



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE 12/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2024

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 25 pages./
*Hierdie nasienriglyne bestaan uit 25 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

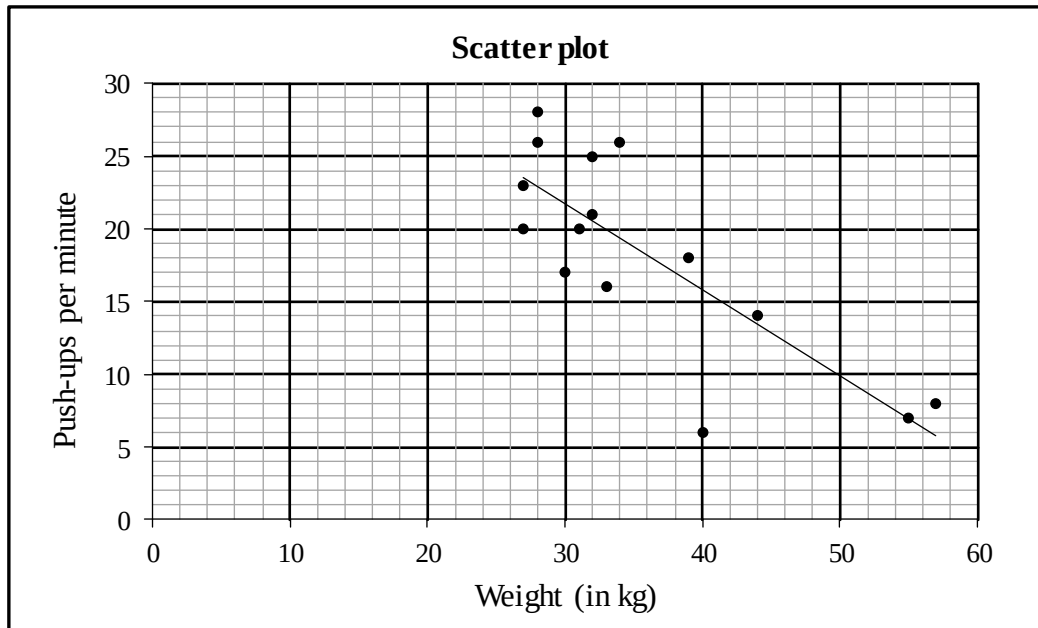
LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct
	Ken 'n punt toe as die bewering EN rede beide korrek is

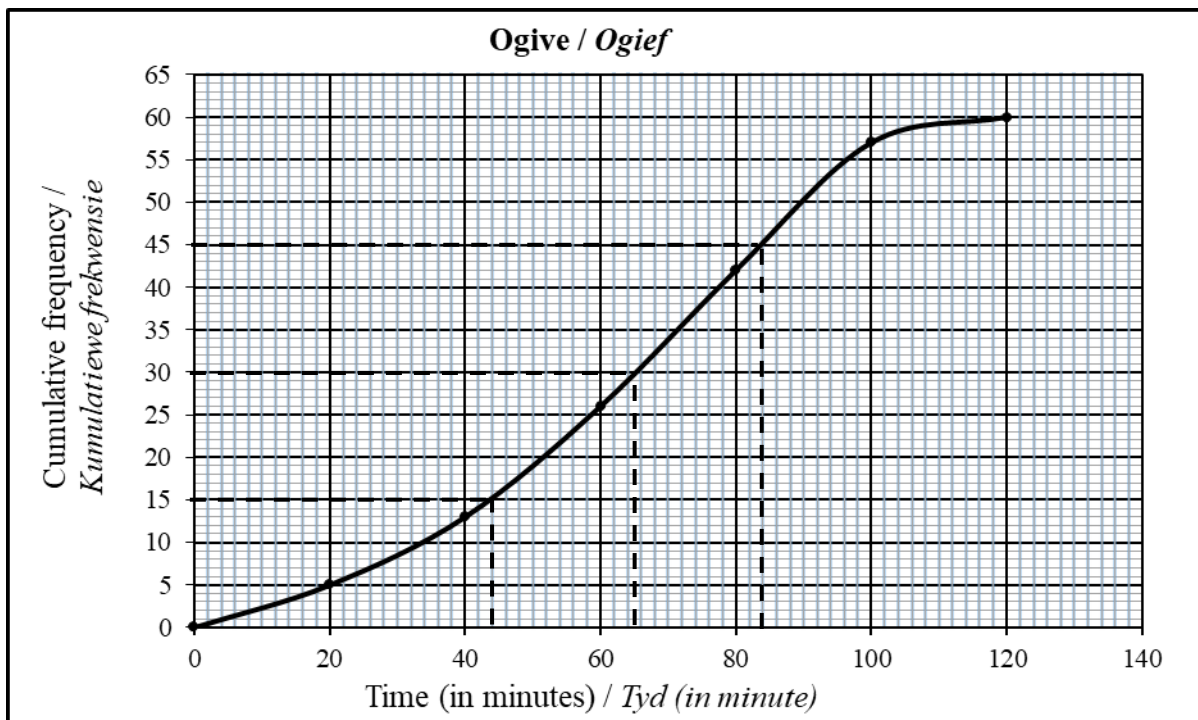
QUESTION/VRAAG 1

Weight (in kg) (x)	34	32	40	27	33	28	27	55	39	44	30	57	28	32	31
Number of push-ups per minute (y)	26	21	6	20	16	26	23	7	18	14	17	8	28	25	20

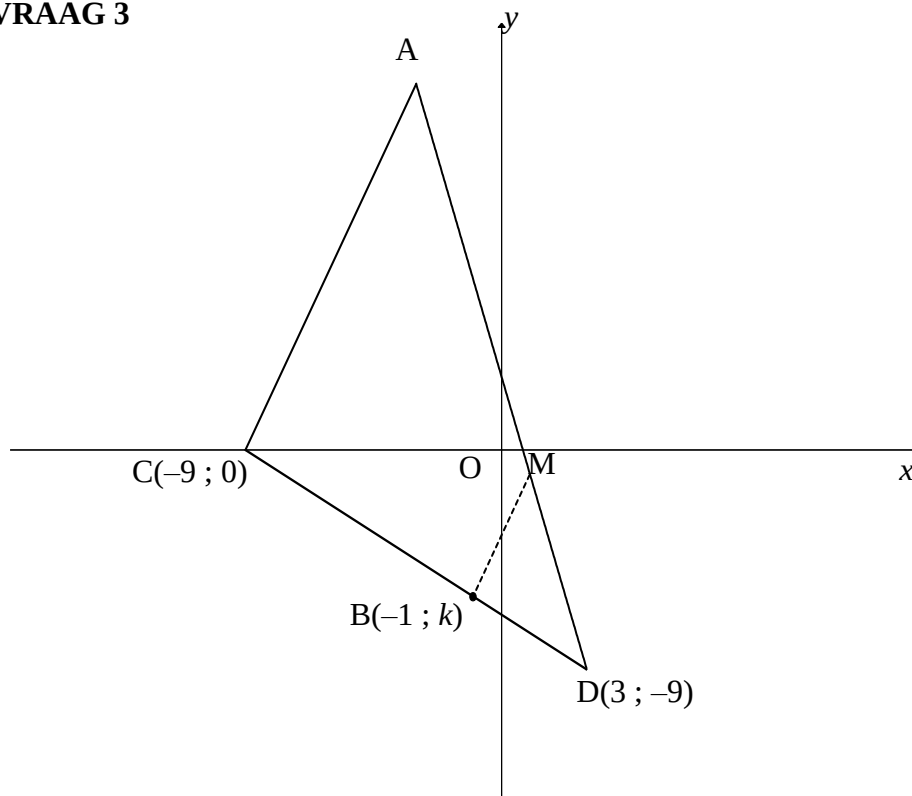


1.1	$a = 39,456001\dots$ $b = -0,590018\dots$ $\hat{y} = 39,46 - 0,59x$ CORRECT ANSWER ONLY: FULL MARKS	✓ $a = 39,46$ ✓ $b = -0,59$ ✓ equation (3)
1.2	$r = -0,8$	✓ (A) $-0,8$ (1)
1.3	$y = 39,46 - 0,59(29)$ $y = 22,35$ OR/OF $y = 22,35$ (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$\bar{y} = 18,33$	✓ (A) $18,33$ (1)
1.5	The increase in the number of push-ups will have no influence . The standard deviation stays the same .	✓ no influence OR standard deviation remains the same <i>geen verandering /</i> <i>bly dieselfde</i> (1)
1.6	6 is furthest y-value below the least squares regression line. An increase of 10 push-ups will get the team member to (40 ; 16), the minimum number of push-ups for a player weighing 40kg.	✓ 6 ✓ difference is 10 (2)
		[10]

QUESTION/VRAAG 2



2.1	Median = 65	✓ 65 (1)
2.2	$Q_1 = 44$	✓ 44 (1)
2.3	$IQR = 84 - 44$ $= 40$	✓ 84 ✓ IQR (2)
2.4		✓ box ✓ (A) whiskers ending at 5 & 120 (2)
2.5	Number of employees who qualify = 34 $\% \text{ of employees who qualify} = \frac{34}{60} \times 100$ $= 56,67\% \text{ of the employees}$ OR/OF Number of employees who qualify = 35 $\% \text{ of employees who qualify} = \frac{35}{60} \times 100$ $= 58,33\% \text{ of the employees}$	✓ 34 ✓ answer ✓ 35 ✓ answer (2)
2.6	Number of intervals = 3 Time allowed to work from home = 3(30 minutes) $= 90 \text{ minutes}$ OR/OF 1,5 hours	✓ 3 ✓ answer (2)
[10]		

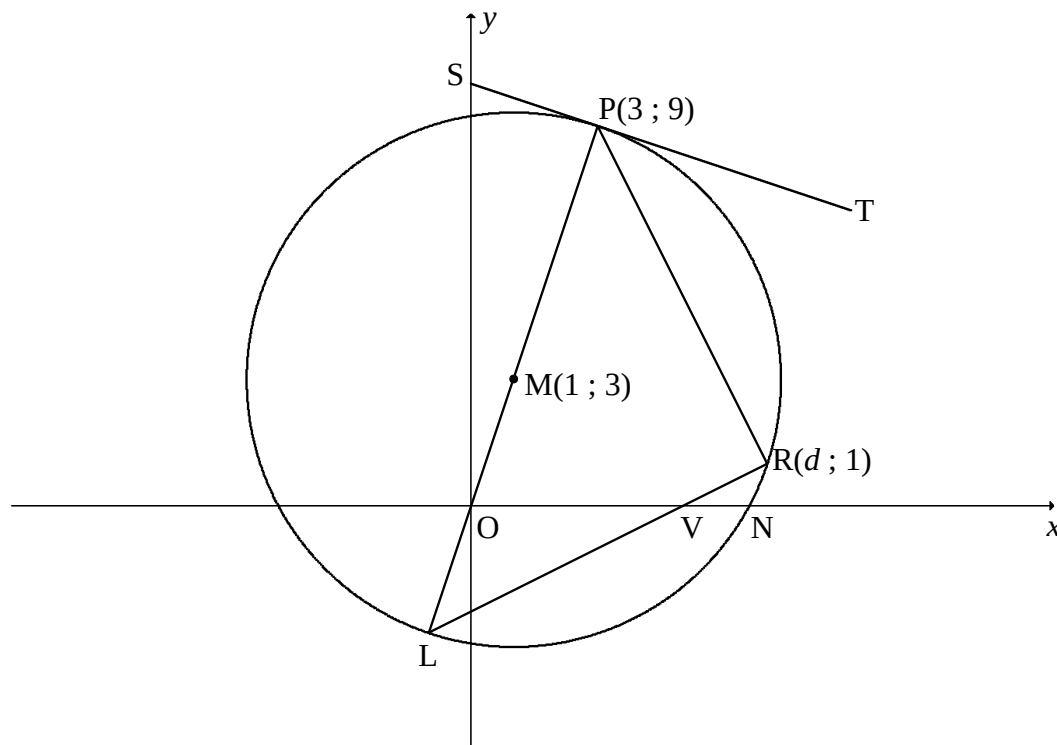
QUESTION / VRAAG 3

3.1	$m_{DC} = \frac{-9-0}{3-(-9)} \quad \text{OR/OF} \quad m_{DC} = \frac{0-(-9)}{-9-3}$ $m_{DC} = -\frac{3}{4} \quad m_{DC} = -\frac{3}{4}$	✓ correct substitution of D(3; -9) & C(-9; 0) into gradient formula ✓ answer (2)
3.2	Equation of DC: $0 = -\frac{3}{4}(-9) + c \quad \text{OR/OF} \quad y - 0 = -\frac{3}{4}(x - (-9))$ $c = \frac{-27}{4} \text{ or } -6\frac{3}{4} \quad y = -\frac{3}{4}(x + 9)$ $y = -\frac{3}{4}x - \frac{27}{4} \quad y = -\frac{3}{4}x - \frac{27}{4}$	✓ correct substitution of C(-9; 0) or D(3; -9) into equation of line ✓ answer (2)
3.3	$k = -\frac{3}{4}(-1) - \frac{27}{4} \quad \text{OR/OF} \quad \frac{k - (-9)}{-1 - 3} = \frac{-3}{4} \quad \text{OR/OF} \quad \frac{k - 0}{-1 - (-9)} = \frac{-3}{4}$ $k = \frac{3}{4} - \frac{27}{4} \quad \text{OR/OF} \quad k + 9 = 3 \quad \text{OR/OF} \quad k = -\frac{3}{4}(8)$ $k = -6 \quad k = -6 \quad k = -6$	✓ substitution of B(-1; k) (1)
3.4	$DC = \sqrt{(3+9)^2 + (-9-0)^2}$ $DC = 15 \text{ units}$	✓ correct substitution of D(3; -9) & C(-9; 0) into distance formula ✓ answer (2)

3.5	$DB = \sqrt{(3 - (-1))^2 + (-9 - (-6))^2}$ $DB = 5$ $\therefore \frac{DB}{DC} = \frac{5}{15} = \frac{1}{3}$	✓ DB = 5 ✓ answer (2)
3.6	$\frac{DM}{DA} = \frac{DB}{DC} = \frac{1}{3}$ $\frac{\text{Area } \triangle MBD}{\text{Area } \triangle ACD} = \frac{\frac{1}{2}(DM)(DB)(\sin \hat{D})}{\frac{1}{2}(DA)(DC)(\sin \hat{D})}$ $= \frac{1}{3} \times \frac{1}{3}$ $= \frac{1}{9}$	✓ $\frac{DM}{DA} = \frac{DB}{DC}$ ✓ correct use of area rule ✓ subst. for $\frac{BD}{DC}$ and $\frac{DM}{DA}$ into correct formula ✓ answer (4)
3.7	$y = -4x + c$ $m_{AD} = -4$ $-9 = -4(3) + c$ $c = 3$ $y = -4x + 3$ $(x-3)^2 + (y+9)^2 = 612$ $(x-3)^2 + (-4x+3+9)^2 = (\sqrt{612})^2$ $(x-3)^2 + (-4x+12)^2 = 612$ $x^2 - 6x + 9 + 16x^2 - 96x + 144 = 612$ $17x^2 - 102x - 459 = 0$ $x^2 - 6x - 27 = 0$ $(x-9)(x+3) = 0$ $x = 9 \text{ or } x = -3$ N/A $y = -4(-3) + 3$ $y = 15$ $A(-3; 15)$	✓ correct substitution of $m_{AD} = -4$ and $D(3; -9)$ ✓ $(x-3)^2 + (y+9)^2 = 612$ ✓ substitution of equation AD into distance formula ✓ standard form ✓ x values with rejection ✓ y coordinate (6)

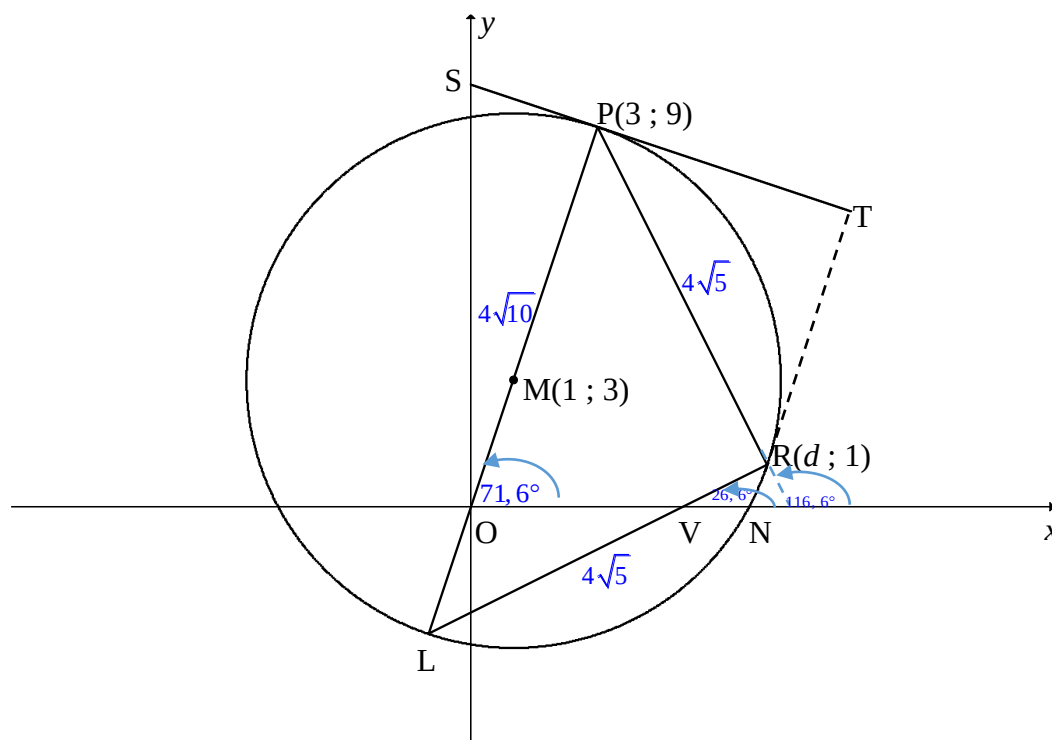
	OR/OF $-9 = -4(3) + c$ $c = 3$ $y = -4x + 3$ $N(0 ; 3)$ $ND = \sqrt{(3-0)^2 + (-9-3)^2}$ $= 3\sqrt{17}$ $AD = 6\sqrt{17}$ $ND = \frac{1}{2}AD$ N is the midpoint of AD A(-3 ; 15)	OR/OF ✓ correct substitution of $m_{AD} = -4$ and D(3 ; -9) ✓ N(0 ; 3) ✓ substitution into distance formula to calculate ND ✓ $ND = \frac{1}{2}AD$ ✓ x – value ✓ y – value (6)
		[19]

QUESTION/VRAAG 4



4.1	$L(-1; -3)$	$\checkmark x = -1 \quad \checkmark y = -3$ (2)
4.2	$m_{MP} = \frac{9-3}{3-1}$ $m_{MP} = 3$ $m_{ST} = -\frac{1}{3}$ $9 = -\frac{1}{3}(3) + c$ $c = 10$ $y = -\frac{1}{3}x + 10$	$\checkmark m_{MP} = 3$ $\checkmark m_{ST} = -\frac{1}{m_{MP}}$ $\checkmark$ substitution of m_{ST} & $P(3; 9)$ into equation of a line $\checkmark$ equation of tangent ST (4)
4.3	$(x-1)^2 + (y-3)^2 = r^2$ $(3-1)^2 + (9-3)^2 = r^2$ $r^2 = 40$ $(x-1)^2 + (y-3)^2 = 40$ $x^2 - 2x + 1 + y^2 - 6y + 9 = 40$ $x^2 + y^2 - 2x - 6y - 30 = 0$	$\checkmark (3-1)^2 + (9-3)^2 = r^2$ $\checkmark$ value of r^2 $\checkmark$ LHS of equation of circle $\checkmark$ expanding LHS (4)

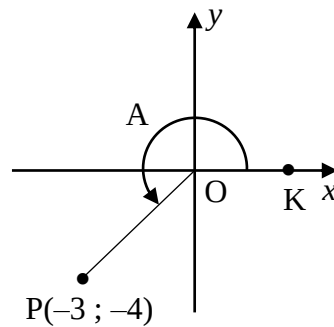
4.4	$d^2 + (1)^2 - 2d - 6(1) - 30 = 0$ $d^2 - 2d - 35 = 0$ $(d - 7)(d + 5) = 0$ $d = 7 \text{ or } d = -5$ $\therefore d = 7$ OR/OF $(x - 1)^2 + (y - 3)^2 = 40$ $(d - 1)^2 + (1 - 3)^2 = 40$ $(d - 1)^2 = 36$ $d - 1 = 6 \text{ or } d - 1 = -6$ $d = 7 \text{ or } d = -5$ $\therefore d = 7$ OR/OF $\hat{PRL} = 90^\circ \quad (\angle \text{ in semi-circle})$ $\frac{9-1}{3-d} \times \frac{1-(-3)}{d-(-1)} = -1$ $d^2 - 2d - 35 = 0$ $(d - 7)(d + 5) = 0$ $d = 7 \text{ or } d = -5$ $\therefore d = 7$	$\checkmark d^2 + (1)^2 - 2d - 6(1) - 30 = 0$ $\checkmark \text{ standard form}$ (2) OR/OF $\checkmark (d - 1)^2 + (1 - 3)^2 = 40$ $\checkmark \text{ standard form}$ (2) OR/OF $\checkmark m_{PR} \times m_{RL} = -1$ $\checkmark \text{ standard form}$ (2)
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4.5 $m_{PO} = 3$ $\therefore \tan \hat{POV} = 3$ $\hat{POV} = 71,565...^\circ$ $m_{RL} = \frac{1 - (-3)}{7 - (-1)}$ $= \frac{1}{2}$ $\therefore \tan \hat{RVN} = \frac{1}{2}$ $\hat{RVN} = 26,565...^\circ$ $\hat{L} = 71,565...^\circ - 26,565...^\circ$ [ext. $\angle$ of Δ / buite $\angle$ van Δ] $\hat{L} = 45^\circ$ OR/OF $\hat{R} = 90^\circ$ [$\angle$ in semi-circle / $\angle$ in 'n halwe sirkel] $PR^2 = (3 - 7)^2 + (9 - 1)^2$ $PR = \sqrt{80} = 4\sqrt{5}$ units $PL^2 = (3 - (-1))^2 + (9 - (-3))^2$ OR $RL^2 = (7 + 1)^2 + (1 + 3)^2$ $PL = \sqrt{160} = 4\sqrt{10}$ $RL = \sqrt{80} = 4\sqrt{5}$ $\sin \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ OR $\cos \hat{L} = \frac{4\sqrt{5}}{4\sqrt{10}}$ OR $\tan \hat{L} = \frac{4\sqrt{5}}{4\sqrt{5}}$ $\hat{L} = 45^\circ$	✓ $\tan \hat{POV} = m_{PO}$ ✓ $\hat{POV}$ ✓ m_{RL} using R(7 ; 1) & L ✓ $\hat{RVN}$ ✓ answer OR/OF ✓ $\hat{R} = 90^\circ$ ✓ $PR = \sqrt{80} = 4\sqrt{5}$ ✓ length of PL OR RL ✓ trig ratio of $\hat{L}$ ✓ answer
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	OR/OF $PL = \sqrt{(3+1)^2 + (9+3)^2} = \sqrt{160} = 4\sqrt{10}$ $PR = \sqrt{(7-3)^2 + (1-9)^2} = \sqrt{80} = 4\sqrt{5}$ $LR = \sqrt{(7+1)^2 + (1+3)^2} = \sqrt{80} = 4\sqrt{5}$ $\cos L = \frac{80+160-80}{2\sqrt{80} \times \sqrt{160}}$ $\cos L = \frac{\sqrt{2}}{2}$ $\hat{L} = 45^\circ$	OR/OF ✓ length of PL ✓ $PR = \sqrt{80} = 4\sqrt{5}$ ✓ length of LR ✓ substitution into the cos rule ✓ answer (5)
4.6	$m_{RM} = \frac{1-3}{7-1}$ $= -\frac{1}{3}$ $m_{RT} = 3 \quad (\tan \perp \text{ rad})$ $m_{PT} = -\frac{1}{3}$ $m_{RT} \times m_{PT} = -1$ PT $\perp$ RT OR/OF $m_{MR} = \frac{3-1}{1-7}$ $= -\frac{1}{3}$ $m_{PT} = -\frac{1}{3} \quad [\text{proved in Q4.2}]$ $m_{PT} = m_{MR}$ $\therefore$ PT $\parallel$ MR $\hat{MRT} = 90^\circ$ [radius $\perp$ tangent / <i>raaklyn $\perp$ radius</i>] $\hat{PTR} = 90^\circ$ [co-int $\angle$s; PT $\parallel$ MR/ooreenkomst. $\angle$e; PT $\parallel$ MR] PT $\perp$ RT OR/OF $\hat{TPR} = \hat{L} = 45^\circ$ [tan-chord theorem/ $\angle$ tussen raaklyn en koord] TP = TR [tans from common pt] $\therefore \hat{TPR} = \hat{TRP} = 45^\circ$ [$\angle$s opp equal sides/ <i>$\angle$e teenoor gelyke sye</i>] $\therefore \hat{PTR} = 90^\circ$ [sum of $\angle$s in Δ / <i>binne $\angle$e van Δ</i>] PT $\perp$ RT	✓ m_{RM} ✓ m_{RT} ✓ $m_{RT} \times m_{PT} = -1$ (3) OR/OF ✓ PT $\parallel$ MR ✓ $\hat{MRT} = 90^\circ$ ✓ $\hat{PTR} = 90^\circ$ (3) OR/OF ✓ $\hat{TPR} = \hat{L}$ ✓ $\hat{TPR} = \hat{TRP}$ ✓ $\hat{PTR} = 90^\circ$ (3) [20]

QUESTION/VRAAG 5



5.1.1	$r = 5$ $\cos A = -\frac{3}{5}$	✓ $r = 5$ ✓ answer (2)
5.1.2	$\cos 2A = 2\cos^2 A - 1$ $= 2\left(-\frac{3}{5}\right)^2 - 1$ $= -\frac{7}{25}$ OR/OF $\cos 2A = \cos^2 A - \sin^2 A$ $= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$ OR/OF $\cos 2A = 1 - 2\sin^2 A$ $= 1 - 2\left(-\frac{4}{5}\right)^2$ $= -\frac{7}{25}$	✓ substitution of $\cos A$ into double angle formula ✓ answer (2) ✓ substitution of $\cos A$ & $\sin A$ into double angle formula ✓ answer (2) ✓ substitution of $\sin A$ into double angle formula ✓ answer (2)
5.1.3	 $x = -3$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) - \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$ $= \frac{12}{25} + \frac{12}{25}$ $= \frac{24}{25}$	✓ $x = -3$ ✓✓ substitution into the compound angle formula ✓ answer (4)

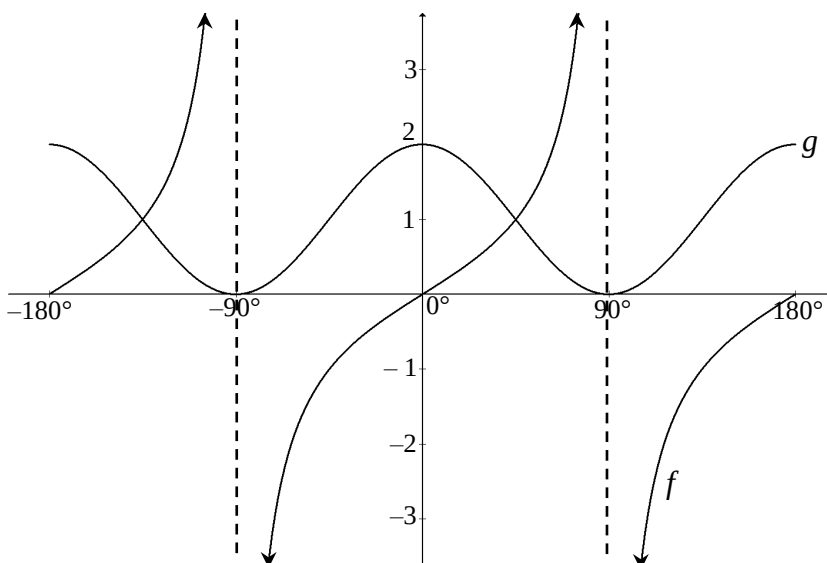
5.2	$\frac{\cos\left(\frac{\alpha}{2} - 45^\circ\right) \sin\left(\frac{\alpha}{2} - 45^\circ\right)}{2}$ $= \frac{2\cos\left(\frac{\alpha}{2} - 45^\circ\right) \sin\left(\frac{\alpha}{2} - 45^\circ\right)}{2 \cdot 2}$ $= \frac{\sin(\alpha - 90^\circ)}{4}$ $= \frac{-\cos \alpha}{4}$ $= \frac{-p}{4} \quad \text{OR/OF} \quad = -\frac{1}{4} p$ OR/OF $\frac{\cos\left(\frac{\alpha}{2} - 45^\circ\right) \sin\left(\frac{\alpha}{2} - 45^\circ\right)}{2}$ $= \frac{\left[\cos \frac{\alpha}{2} \cos 45^\circ + \sin \frac{\alpha}{2} \sin 45^\circ\right] \left[\sin \frac{\alpha}{2} \cos 45^\circ - \cos \frac{\alpha}{2} \sin 45^\circ\right]}{2}$ $= \frac{\left[\frac{\sqrt{2}}{2} \cos \frac{\alpha}{2} + \frac{\sqrt{2}}{2} \sin \frac{\alpha}{2}\right] \left[\frac{\sqrt{2}}{2} \sin \frac{\alpha}{2} - \frac{\sqrt{2}}{2} \cos \frac{\alpha}{2}\right]}{2}$ $= \frac{\frac{1}{2} \sin^2 \frac{\alpha}{2} - \frac{1}{2} \cos^2 \frac{\alpha}{2}}{2}$ $= \frac{-\frac{1}{2} \left(\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}\right)}{2}$ $= -\frac{\cos 2\left(\frac{\alpha}{2}\right)}{4}$ $= -\frac{\cos \alpha}{4}$ $= -\frac{1}{4} p$	✓ multiply by $\frac{2}{2}$ ✓ double angle ✓ co function ✓ answer (4) OR/OF ✓ expansion ✓ special angles ✓ double angle ✓ answer (4)
		[12]

QUESTION/VRAAG 6

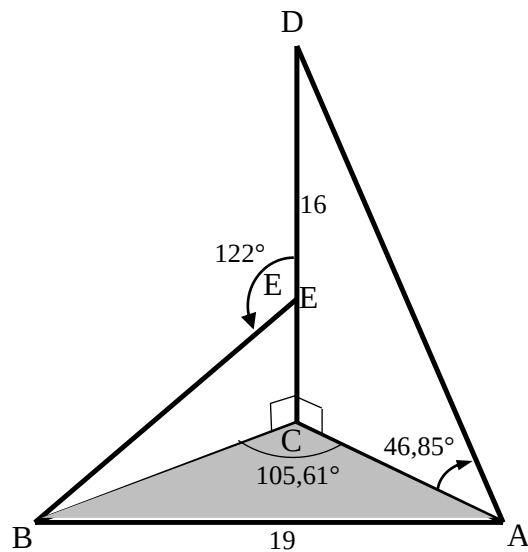
6.1.1	$\cos(x + y) = \cos(x - (-y))$ $= \cos x \cos(-y) + \sin x \sin(-y)$ $= \cos x \cos y - \sin x \sin y$	✓ $(x + y) = (x - (-y))$ ✓ correct expansion (2)
6.1.2	$\text{LHS} = \frac{\cos(90^\circ - x)\cos y + \sin(-y)\cos(180^\circ + x)}{\cos x \cos(360^\circ + y) + \sin(360^\circ - x)\sin y}$ $= \frac{(\sin x)\cos y + (-\sin y)(-\cos x)}{\cos x(\cos y) + (-\sin x)\sin y}$ $= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$ $= \frac{\sin(x + y)}{\cos(x + y)}$ $= \tan(x + y)$ $= \text{RHS}$	✓ $\cos(90^\circ - x) = \sin x$ ✓ $\sin(-y) = -\sin y$ ✓ $\cos(180^\circ + x) = -\cos x$ ✓ $\cos(360^\circ + y) = \cos y$ ✓ $\sin(360^\circ - x) = -\sin x$ ✓ compound angle formulae (6)
6.2	$\sqrt{6\sin^2 x - 11\cos(90^\circ + x) + 7} = 2$ $6\sin^2 x - 11\cos(90^\circ + x) + 7 = 4$ $6\sin^2 x - 11(-\sin x) + 7 = 4$ $6\sin^2 x + 11\sin x + 3 = 0$ $(3\sin x + 1)(2\sin x + 3) = 0$ $\sin x = -\frac{1}{3} \quad \text{OR/OR} \quad \sin x = -\frac{3}{2}$ $\text{ref } \angle = 19,47^\circ \quad \text{no solution}$ $x = 199,47^\circ \text{ or } x = 340,53^\circ$	✓ squaring both sides ✓ $\cos(90^\circ + x) = -\sin x$ ✓ factors ✓ both equations ✓ ✓ answers (6)
6.3.1	$g(x) = \frac{4 - 8\sin^2 x}{3}$ $= \frac{4(1 - 2\sin^2 x)}{3}$ $= \frac{4\cos 2x}{3}$ Maximum value of $\cos 2x$ is 1 $\therefore$ maximum value of $g(x) = \frac{4}{3}$	✓ factors ✓ $\frac{4\cos 2x}{3}$ ✓ answer (3)

	OR/OF $4 - 8\sin^2 x$ is a maximum when $\sin^2 x$ is a minimum Minimum value of $\sin^2 x$ is 0 $\therefore$ max. value of $g(x) = \frac{4-8(0)}{3}$ $g(x) = \frac{4}{3}$ OR/OF $\sin x = \frac{-(0)}{2\left(-\frac{8}{3}\right)}$ $\sin x = 0$ $\therefore$ max. value of $g(x) = \frac{4-8(0)}{3}$ $g(x) = \frac{4}{3}$	OR/OF ✓ min of $\sin^2 x = 0$ ✓ $g(x) = \frac{4-8(0)}{3}$ ✓ answer (3) OR/OF ✓ $\sin x = \frac{-(0)}{2\left(-\frac{8}{3}\right)}$ ✓ $\sin x = 0$ ✓ answer (3)
6.3.2	$x = 180^\circ$	✓ 180° (1)
[18]		

QUESTION/VRAAG 7

7.1	$x = 90^\circ$	✓ $x = 90^\circ$ (1)
7.2	$x = -180^\circ$ or $x \in (-90^\circ ; 0^\circ]$ OR/OF $x = -180^\circ$ or $-90^\circ < x \leq 0^\circ$	✓✓ answer (2) ✓✓ answer (2)
7.3.1	180°	✓ answer (1)
7.3.2		✓ turning points on x-axis: $x = -90^\circ ; 90^\circ$ ✓ shape ✓ turning point on y-axis at $(0 ; 2)$ (3)
7.4	$2 \cos^3 x - \sin x = 0$ $2 \cos^3 x = \sin x$ $2 \cos^2 x = \frac{\sin x}{\cos x}$ $2 \cos^2 x = \tan x$ $2 \cos^2 x - 1 = \tan x - 1$ $\cos 2x + 1 = \tan x$ $x = 45^\circ + k \cdot 180^\circ ; k \in \mathbb{Z}$ OR/OF $2 \cos^3 x - \sin x = 0$ $\cos x (2 \cos^2 x - \tan x) = 0$ $\cos x = 0$ or $2 \cos^2 x = \tan x$ not valid $2 \cos^2 x - 1 + 1 = \tan x$ $\cos 2x + 1 = \tan x$ $x = 45^\circ + k \cdot 180^\circ ; k \in \mathbb{Z}$	✓ $2 \cos^2 x = \tan x$ ✓ $2 \cos^2 x - 1 = \tan x - 1$ ✓ $\cos 2x + 1 = \tan x$ ✓ answer (4) OR/OF ✓ $2 \cos^2 x = \tan x$ ✓ $2 \cos^2 x - 1 + 1 = \tan x$ ✓ $\cos 2x + 1 = \tan x$ ✓ answer (4)

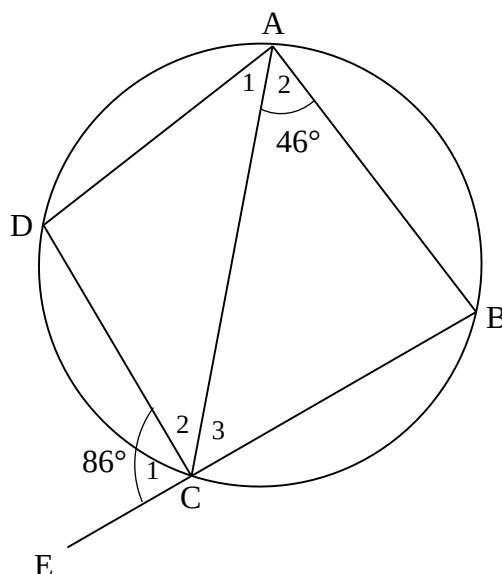
[11]

QUESTION/VRAAG 8

8.1	$\tan \hat{D}AC = \frac{DC}{AC}$ $AC = \frac{16}{\tan 46,85^\circ}$ $AC = 15 \text{ m}$	✓ correct subs into trig ratio ✓ answer (2)
8.2	$(AB)^2 = (BC)^2 + (AC)^2 - 2(BC)(AC)\cos \hat{B}CA$ $(19)^2 = x^2 + (15)^2 - 2x(15)\cos 105,61^\circ$ $x^2 + 8,07x - 136 = 0$ $x = \frac{-8,07 \pm \sqrt{(8,07)^2 - 4(1)(-136)}}{2(1)}$ $x = 8,30 \text{ m or } x \neq -16,38 \text{ m}$ $\hat{B}EC = 58^\circ \quad \text{OR/OF} \quad \hat{E}BC = 32^\circ$ $\tan \hat{B}EC = \frac{BC}{EC}$ $EC = \frac{8,3}{\tan 58^\circ}$ $EC = 5,19 \text{ m}$ $DE = 10,81 \text{ m}$ $\tan \hat{E}BC = \frac{EC}{BC}$ $EC = 8,3 \tan 32^\circ$ $EC = 5,19 \text{ m}$ $DE = 10,81 \text{ m}$	✓ correct subst. into cosine rule ✓ quadratic equation in std form ✓ correct subst. into quadratic formula ✓ length of BC ✓ size of $\hat{B}EC$ OR/OF $\hat{E}BC$ ✓ length of EC ✓ answer (7)

	OR/OF $\frac{\sin 105,61^\circ}{19} = \frac{\sin \hat{C}BA}{15}$ $\hat{C}BA = 49,5^\circ$ $\hat{B}AC = 24,89^\circ$ $\frac{BC}{\sin 24,89^\circ} = \frac{19}{\sin 105,61^\circ}$ $BC = 8,3 \text{ m}$ $\hat{B}EC = 58^\circ$ $\tan \hat{B}EC = \frac{BC}{EC}$ $EC = \frac{8,3}{\tan 58^\circ}$ $EC = 5,19 \text{ m}$ $DE = 10,81 \text{ m}$	OR/OF ✓ correct subst. into sine rule ✓ $\hat{B}AC$ ✓ correct subst. into sine formula ✓ length of BC ✓ size of $\hat{B}EC$ OR/OF $\hat{E}BC$ ✓ length of EC ✓ answer (7)
[9]		

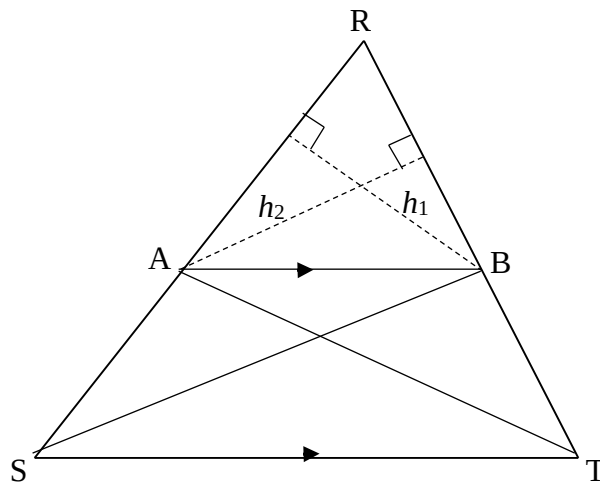
QUESTION/VRAAG 9



9.1	$\hat{A}_1 = 40^\circ$ [ext. $\angle$ of a cyclic quad / buite $\angle$ van kvh]	✓ S ✓ R (2)
9.2	$\hat{B} = 80^\circ$ $\left[\hat{A}_1 = \frac{1}{2} \hat{B} \right]$ $\hat{D} = 100^\circ$ [opp $\angle$ s of cyclic quad / teenoorst. $\angle$ e van kvh] $\therefore \hat{C}_2 = 40^\circ$ [sum of $\angle$ s in Δ / binne $\angle$ e van Δ] $\therefore \hat{C}_2 = \hat{A}_1 = 40^\circ$ $\therefore AD = DC$ [sides opp = $\angle$ s / sye teenoor gelyke $\angle$] OR/OF $\hat{B} = 80^\circ$ $\left[\hat{A}_1 = \frac{1}{2} \hat{B} \right]$ $\angle ACE = \hat{A}_2 + \hat{B}$ [ext $\angle$ of Δ / buite $\angle$ van Δ] $\therefore \hat{C}_2 = 40^\circ$ $\therefore \hat{C}_2 = \hat{A}_1 = 40^\circ$ $\therefore AD = DC$ [sides opp = $\angle$ s / sye teenoor gelyke $\angle$] OR/OF $\hat{B} = 80^\circ$ $\left[\hat{A}_1 = \frac{1}{2} \hat{B} \right]$ $\therefore \hat{C}_3 = 180^\circ - 46^\circ - 80^\circ$ [sum of $\angle$ s in Δ / binne $\angle$ e van Δ] $\therefore \hat{C}_3 = 54^\circ$ $\therefore \hat{C}_2 = 180^\circ - 86^\circ - 54^\circ$ [$\angle$ s on a str. line / $\angle$ e op 'n reguitlyn] $\therefore \hat{C}_2 = 40^\circ$ $\therefore \hat{C}_2 = \hat{A}_1 = 40^\circ$ $\therefore AD = DC$ [sides opp = $\angle$ s / sye teenoor gelyke $\angle$] 	✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S/R ✓ S ✓ R (4)
[6]		

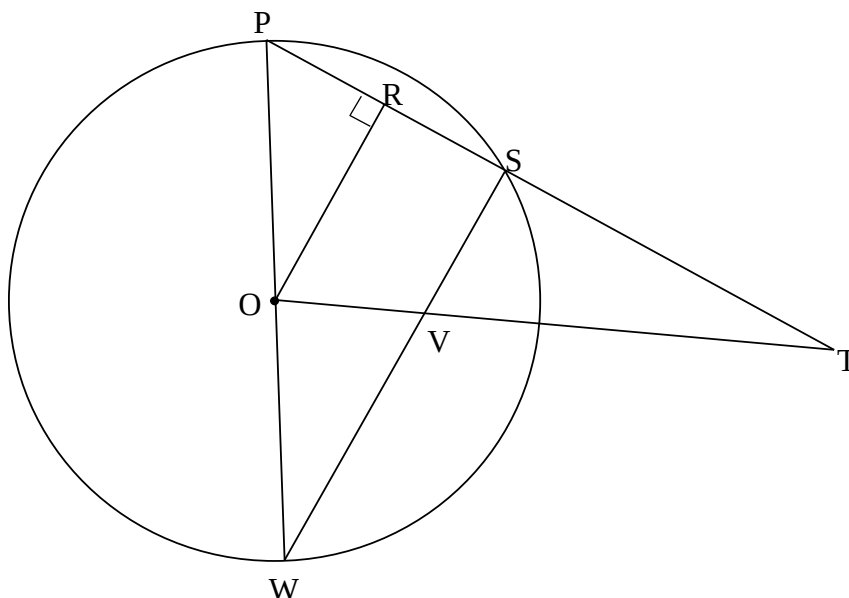
QUESTION/VRAAG 10

10.1



10.1	Construction: Join SB and TA and draw h_1 from B $\perp$ AR and h_2 from A $\perp$ RB Konstruksie: Verbind SB en TA en trek h_1 vanaf B $\perp$ AR en h_2 vanaf A $\perp$ RB Proof/Bewys: $\frac{\text{area } \triangle RAB}{\text{area } \triangle ASB} = \frac{\frac{1}{2} RA \times h_1}{\frac{1}{2} AS \times h_1} = \frac{RA}{AS}$ $\frac{\text{area } \triangle RAB}{\text{area } \triangle ABT} = \frac{\frac{1}{2} RB \times h_2}{\frac{1}{2} BT \times h_2} = \frac{RB}{BT}$ area $\triangle RAB$ = area $\triangle RAB$ [common/gemeenskaplik] But area $\triangle ASB$ = area $\triangle ABT$ [same base & height; AB $\parallel$ ST/ dies. basis & hoogte; AB $\parallel$ ST] $\therefore \frac{\text{area } \triangle RAB}{\text{area } \triangle ASB} = \frac{\text{area } \triangle RAB}{\text{area } \triangle ABT}$ $\therefore \frac{RA}{AS} = \frac{RB}{BT}$	✓ construction $\checkmark \frac{\text{area } \triangle RAB}{\text{area } \triangle ASB} = \frac{\frac{1}{2} RA \times h_1}{\frac{1}{2} AS \times h_1}$ $\checkmark \frac{RA}{AS}$ $\checkmark \frac{\text{area } \triangle RAB}{\text{area } \triangle ABT} = \frac{RB}{BT}$ ✓ S ✓ R (6)
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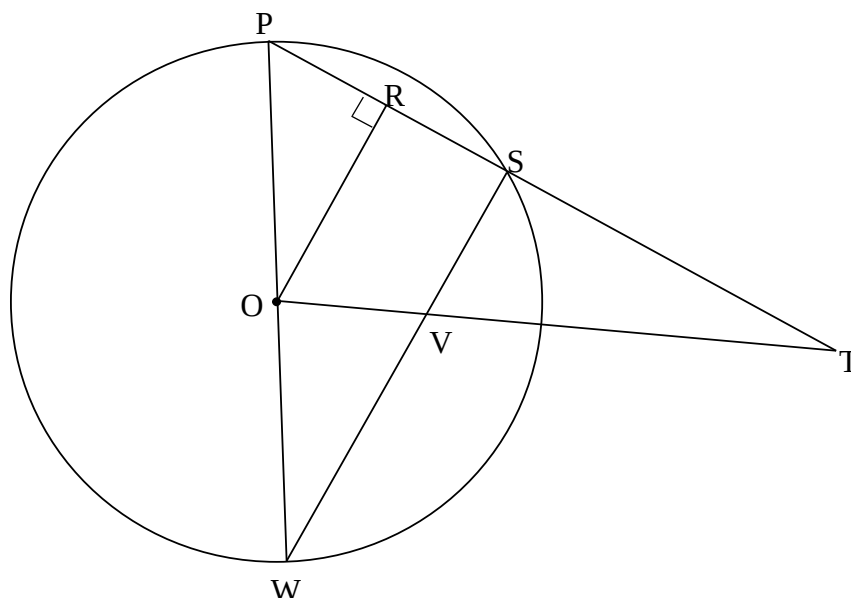
10.2



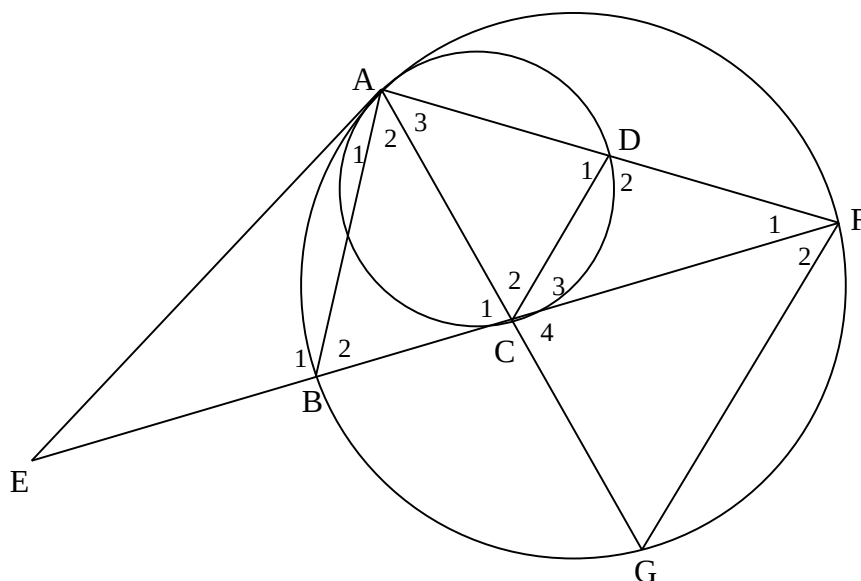
10.2.1	PR = RS PO = OW $\therefore OR = \frac{1}{2} WS$ $\therefore OR : WS = 1 : 2$ OR/OF $\hat{P}SW = 90^\circ$ $\hat{P}RO = 90^\circ$ $\therefore \hat{P}RO = \hat{P}SW$ $\therefore RO \parallel SW$ $\frac{PO}{OW} = \frac{PR}{RS}$ PO = OW $\therefore PR = RS$ $\therefore OR : WS = 1 : 2$	[line from centre $\perp$ to chord/ lyn vanuit midpt. sirkel $\perp$ op koord] [radii / radiusse] [midpt theorem/midpt. stelling] [$\angle$ in semi circle/$\angle$ in halwe sirkel] [given] [corresp $\angle$s = / ooreenk. $\angle$e =] OR/OF [co-int. $\angle$s suppl / ko-binne $\angle$e suppl] [prop theorem; $RO \parallel SW$/ lyn $\parallel$ een sy van Δ] [radii / radiusse] [midpt theorem/ midpt. stelling]	✓ S ✓ R ✓ S ✓ S ✓ R (5) ✓ S ✓ S ✓ S ✓ R ✓ R (5)
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	OR/OF ΔPRO and ΔPSW $\hat{P}\hat{S}W = 90^\circ$ [∠ in semi circle/∠ in halwe sirkel] $\hat{P}\hat{R}O = 90^\circ$ [given] $\therefore \hat{P}\hat{R}O = \hat{P}\hat{S}W$ $\hat{P}$ is common $\hat{P}\hat{O}R = \hat{P}\hat{W}S$ [sum of ∠s in Δ/ som van ∠e in Δ] $\therefore \Delta PRO \parallel \Delta PSW$ [∠∠∠] $\therefore \frac{PO}{PW} = \frac{RO}{SW}$ [∥ Δs / ∥ Δe] but $PW = 2 PO$ [diameter = 2 radius/middellyn = 2 radius] $\therefore \frac{RO}{SW} = \frac{PO}{2PO}$ $= \frac{1}{2}$ $\therefore OR : WS = 1 : 2$	✓ S ✓ R ✓ S ✓ S ✓ S (5)
10.2.2	$\frac{OV}{VT} = \frac{RS}{ST} = \frac{1}{3}$ [prop theorem; $RO \parallel SW$/ lyn een sy van Δ] $\frac{RS}{15} = \frac{1}{3}$ $RS = 5$ units $PR = RS = 5$ units [line from centre ⊥ to chord / lyn vanuit midpt. sirkel ⊥ op koord] $\therefore PT = 25$ units	✓ S / R ✓ S ✓ S ✓ answer (4)
[15]		

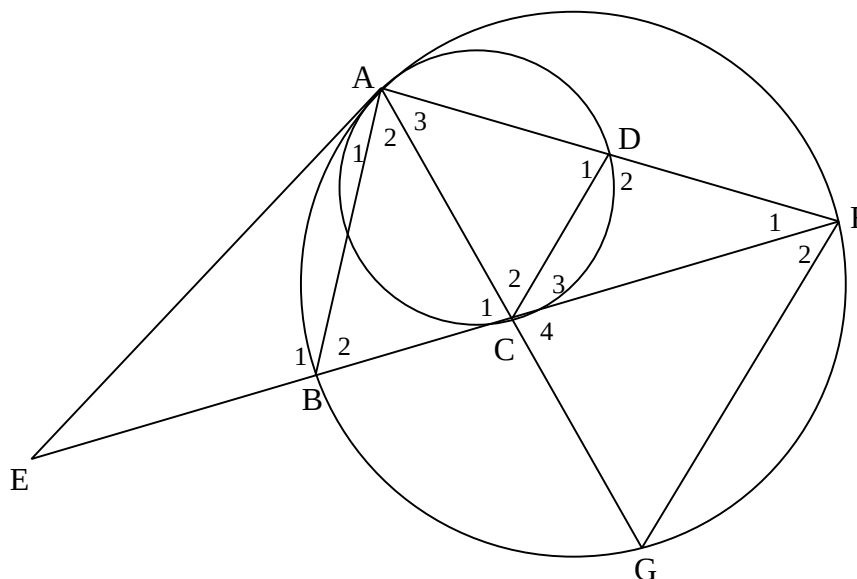
10.2



QUESTION/VRAAG 11



11.1	$\hat{D}_1 = \hat{E}AG = x$	[tan-chord theorem/ $\angle$ tussen raaklyn en koord]	✓ S ✓ R	(6)
	$\hat{C}_1 = \hat{D}_1 = x$	[tan-chord theorem/ $\angle$ tussen raaklyn en koord]	✓ S ✓ R	
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$ s = / regoorst. $\angle$ e]	✓ S/R	
	$\hat{A}FG = \hat{E}AG = x$	[tan-chord theorem/ $\angle$ tussen raaklyn en koord]	✓ S	
	OR/OF			(6)
	$EA = EC$	[tans from common pt/ raaklyne vanuit dies. punt]	✓ S/R	
	$\hat{C}_1 = \hat{E}AG = x$	[$\angle$ s opp equal sides/ $\angle$ e teenoor gelyke sye]	✓ S	
	$\hat{C}_4 = \hat{C}_1 = x$	[vert opp $\angle$ s = / regoorst. $\angle$ e]	✓ S/R	
	$\hat{D}_1 = \hat{E}AG = x$	[tan-chord theorem/ $\angle$ tussen raaklyn en koord]	✓ S ✓ R	
	$\hat{A}FG = \hat{E}AG = x$	[tan-chord theorem $\angle$ tussen raaklyn en koord]	✓ S	



11.2	$\hat{D}_1 = \hat{A}FG = x$ $\therefore DC \parallel FG$ [corresp $\angle$ s = / ooreenk $\angle$ e =] $\frac{AG}{AC} = \frac{AF}{AD}$ [prop theorem; $DC \parallel FG$ / lyn een sy van Δ] $\therefore AG \cdot AD = AC \cdot AF$ OR/OF In ΔACD and ΔAGF $\hat{A}_3$ is common $\hat{A}FG = \hat{D}_1 = x$ [proved in 11.1 / reeds bewys] $\hat{C}_2 = \hat{A}GF = x$ [sum $\angle$ Δ s/binne $\angle$ e Δ] $\Delta ACD \parallel \Delta AGF$ [$\angle \angle \angle$] $\frac{AC}{AG} = \frac{AD}{AF}$ [$\parallel \Delta$ s $\therefore$ sides in proportion / $\parallel \Delta$ e $\therefore$ sye in dieselfde verhouding] $\therefore AG \cdot AD = AC \cdot AF$	✓ S ✓ S/R ✓ S ✓ R (4) ✓ S ✓ S ✓ S/R ✓ S (4)
11.3	In ΔAGF and ΔABC $\hat{G} = \hat{B}_2$ [$\angle$ s in the same seg / $\angle$ e in dies. segment] $\hat{A}FG = \hat{C}_1 = x$ [proved in 11.1 / reeds bewys] $\hat{A}_3 = \hat{A}_2$ [sum of $\angle$ s in Δ /binne $\angle$ e van Δ] $\Delta AGF \parallel \Delta ABC$ [$\angle \angle \angle$]	✓ S ✓ R ✓ S ✓ S OR/OF R (4)

11.4	$\frac{GF}{BC} = \frac{AF}{AC} \quad [\Delta AGF \parallel \Delta ABC]$ $\therefore GF = \frac{BC \cdot AF}{AC}$ $\Delta ACD \parallel \Delta FGC \quad [\angle \angle \angle]$ $\therefore \frac{AC}{GF} = \frac{AD}{FC}$ $\therefore AC = \frac{AD \cdot FG}{FC}$ $\therefore GF = BC \cdot AF \div \frac{AD \cdot FG}{FC}$ $GF = BC \cdot AF \times \frac{FC}{AD \cdot FG}$ $\therefore GF^2 = \frac{BC \cdot FC \cdot AF}{AD}$ OR/OF $\Delta AGF \parallel \Delta ABC \quad [\angle \angle \angle]$ $\frac{GF}{BC} = \frac{AF}{AC}$ $GF = \frac{AF \cdot BC}{AC}$ $\Delta ACD \parallel \Delta AGF \quad [\angle \angle \angle]$ $\frac{AD}{AF} = \frac{CD}{GF}$ $GF = \frac{AF \cdot CD}{AD}$ $GF \times GF = \frac{AF \cdot BC}{AC} \cdot \frac{AF \cdot CD}{AD}$ $\Delta FCD \parallel \Delta FAC \quad [\angle \angle \angle]$ $\frac{FC}{FA} = \frac{CD}{AC} \quad \text{from } \parallel \Delta \text{'s}$ $FC = \frac{CD \cdot AF}{AC}$ $GF^2 = \frac{AF \cdot FC \cdot BC}{AD}$	✓ S / R ✓ S ✓ S ✓ S ✓ S ✓ S ✓ S (6) OR/OF ✓ S ✓ S ✓ S ✓ S ✓ S ✓ S (6) [20]
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TOTAL/TOTAAL: 150