$\frac{BG}{BD} = \frac{AB}{BE}$ [prop theorem, ED $\parallel$ AG] [eweredigheidstelling] $\frac{BG}{BD} = \frac{3}{1}$ $\frac{\text{Area of } \Delta ABG}{\text{Area of } \Delta CAG} = \frac{\frac{1}{2} \cdot BG \cdot h}{\frac{1}{2} \cdot GC \cdot h} = \frac{\frac{9}{2}}{\frac{2}{1}} = \frac{9}{4}$ OR /OF $\frac{3}{\frac{1}{4}} = \frac{12}{1}$	✓ S/ R ✓ S/ R ✓ answer/antwoord (3)
		[5]

QUESTION 11 / VRAAG 11



chord theorem] [raak
oved] [bewys]
s, verw $\angle e$, QM||R
 $\hat{T}_2$] anmorephysics.com

	TOTAL/TOTAAL:150
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