



education

Department:
Education
North West Provincial Government
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2
SEPTEMBER 2024

MARKS: 150

TIME: 3 hours



**This question paper consists of 12 pages, 1 information sheet
and an answer book of 20 pages.**

INSTRUCTIONS AND INFORMATION

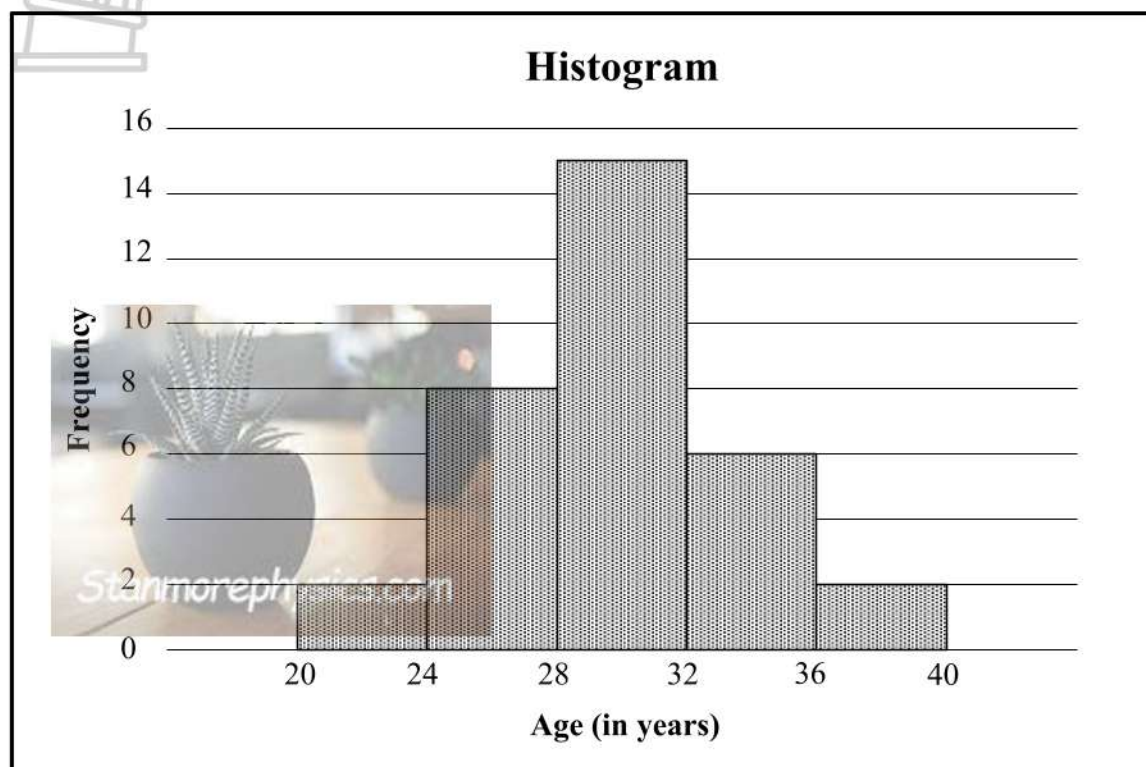
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.



QUESTION 1

During the Rugby World Cup of 2023, the ages (in years) of the players of the Springbok rugby squad were recorded. The data is represented in the histogram below.



- 1.1 How many players were in this rugby squad? (1)
- 1.2 Calculate the estimated mean age of these rugby players. (2)
- 1.3 Use the histogram to:
 - 1.3.1 Complete the cumulative frequency column in the ANSWER BOOK (2)
 - 1.3.2 Draw an ogive (cumulative frequency graph) of the above data on the grid that is provided in the ANSWER BOOK (3)
- 1.4 Write down the estimated median of the above data. (2)
- 1.5 It was discovered that the frequency of the age data for k player(s) in the modal age interval was recorded incorrectly. The mistake is corrected and the frequency of TWO other intervals are increased. The number of players in the squad remains unchanged. Determine the minimum value of k , if the data of the new histogram is symmetrical. (3)

[13]

QUESTION 2

Mrs Mochini wants to use mathematical modelling to predict the final results of her grade 12 Mathematics learners. She decides to use the Preparatory and Final Mathematics examination results of the previous year to help her in developing such a possible model.

She records in the table below, 10 learners' previous year's results (in %) as follows:

Preparatory examination (x)	38	65	78	23	67	93	39	83	51	66
Final examination (y)	57	72	81	27	59	94	41	85	54	79

2.1 Determine the equation of the least squares regression line. (3)

2.2 A learner obtained 46% for the Preparatory examination.

2.2.1 Calculate the possible final examination results that Mrs Mochine can expect from this learner. (2)

2.2.2 Is the answer in QUESTION 2.2.1 a good indication of the expected final examination result? Motivate your answer. (2)

2.3 The point $(\bar{x}; q)$ lies on the regression line of QUESTION 2.1.

Only ONE of the options below correctly reflects the value of q .
Write only the letter of the correct option as your answer.

A $\sqrt{\bar{x}}$

B $\frac{\sum y}{10}$

C σ_x

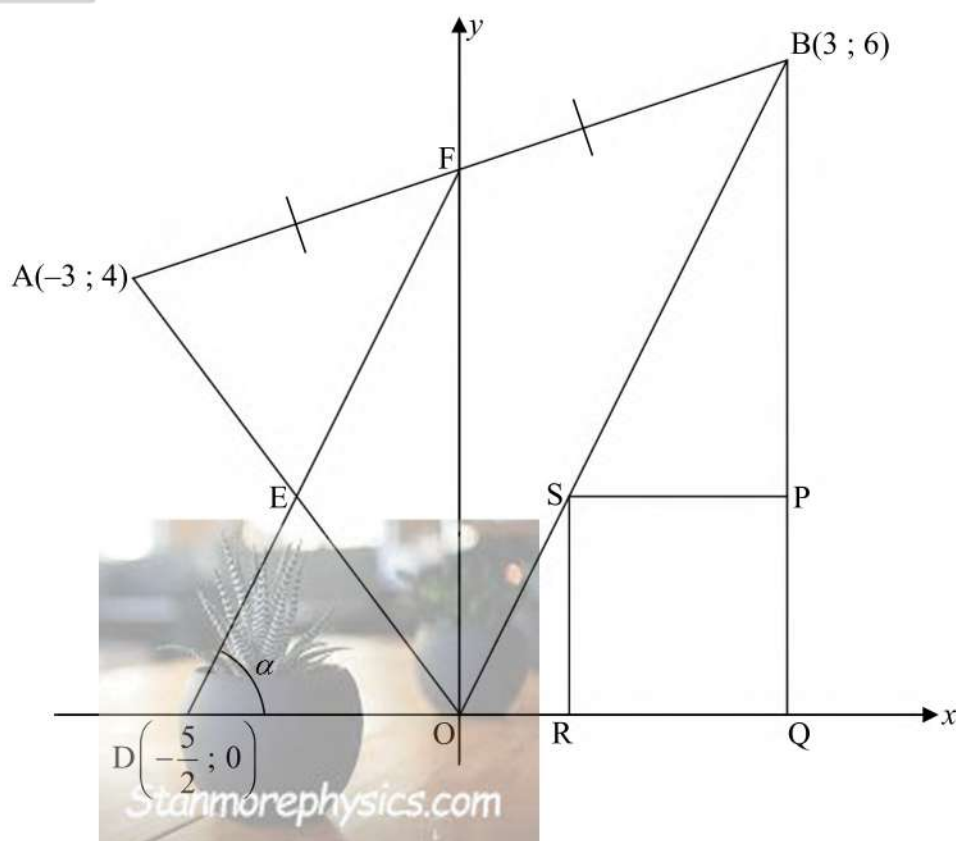
D σ_y

(1)

[8]

QUESTION 3

In the diagram, $A(-3 ; 4)$, $B(3 ; 6)$ and O (origin) are vertices of $\triangle ABO$. F is the midpoint of AB and is joined with $D\left(-\frac{5}{2} ; 0\right)$. The angle of inclination of FD is α . The lines AO and DF intersect at E . A quadrilateral $PQRS$ is drawn with QR on the x -axis and S is a point on OB . The side QP is produced to B .

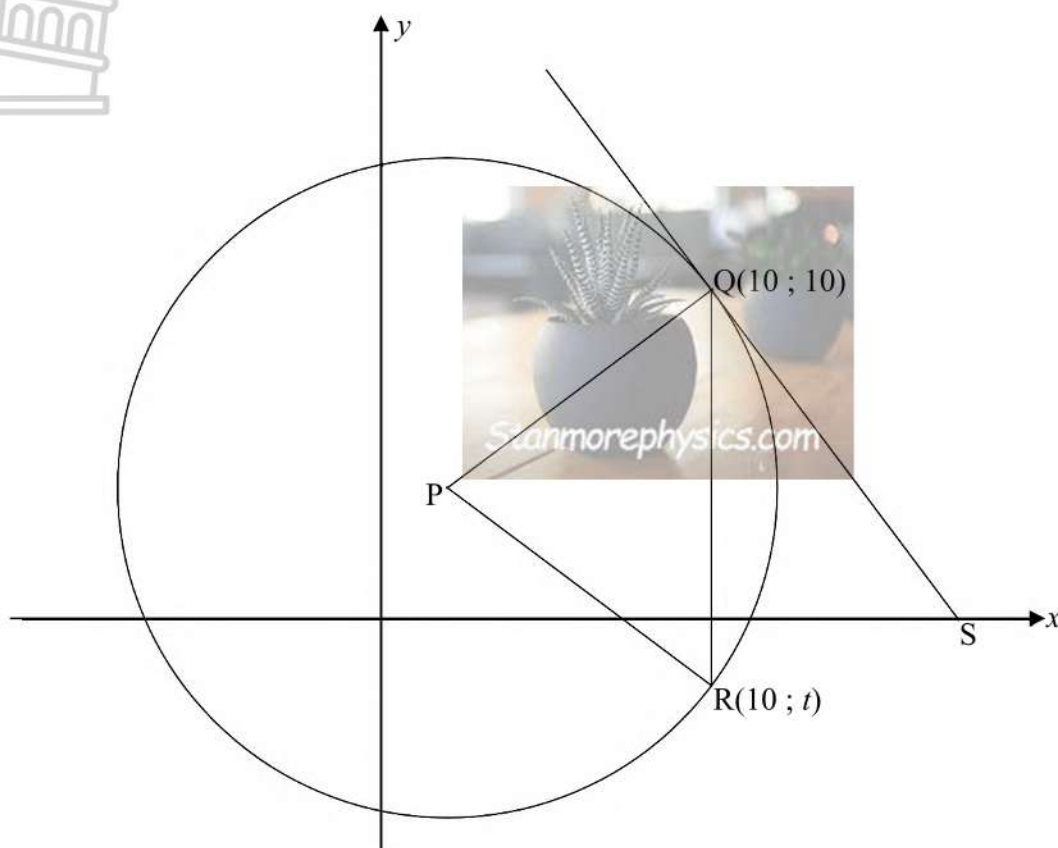


- 3.1 Calculate the:
 - 3.1.1 Coordinates of F (2)
 - 3.1.2 Gradient of DF (2)
 - 3.1.3 Size of α (2)
- 3.2 Write down the equation of OB . (1)
- 3.3 Give a reason why $DF \parallel OB$. (1)
- 3.4 It is given that $PQRS$ is a square with an area of $9x^2$ squared units. Calculate the coordinates of S . (6)
- 3.5 Prove that $EDOS$ forms a parallelogram. (4)

[18]

QUESTION 4

In the diagram below, the circle with centre P has the equation $x^2 - 4x + y^2 - 8y = 80$. QS is a tangent that touches the circle at Q and intersects the x -axis at S. The point R(10 ; t) lies on the circle.

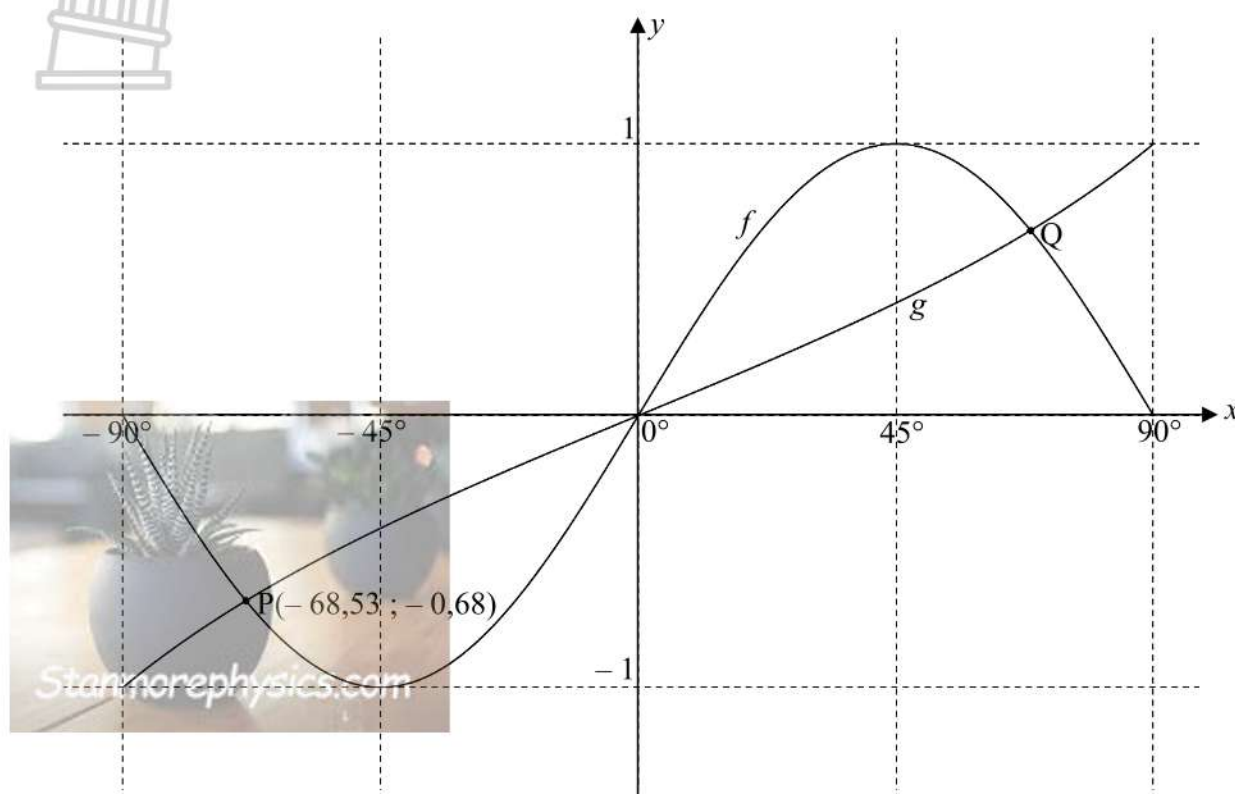


- 4.1 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (2)
- 4.2 Write down the:
 - 4.2.1 Coordinates of P (2)
 - 4.2.2 Equation of QR (1)
- 4.3 Determine the equation of the tangent QS. (5)
- 4.4 Calculate the size of $\hat{RQS}$. (3)
- 4.5 Calculate the area of $\triangle PQR$. (4)
- 4.6 A function h is formed by restricting the range of the circle to $y \geq 4$.
If $\sum_{x=-8}^2 h(x) = k$, determine the value of $\sum_{x=3}^{12} h(x)$ in terms of k . (4)

[21]

QUESTION 5

In the diagram below, the graphs of $f(x) = a \sin 2x$ and $g(x) = \tan bx$ for $x \in [-90^\circ; 90^\circ]$ are drawn. $P(-68,53; -0,68)$ and Q are points of intersection of f and g .



5.1 Write down the:

5.1.1 Value of a (1)

5.1.2 Coordinates of Q (2)

5.1.3 x values of the turning points of h , if $h(x) = f(x + 30^\circ)$ (2)

5.1.4 Value(s) of x where $-0,68 < g(x) \leq 1$ (2)

5.1.5 Value of m where $f(x + m) = -\cos 2x$ (2)

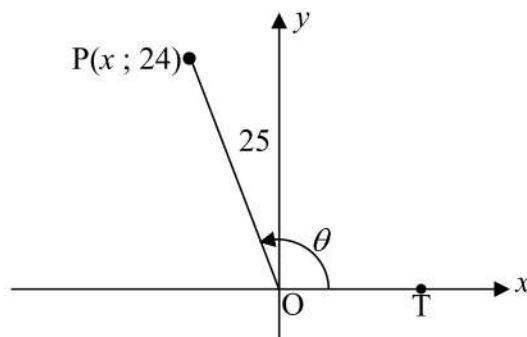
5.1.6 Value of b (1)

5.2 For which value(s) of x , in the given interval, will $x \cdot \sqrt{g(x) - f(x)} > 0$? (2)

[12]

QUESTION 6

- 6.1 In the diagram below, the point $P(x; 24)$ is 25 units from the origin O . T is a point on the x -axis such that $\hat{TOP} = \theta$.

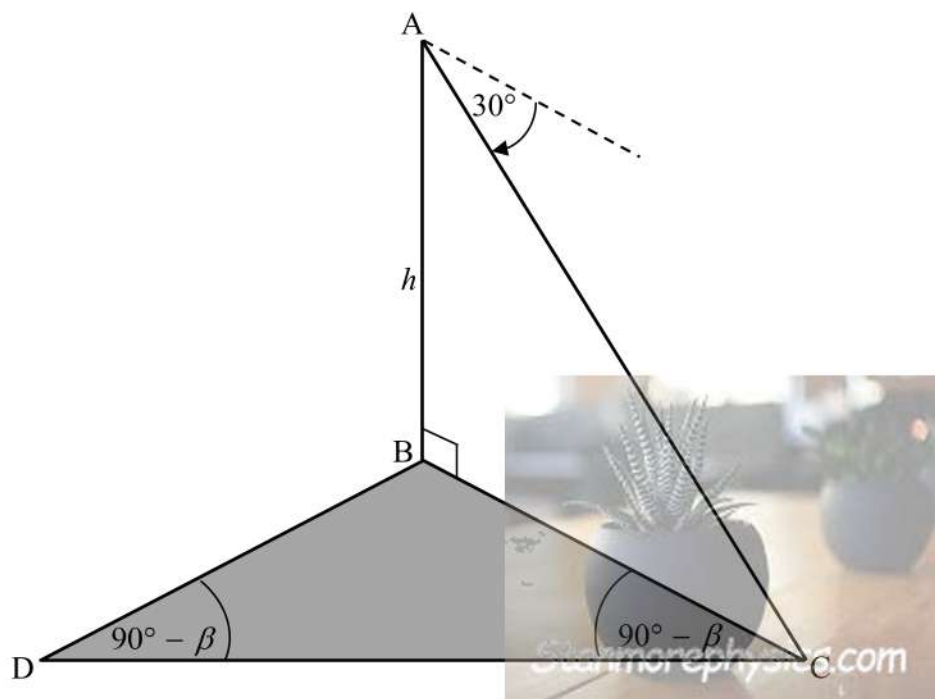


- 6.1.1 Calculate the value of x . (2)
- 6.1.2 **Without using a calculator**, determine the value of $\tan(360^\circ - \theta)$ (2)
- 6.1.3 Calculate the size of $\hat{POT}$ (2)
- 6.2 **Without using a calculator**, calculate the value of the following expressions:
- 6.2.1 $\sin 20^\circ + \cos 120^\circ \cdot \tan 405^\circ + \cos 110^\circ$ (4)
- 6.2.2
$$\frac{(\sqrt{2} \cos 15^\circ + 1)(\sqrt{2} \cos 15^\circ - 1) \sin(-2x)}{4 \sin x \cos x}$$
 (4)
- 6.3 If $\sin(x + y) \cdot \cos(x + y) = t$, express the following in terms of t :
 $4 \cos(90^\circ - 2y) \cdot \cos 2x + 4 \sin 2x \cdot \cos(360^\circ + 2y)$ (5)
- 6.4 Given: $\sin^2 x + \cos^2 x + \tan^2 x$
- 6.4.1 Prove that $\sin^2 x + \cos^2 x + \tan^2 x = \frac{1}{\cos^2 x}$ (3)
- 6.4.2 Attie, a grade 12 learner, argues that $\sqrt{\sin^2 x + \cos^2 x + \tan^2 x} \neq \frac{1}{\cos x}$, if $x \in (180^\circ; 270^\circ)$.
 Is Attie's argument correct? Motivate your answer. (2)
- 6.5 Given: $2^{2 \sin^2 x} - 5 \cdot 2^{\cos 2x} = -3$, $x \in (0^\circ; 90^\circ)$
 Show, **without using a calculator**, that $\sin x = \frac{1}{\sqrt{2}}$. (6)

[30]

QUESTION 7

In the diagram, D, B and C lie in the same horizontal plane. AB is a vertical tower. The angle of depression from A to C is 30° . $\hat{BCD} = \hat{BDC} = 90^\circ - \beta$ and $AB = h$ metres.



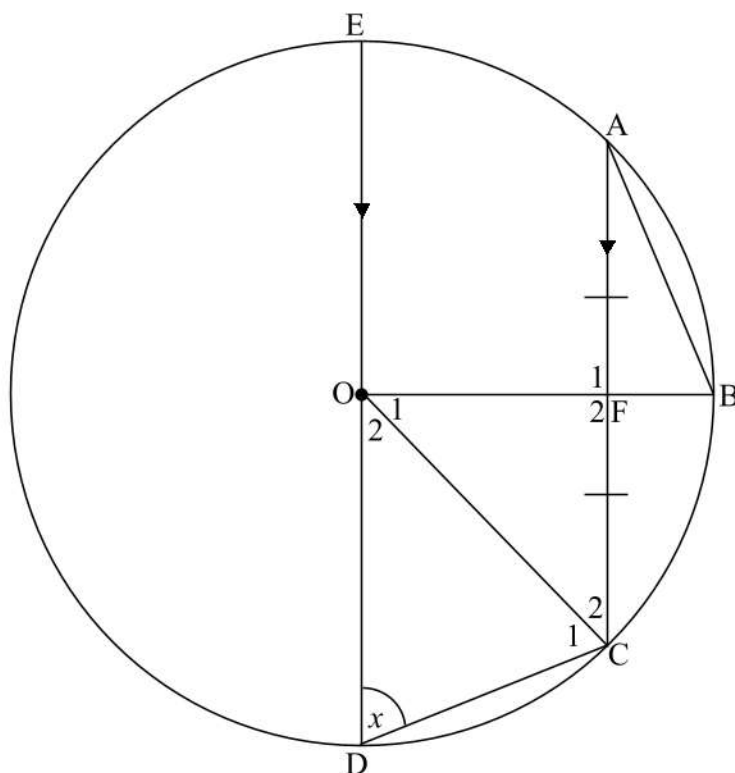
- 7.1 Determine the length of BC in terms of h . (2)
- 7.2 Write down the size of $\hat{DBC}$ in terms of β . (1)
- 7.3 Show that $DC = \sqrt{12} h \sin \beta$ (5)
- [8]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

DE is the diameter of the circle having centre O. Points A, B and C lie on the circle.

$AC \parallel DE$, $AF = FC$ and $\angle ODC = x$.



8.1 Calculate the size of $\hat{F}_2$. (2)

8.2 Determine in terms of x , giving reasons, the size of the following angles:

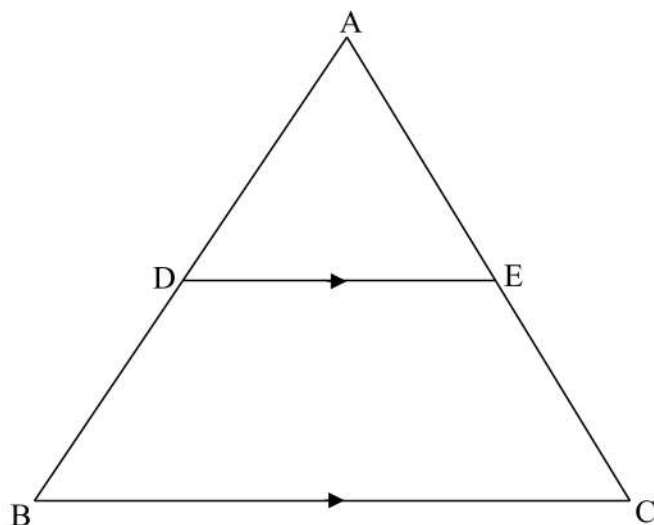
8.2.1 $\hat{O}_2$ (2)

8.2.2 $\hat{B}$ (6)

[10]

QUESTION 9

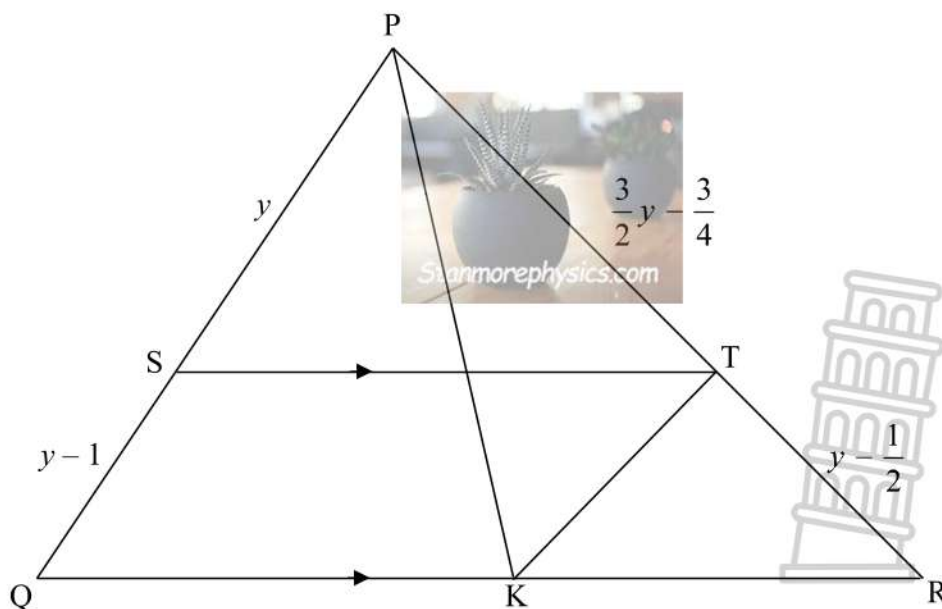
9.1 In the diagram is $\triangle ABC$ with $DE \parallel BC$.



Prove the theorem that states that $\frac{AD}{DB} = \frac{AE}{EC}$. (5)

9.2 In the diagram is $\triangle PQR$ with $ST \parallel QR$. K is a point on QR and is joined to T and P.

$PS = y$, $SQ = y - 1$, $TR = y - \frac{1}{2}$ and $PT = \frac{3}{2}y - \frac{3}{4}$ units.



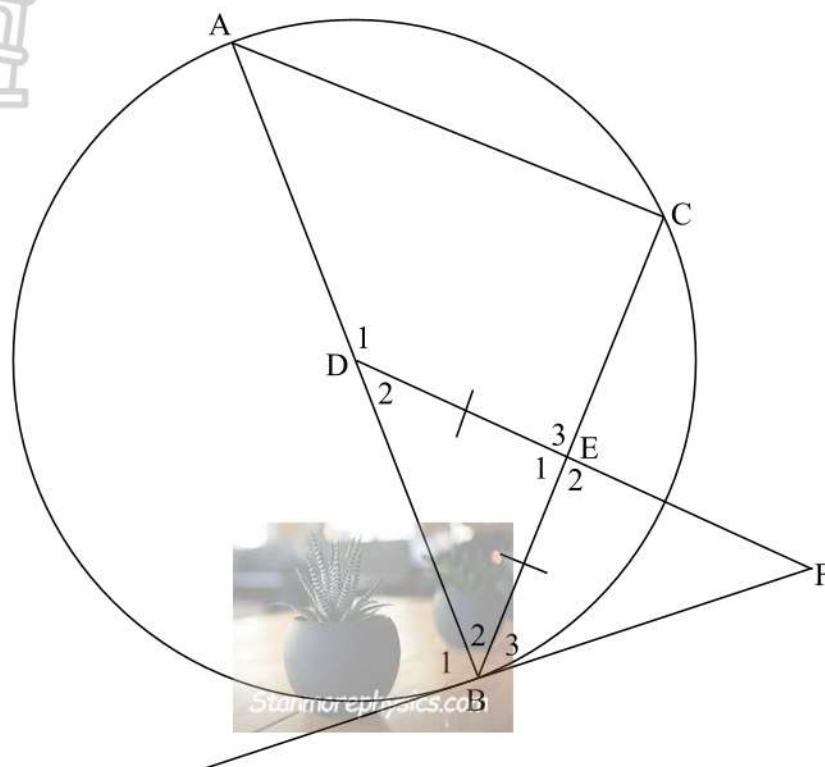
9.2.1 Calculate, with reasons, the value of y . (5)

9.2.2 If $\hat{PKT} = Q$, prove that PSKT is a cyclic quadrilateral. (2)

[12]

QUESTION 10

In the diagram, D is the centre of the circle. BF is a tangent that touches the circle at B. C lies on the circle to form chords AC and BC. D is joined with F and $BE = DE$.



10.1 Prove that:

10.1.1 $\hat{C} = \hat{ABF}$ (4)

10.1.2 $DF \cdot BC = AB \cdot BD$ (4)

10.1.3 E is the midpoint of the circle that passes through the points D, B and F. (4)

10.2 Prove that: $\frac{1}{BC^2 - AB^2} = -\frac{AD \cdot FB}{EF \cdot AC^3}$ (6)

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{n}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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LEARNER'S NAME/NAAM VAN LEERDER

NATIONAL SENIOR CERTIFICATE/NASIONALE SENIORCERTIFIKAAT

MATHEMATICS P2/WISKUNDE V2

GRADE 12/GRAAD 12

SEPTEMBER 2024

**SPECIAL ANSWER BOOK/
SPESIALE ANTWOORDEBOEK**

QUESTION/ VRAAG	MARK/ PUNT	INITIAL/ PARAAF	MODERATOR
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
TOTAL/ TOTAAL			

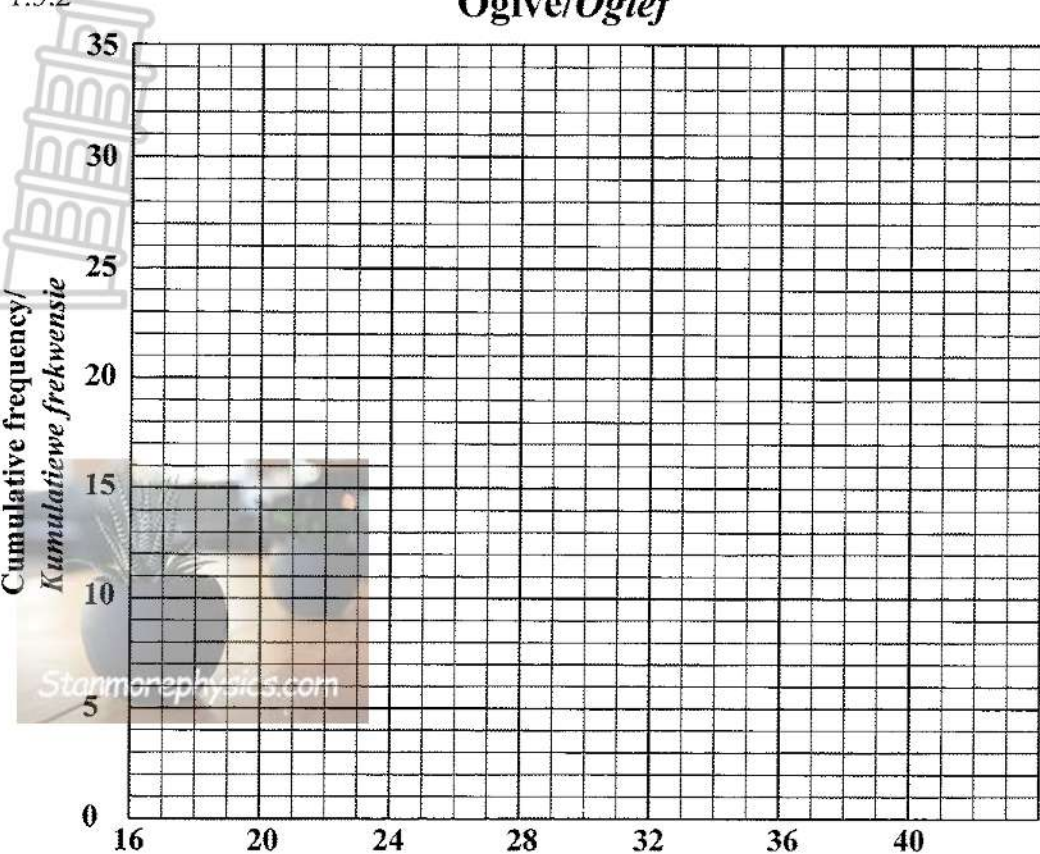
**This answer book consists of 20 pages. /
Hierdie antwoordeboek bestaan uit 20 bladsye.**

FOLLOW THESE INSTRUCTIONS CAREFULLY/ VOLG HIERDIE INSTRUKSIES NOUKEURIG

1. Clearly write your name in the space provided./ Skryf jou naam in die ruimte soos verskaf.
2. NO pages may be torn from this answer book./ GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.
3. Answer ALL questions in the spaces provided./ Beantwoord ALLE vrae in die ruimtes wat voorsien is.
4. Read the instructions given in the examination paper carefully./ Lees die instruksies wat in die eksamenvraestel gegee word, sorgvuldig deur.
5. Candidates may NOT retain any answer book or remove them from the examination room./ Kandidate mag GEEN antwoordeboek behou of uit die eksamenlokaal verwyder NIE.
6. Answers must be written in black/blue ink as distinctly as possible./ Skryf die antwoorde so duidelik moontlik met swart/blou ink.
7. Draw a neat line through any work/rough work that must NOT be marked./ Trek 'n netjiese lyn deur enige werk/rofwerk wat NIE nagesien moet word NIE.

QUESTION/VRAAG 1

	Solution/Oplissing	Marks/ Punte																		
1.1		(1)																		
1.2		(2)																		
1.3.1	<table border="1"> <thead> <tr> <th>Age class <i>Ouderdomklas</i></th><th>Frequency <i>Frekwensie</i></th><th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th></tr> </thead> <tbody> <tr> <td>$20 < x \leq 24$</td><td>2</td><td></td></tr> <tr> <td>$24 < x \leq 28$</td><td>8</td><td></td></tr> <tr> <td>$28 < x \leq 32$</td><td>15</td><td></td></tr> <tr> <td>$32 < x \leq 36$</td><td>6</td><td></td></tr> <tr> <td>$36 < x \leq 40$</td><td>2</td><td></td></tr> </tbody> </table>	Age class <i>Ouderdomklas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$20 < x \leq 24$	2		$24 < x \leq 28$	8		$28 < x \leq 32$	15		$32 < x \leq 36$	6		$36 < x \leq 40$	2		(2)
Age class <i>Ouderdomklas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																		
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$28 < x \leq 32$	15																			
$32 < x \leq 36$	6																			
$36 < x \leq 40$	2																			

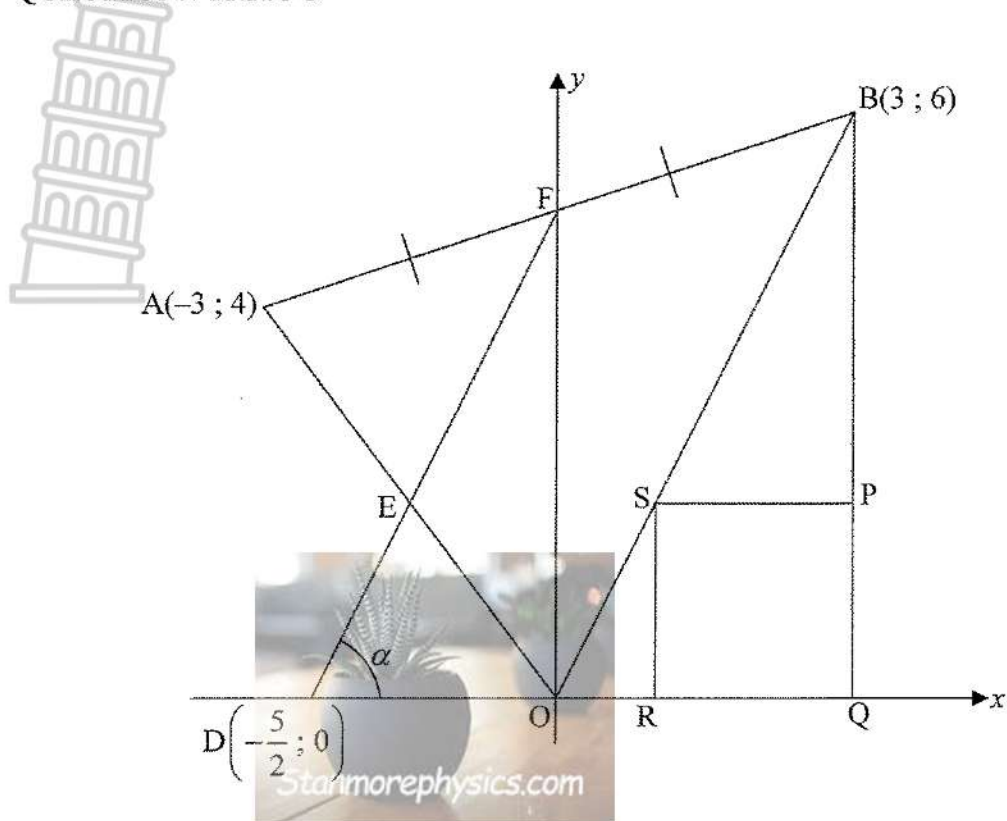
1.3.2	Ogive/Ogief 	(3)						
1.4	<table border="1" style="width: 100%; height: 100%;"> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> </table>					(2)		
1.5	<table border="1" style="width: 100%; height: 100%;"> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> <tr><td> </td></tr> </table>							(3)
[13]								

QUESTION/VRAAG 2

Preparatory examination (x)/ Voorbereidende eksamen (x)	38	65	78	23	67	93	39	83	51	66
Final examination (y)/ Finale eksamen (y)	57	72	81	27	59	94	41	85	54	79

	Solution/Oplissing	Marks/ Punte
2.1		(3)
2.2.1		(2)
2.2.2		(2)
2.3		(1)
		[8]

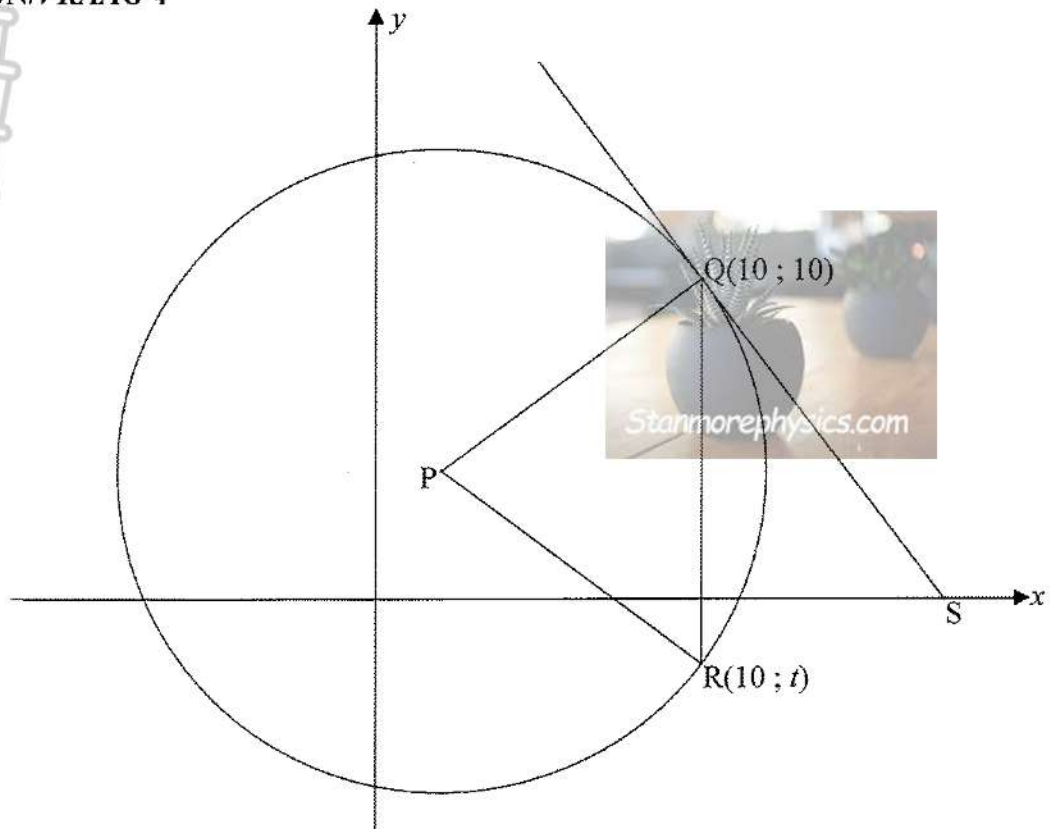
QUESTION/VRAAG 3



	Solution/Oplissing	Marks/ Punte
3.1.1		(2)
3.1.2		(2)
3.1.3		(2)
3.2		(1)

	Solution/Oplossing	Marks/ Punte
3.3		(1)
3.4		(6)
		(6)
3.5		(4)
		(4)
		[18]

QUESTION/VRAAG 4

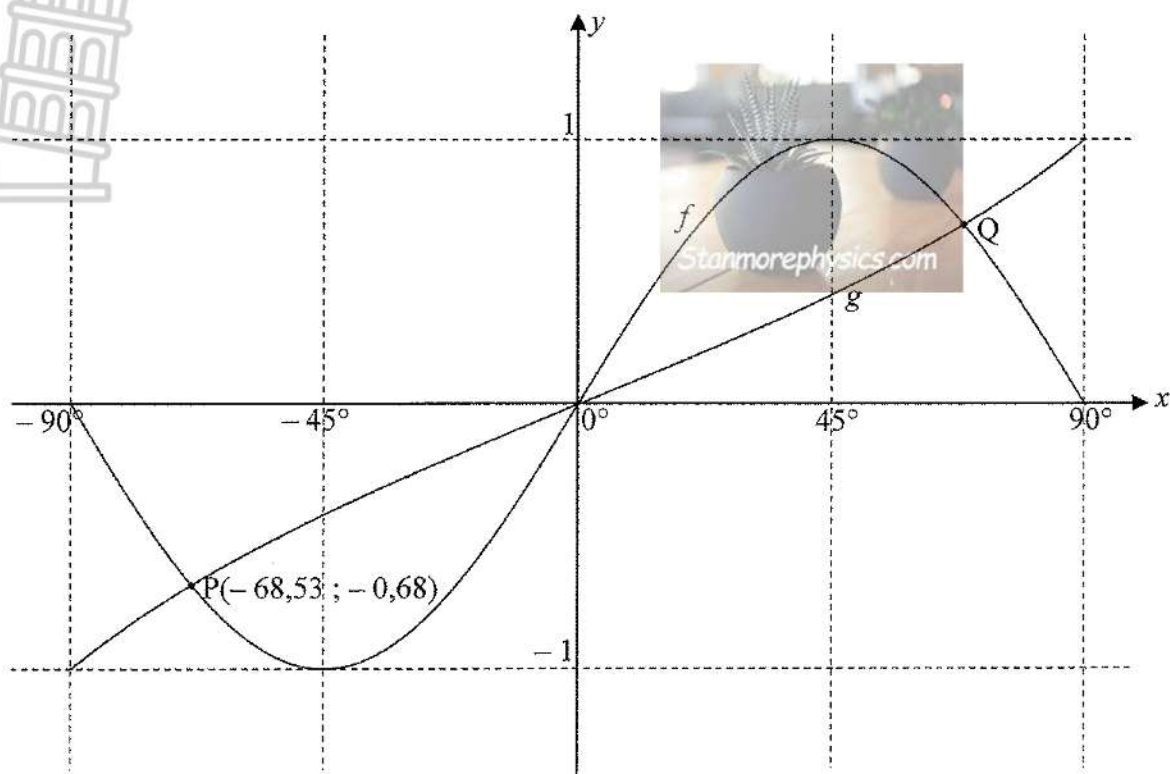


	Solution/Oplissing	Marks/ Punte
4.1		(2)
4.2.1		(2)
4.2.2		(1)



	Solution/Oplissing	Marks/ Punte
4.3		(5)
4.4		(3)
4.5		(4)
4.6		(4)
		[21]

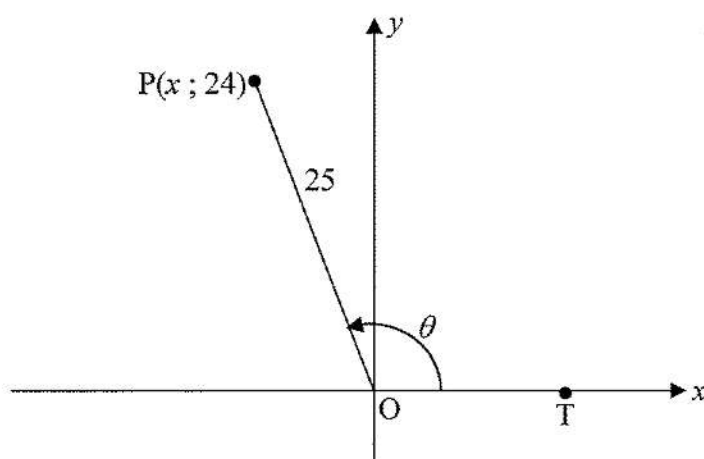
QUESTION/VRAAG 5



	Solution/Oplissing	Marks/ Punte
5.1.1		(1)
5.1.2		(2)
5.1.3		(2)
5.1.4		(2)
5.1.5		(2)

5.1.6		
		(1)
5.2		
		(2)
		[12]

QUESTION/VRAAG 6

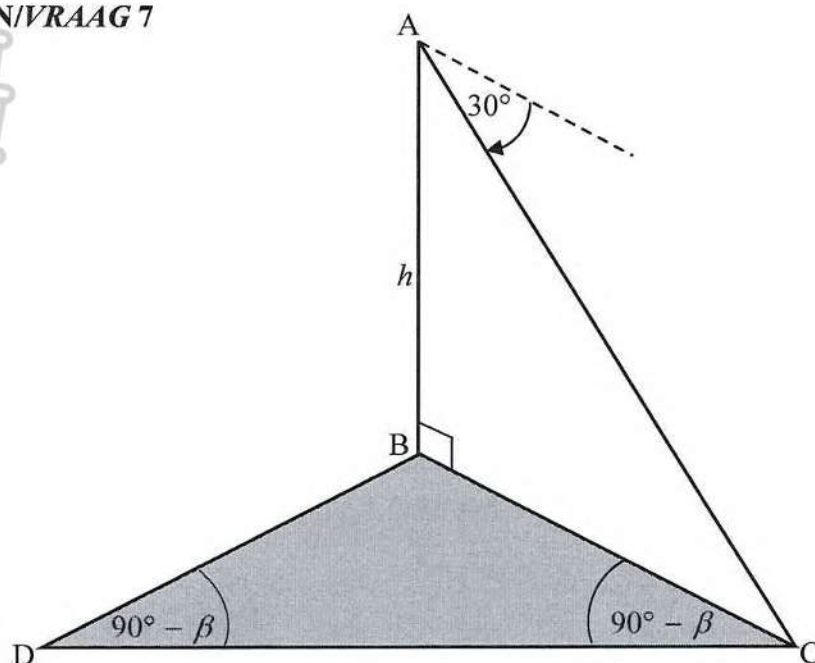


	Solution/Oplissing	Marks/ Punte
6.1.1		
		(2)
6.1.2		
		(2)

	Solution/Oplissing	Marks/ Punte
6.1.3		
		(2)
6.2.1		
		(4)
6.2.2		
		(4)
6.3		
		(5)

	Solution/Oplissing	Marks/ Punte
6.4.1		(3)
6.4.2		(2)
6.5		(6)
		[30]

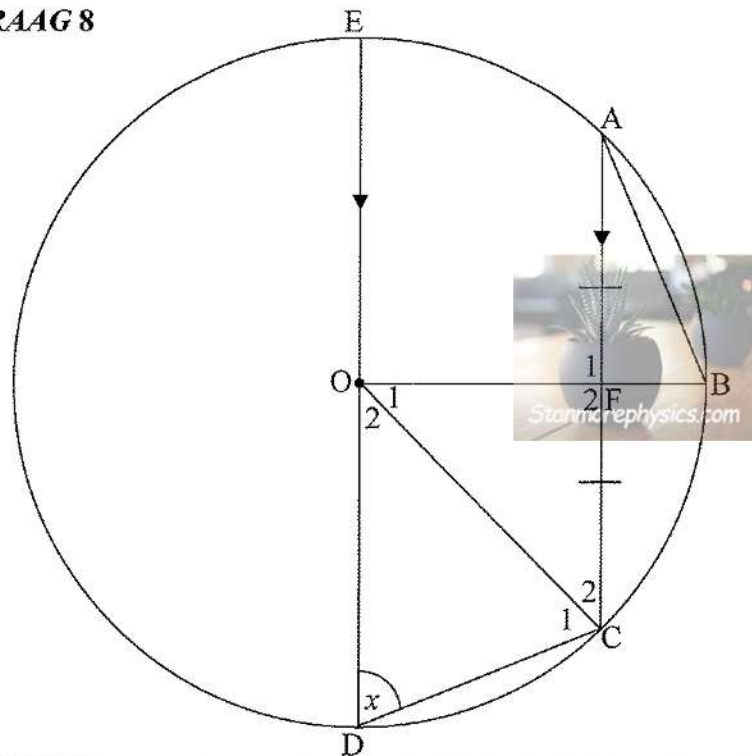
QUESTION/VRAAG 7



	Solution/Oplissing	Marks/ Punte
7.1		(2)
7.2		(1)
7.3		(5)
		[8]



QUESTION/VRAAG 8



	Solution/Oplissing	Marks/ Punte
8.1		(2)
8.2.1		(2)
8.2.2		(6)
		[10]

9.1

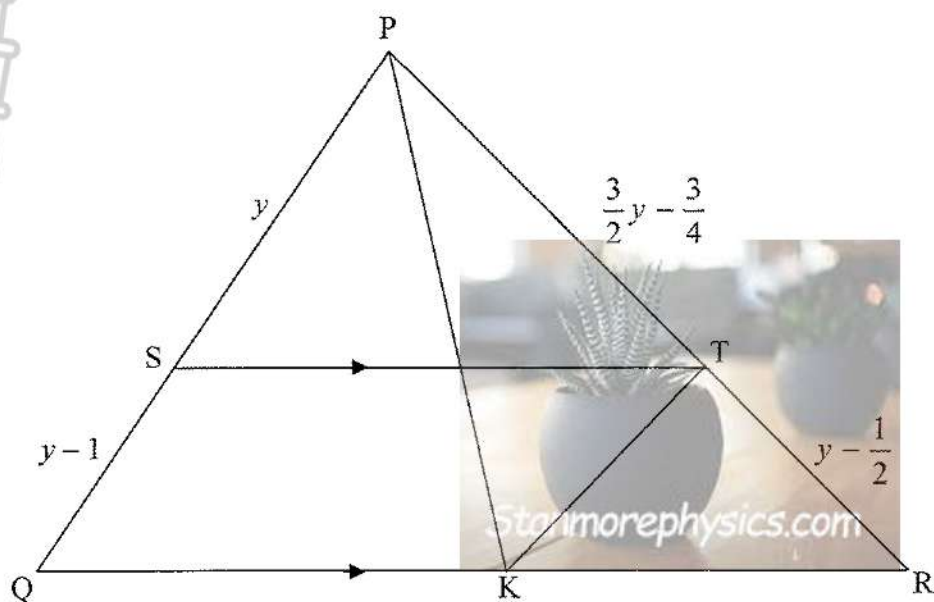
9.1



(5)

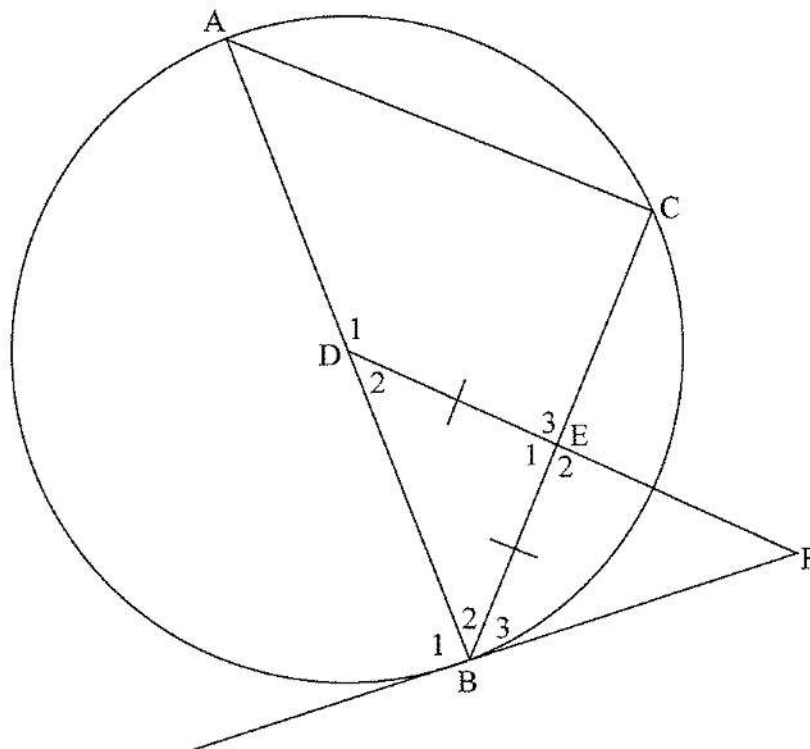


9.2



	Solution/Oplossing	Marks/ Punte
9.2.1		(5)
9.2.2		(2)
		[12]

QUESTION /VRAAG 10



	Solution/Oplissing	Marks/ Punte
10.1.1		(4)
10.1.2		(4)



10.1.3		
10.2		(4)
		(6)
		[18]







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GRADE/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

SEPTEMBER 2024

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150



**These marking guidelines consist of 19 pages and 3 pages containing the cognitive grid./
Hierdie nasienriglyne bestaan uit 19 bladsye en 3 bladsye wat die kognitiewe tabel bevat.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.
- Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.

QUESTION/VRAAG 1

1.1	33 players/spelers	✓ 33 (1)																		
1.2	$\bar{x} = \frac{(2 \times 22) + (8 \times 26) + (15 \times 30) + (6 \times 34) + (2 \times 38)}{33}$ $= \frac{982}{33}$ $= 29,76$ Answer only: full marks	✓ 982 ✓ answer/antwoord (CA if/as ÷ 33) (2)																		
1.3.1	<table border="1"> <thead> <tr> <th>Age class <i>Ouderdomklas</i></th><th>Frequency <i>Frekwensie</i></th><th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th></tr> </thead> <tbody> <tr> <td>$20 < x \leq 24$</td><td>2</td><td>2</td></tr> <tr> <td>$24 < x \leq 28$</td><td>8</td><td>10</td></tr> <tr> <td>$28 < x \leq 32$</td><td>15</td><td>25</td></tr> <tr> <td>$32 < x \leq 36$</td><td>6</td><td>31</td></tr> <tr> <td>$36 < x \leq 40$</td><td>2</td><td>33</td></tr> </tbody> </table>	Age class <i>Ouderdomklas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$20 < x \leq 24$	2	2	$24 < x \leq 28$	8	10	$28 < x \leq 32$	15	25	$32 < x \leq 36$	6	31	$36 < x \leq 40$	2	33	✓ 2 & 10 ✓ 33 (2)
Age class <i>Ouderdomklas</i>	Frequency <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																		
$20 < x \leq 24$	2	2																		
$24 < x \leq 28$	8	10																		
$28 < x \leq 32$	15	25																		
$32 < x \leq 36$	6	31																		
$36 < x \leq 40$	2	33																		

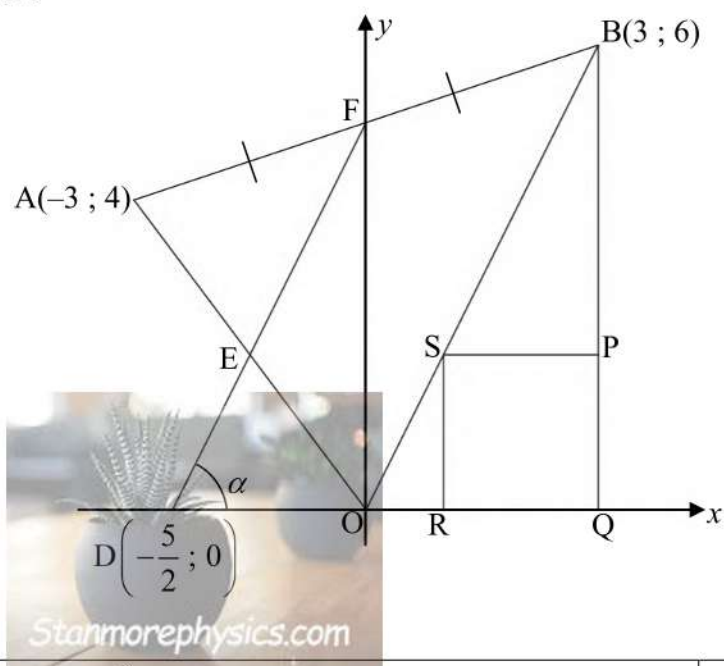
1.3.2	Ogive/Ogief	✓ grounding/anker (20 ; 0) ✓ plotting ogive at upper limits correctly/korrekte plot van ogief by boonste limiete ✓ shape (smooth)/ vorm (vryhand)
1.4	On graph the y-value at 16,5 / y-waarde van 16,5 op die grafiek. median / mediaan $\approx$ 29,8 years / jare (accept values/aanvaar waardes: $29,4 \leq x < 30$)	✓ graph/grafiek ✓ 29,8
1.5	The affected intervals are/ die geaffekteerde intervale is: $24 < x \leq 28$ and / en $32 < x \leq 36$. If the frequency of both intervals are increased to 9 players, the data will be symmetrical/as die frekwensie van beide intervale tot 9 spelers verhoog word, is die data simmetries. $\therefore k = 4$ Answer only: full marks	✓ identify affected intervals/ identifiseer geaffekteerde intervale ✓ increase frequency to 9 players/verhoog frekwensie na 9 spelers ✓ $k = 4$
		[13]

QUESTION/VRAAG 2

Preparatory examination (x)/ Voorbereidende eksamen (x)	38	65	78	23	67	93	39	83	51	66
Final examination (y)/ Finale eksamen (y)	57	72	81	27	59	94	41	85	54	79

2.1	$a = 10,85$ $b = 0,90$ $\hat{y} = 10,85 + 0,9x$	Answer only: full marks, but if a and b are swopped only 1/3 marks/ maar as a en b omgeruil is, slegs 1/3 punte.	✓ a ✓ b ✓ equation/ vergelyking (3)
2.2.1	$\hat{y} = 10,85 + 0,9(46)$ $= 52,25\%$ OR/OF $46\hat{y} = 52,08\%$ (calculator / sakrekenaar)	Answer only: full marks	✓ substitute 46 into eq. / vervang 46 in vgl. ✓ answer/antwoord (2) ✓✓ 52,08 % (2)
2.2.2	$r = 0,94$ Yes, because there is a very strong correlation between the data/ Ja, want die korrelasie tussen die data is baie sterk.		✓ yes/ja ✓ very strong/baie sterk (2)
2.3	B		✓ B (1)
			[8]

QUESTION/VRAAG 3



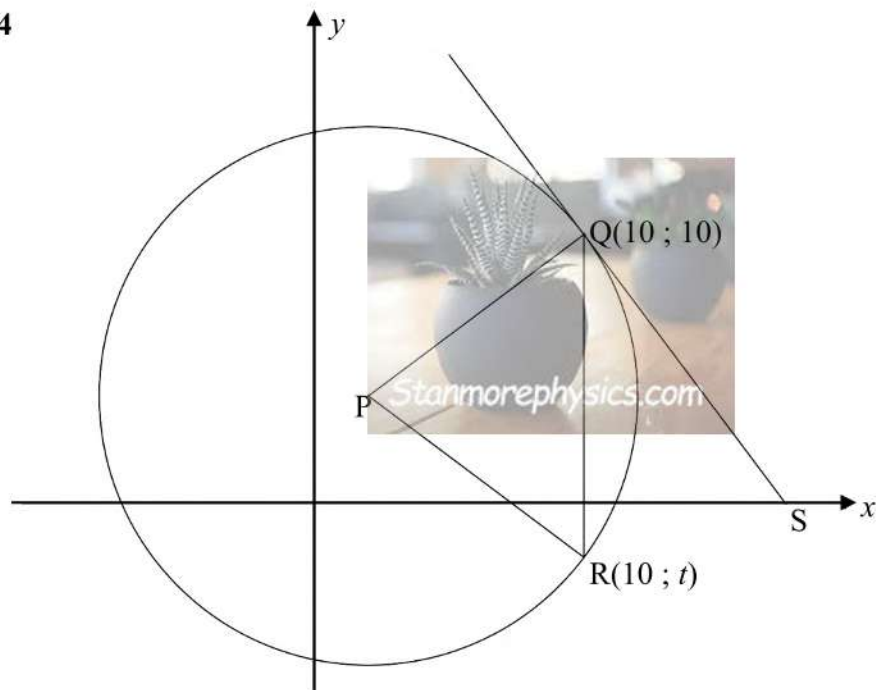
3.1.1	$F\left(\frac{-3+3}{2}; \frac{4+6}{2}\right)$ $= F(0; 5)$	✓ x-value/waarde ✓ y-value/waarde (2)
3.1.2	$m_{DF} = \frac{5-0}{0-\left(-\frac{5}{2}\right)}$ $= 2$ Answer only: full marks	✓ subst./vervang ✓ answer/antwoord (2)
3.1.3	$\tan \alpha = 2$ $\therefore \alpha = 63,43^\circ$	✓ $\tan \alpha = m_{DF}$ ✓ answer/antwoord (2)
3.2	$y = 2x$	✓ answer/antwoord (1)
3.3	$m_{DF} = m_{OB} = 2$ $\therefore$ Lines have the same gradient/lyne het dieselfde gradiënte	✓ answer/antwoord (1)

3.4	$QR = PQ = 3x$ $\therefore R(3 - 3x; 0)$ $QP = 3x$ $\therefore S(3 - 3x; 3x)$ Equation / vergelyking : OB $y = 2x$ $3x = 2(3 - 3x)$ $3x = 6 - 6x$ $x = \frac{2}{3}$ Substitute/vervang $S\left(3 - 3\left(\frac{2}{3}\right); 3\left(\frac{2}{3}\right)\right)$ $\therefore S(1; 2)$ OR/OF $SR = 3x$ $\tan \hat{SOR} = \tan 63,43^\circ = \frac{SR}{OR}$ $OR = \frac{3x}{2}$ $OQ = 3 \text{ units / eenhede}$ $RQ = 3x$ $\frac{3x}{2} + 3x = 3$ $x = \frac{2}{3}$ $S(3 - 3x; 3x)$ $S\left(3 - 3\left(\frac{2}{3}\right); 3\left(\frac{2}{3}\right)\right)$ $S(1; 2)$	$\checkmark QR = PQ = 3x$ $\checkmark$ coordinates of/ <i>koördinate van R</i> $\checkmark$ coordinates of/ <i>koördinate van S</i> $\checkmark$ value of/ <i>waarde van x</i> $\checkmark x_s$-value/waarde $\checkmark y_s$-value/waarde (6) $\checkmark SR = 3x$ $\checkmark OR$ $\checkmark$ value of/ <i>waarde van x</i> $\checkmark$ coordinates of/ <i>koördinate van S</i> $\checkmark x_s$-value/waarde $\checkmark y_s$-value/waarde (6)
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3.5	E is midpt. of / van $\parallel$ AO $\left[\begin{array}{l} \text{converse midpt.theorem /} \\ \text{omgekeerde midpt. – stelling} \end{array} \right]$ $E\left(\frac{-3}{2}; 2\right)$ $ES \parallel DO$ (same y – coordinates / <i>dieselfde y – koördinate</i>) $\therefore$ EDOS is $\parallel^m$ [both pairs opp. sides $\parallel$ / <i>beide pare teenoorst. sye $\parallel$</i>] OR/OF E is midpt. of / van $\parallel$ AO $\left[\begin{array}{l} \text{converse midpt.theorem /} \\ \text{omgekeerde midpt. – stelling} \end{array} \right]$ $E\left(\frac{-3}{2}; 2\right)$ Midpt. EO = $\left(\frac{-3}{4}; 1\right)$ & Midpt. DS = $\left(\frac{-3}{4}; 1\right)$ $\therefore$ EDOS is $\parallel^m$ [converse diag. of/omgekeerde hoeklyne van $\parallel^m$]	✓S ✓coordinates of E/ <i>koördinate van E</i> ✓S/R ✓reason/rede (4)
		(4) [18]



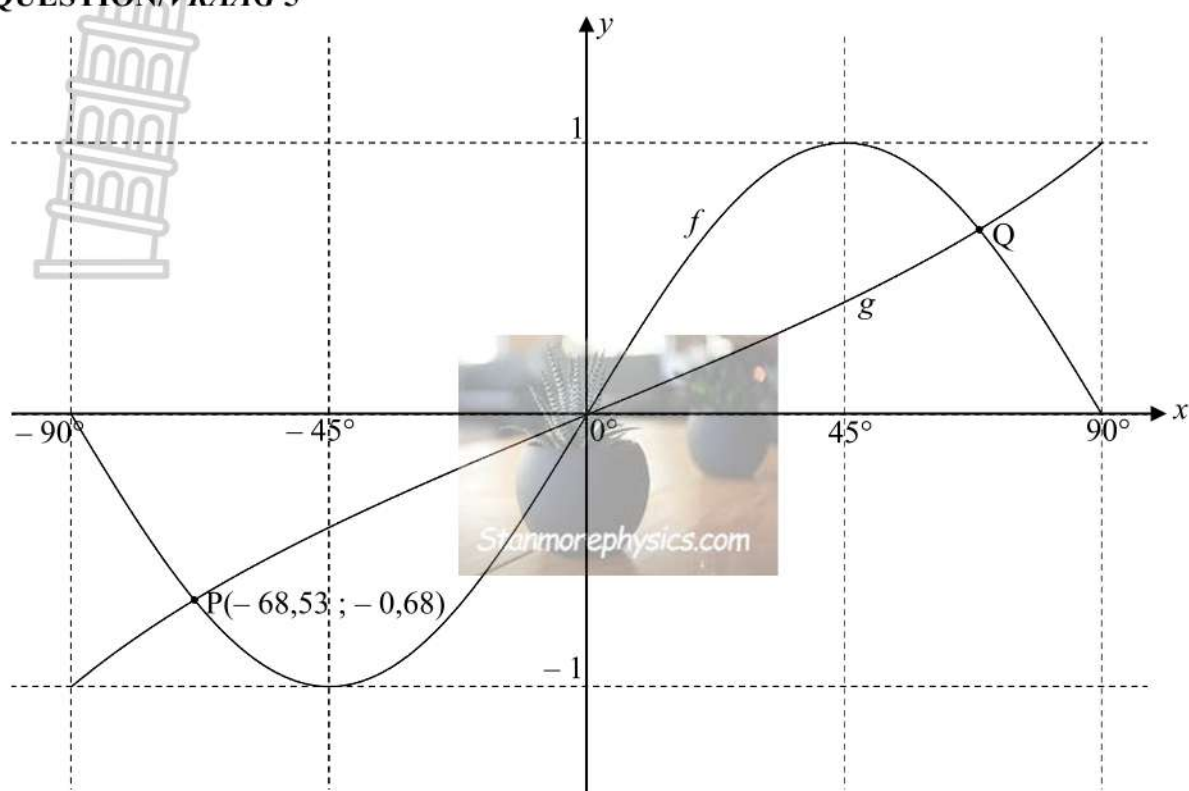
QUESTION/VRAAG 4



4.1	$x^2 - 4x + (-2)^2 + y^2 - 8y + (-4)^2 = 80 + (-2)^2 + (-4)^2$ $(x - 2)^2 + (y - 4)^2 = 100$	✓LHS/LK ✓RHS/RK (2)
4.2.1	P(2 ; 4)	✓ $x = 2$ ✓ $y = 4$ (2)
4.2.2	$x = 10$	✓equation/vergelyking (1)
4.3	$m_{QP} = \frac{10-4}{10-2}$ $= \frac{3}{4}$ $\therefore m_{QS} = -\frac{4}{3} \text{ [rad } \perp \text{ tangent / rad } \perp \text{ raaklyn]}$ $10 = -\frac{4}{3}(10) + c \quad \text{OR / OF} \quad y - 10 = -\frac{4}{3}(x - 10)$ $c = \frac{70}{3}$ $y = -\frac{4}{3}x + \frac{70}{3}$	✓subst./vervang $\checkmark m_{QP} = \frac{3}{4}$ $\checkmark m_{QS} = -\frac{4}{3}$ ✓subst./vervang m & (10 ; 10) ✓equation/vergelyking (5)

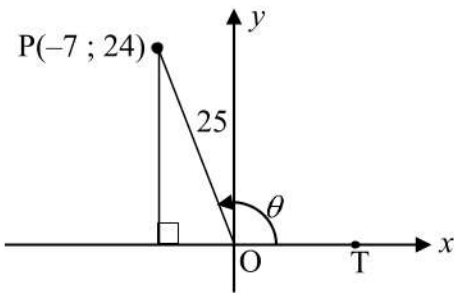
4.4	$\tan \hat{Q}\hat{S}x = -\frac{4}{3}$ $\hat{Q}\hat{S}x = 126,87^\circ$ $126,87^\circ = \hat{R}\hat{Q}S + 90^\circ$ [ext $\angle$ of Δ / buite $\angle$ van Δ] $\hat{R}\hat{Q}S = 36,87^\circ$	✓ $\hat{Q}\hat{S}x$ ✓ method/metode ✓ answer/antwoord (3)
4.5	$\hat{P}\hat{Q}R = 53,13^\circ$ [radius $\perp$ tangent / raaklyn] $\hat{P}\hat{Q}R = \hat{Q}\hat{R}P = 53,13^\circ$ [$\angle$ s opp. = radii / $\angle$ e teenoor = radiusse] $\hat{Q}\hat{P}R = 73,74^\circ$ [$\angle$ s of Δ / $\angle$ e van Δ] $\text{area } \Delta PQR = \frac{1}{2}(PQ)(PR)\sin \hat{Q}\hat{P}R$ $= \frac{1}{2}(10)(10)\sin 73,74^\circ$ $= 48 \text{ units}^2 / \text{eenhede}^2$ OR/OF $\hat{P}\hat{Q}R = 53,13^\circ$ [radius $\perp$ tangent / raaklyn] Q and R are symmetrical about the line $y = 4$ / <i>Q en R is simmetries rondom die lyn $y = 4$</i> $\therefore t = -2$ & $QR = 12$ $\text{area } \Delta PQR = \frac{1}{2}(PQ)(QR)\sin \hat{P}\hat{Q}R$ $= \frac{1}{2}(10)(12)\sin 53,13^\circ$ $= 48 \text{ units}^2 / \text{eenhede}^2$	✓ $\hat{P}\hat{Q}R$ ✓ $\hat{Q}\hat{P}R$ ✓ subst. in area rule correctly/vervang korrek in oppv.-reël ✓ answer/antwoord (4) ✓ $\hat{P}\hat{Q}R$ ✓ QR ✓ subst. in area rule correctly/vervang korrek in oppv.-reël ✓ answer/antwoord (4)
4.6	$\sum_{x=-8}^2 h(x) = \sum_{x=2}^{12} h(x) = k$ $h(2) = 14$ $\sum_{x=3}^{12} h(x) = \sum_{x=2}^{12} h(x) - h(2)$ $\therefore \sum_{x=3}^{12} h(x) = k - 14$	✓ $\sum_{x=-8}^2 h(x) = \sum_{x=2}^{12} h(x) = k$ ✓ $h(2) = 14$ ✓ $\sum_{x=3}^{12} h(x) = \sum_{x=2}^{12} h(x) - h(2)$ ✓ answer/antwoord (4)
		[21]

QUESTION/VRAAG 5



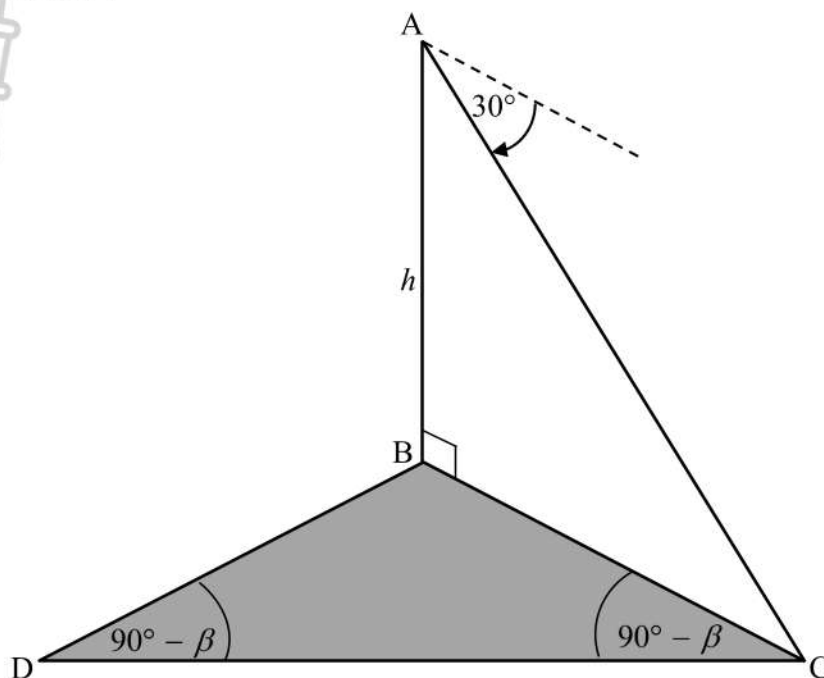
5.1.1	$a = 1$	✓ answer/antwoord (1)
5.1.2	$Q(68,53^\circ; 0,68)$	✓ x -value/waarde ✓ y -value/waarde (2)
5.1.3	$x = -75^\circ$ or / of $x = 15^\circ$	✓ -75° ✓ 15° (2)
5.1.4	$x \in (-68,53^\circ; 90^\circ]$ OR / OF $-68,53^\circ < x \leq 90^\circ$	✓ $-68,53^\circ$ & 90° ✓ notation/notasie (2)
5.1.5	$f(x - 45^\circ)$ $m = -45^\circ$	✓ $f(x - 45^\circ)$ ✓ -45° (2)
5.1.6	$b = \frac{1}{2}$	✓ $\frac{1}{2}$ (1)
5.2	$x \in (68,53^\circ; 90^\circ]$ OR / OF $68,53^\circ < x \leq 90^\circ$	✓ ✓ accuracy/akkuraatheid (2)
		[12]

QUESTION/VRAAG 6

6.1.1	$x^2 + 24^2 = 25^2$ $x^2 = 49$ $x = -7$		$\checkmark x^2 + y^2 = r^2$ $\checkmark x = -7$	(2)
6.1.2	$\tan(360^\circ - \theta)$ $= -\tan \theta$ $= -\left(-\frac{24}{7}\right)$ $= \frac{24}{7}$ Answer only: full marks		$\checkmark -\tan \theta$ $\checkmark \text{answer/antwoord}$	(2)
6.1.3	$\tan \theta = \frac{24}{7}$ $\theta = 73,74^\circ$ $\text{P}\hat{\text{O}}\text{T} = 180^\circ - 73,74^\circ$ $= 106,26^\circ$		$\checkmark \text{size of/grootte van } \theta$ $\checkmark \text{answer/antwoord}$	(2)
6.2.1	$\sin 20^\circ + \cos 120^\circ \cdot \tan 405^\circ + \cos 110^\circ$ $= \sin 20^\circ + \cos(180^\circ - 60^\circ) \cdot \tan(360^\circ + 45^\circ) + \cos(90^\circ + 20^\circ)$ $= \sin 20^\circ - \cos 60^\circ \cdot \tan 45^\circ - \sin 20^\circ$ $= -\frac{1}{2}$		$\checkmark -\cos 60^\circ$ $\checkmark \tan 45^\circ$ $\checkmark -\sin 20^\circ$ $\checkmark \text{answer/antwoord}$	(4)
6.2.2	$\frac{(\sqrt{2} \cos 15^\circ + 1)(\sqrt{2} \cos 15^\circ - 1) \sin(-2x)}{4 \sin x \cos x}$ $= \frac{(2 \cos^2 15^\circ - 1) \cdot (-\sin 2x)}{4 \sin x \cos x}$ $= \frac{(\cos 30^\circ) \cdot (-2 \sin x \cos x)}{4 \sin x \cos x}$ $= -\frac{\sqrt{3}}{4}$		$\checkmark 2 \cos^2 15^\circ - 1$ $\checkmark -2 \sin x \cos x$ $\checkmark \cos 30^\circ$ $\checkmark \text{answer/antwoord}$	(4)

6.3	$4 \cos(90^\circ - 2y) \cdot \cos 2x + 4 \sin 2x \cdot \cos(360^\circ + 2y)$ $= 4 \sin 2y \cdot \cos 2x + 4 \sin 2x \cdot \cos 2y$ $= 4 [\sin(2y + 2x)]$ $= 4 [\sin 2(y + x)]$ $= 4 \cdot 2 \sin(y + x) \cdot \cos(y + x)$ $= 8t$	✓ $\sin 2y$ ✓ $\cos 2y$ ✓ compound angle/ saamgestelde hoek ✓ double angle/dubbelhoek ✓ answer/antwoord (5)
6.4.1	$\text{LHS} / LK = \sin^2 x + \cos^2 x + \tan^2 x$ $= 1 + \tan^2 x$ $= 1 + \frac{\sin^2 x}{\cos^2 x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ $= \frac{1}{\cos^2 x}$	✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\frac{\sin^2 x}{\cos^2 x}$ ✓ $\frac{\cos^2 x + \sin^2 x}{\cos^2 x}$ (3)
6.4.2	If/as $x \in (180^\circ ; 270^\circ)$, then/dan is $\cos x < 0$ $\sqrt{\sin^2 x + \cos^2 x + \tan^2 x} \neq \text{negative} / \text{negatief}$ $\therefore$ Attie is correct / korrek	✓ statement/bewering ✓ answer/antwoord (2)
6.5	$2^{2\sin^2 x} - 5 \cdot 2^{\cos 2x} = -3$ $2^{2\sin^2 x} - 5 \cdot 2^{1 - 2\sin^2 x} = -3$ $2^{2\sin^2 x} - 5 \cdot 2 \cdot 2^{-2\sin^2 x} = -3$ $2^{2\sin^2 x} - \frac{10}{2^{2\sin^2 x}} + 3 = 0$ $2^{2\sin^2 x} \cdot 2^{2\sin^2 x} + 3 \cdot 2^{2\sin^2 x} - 10 = 0$ $(2^{2\sin^2 x} + 5)(2^{2\sin^2 x} - 2) = 0$ $2^{2\sin^2 x} \neq -5 \text{ or / of } 2^{2\sin^2 x} = 2^1$ $2\sin^2 x = 1$ $\sin^2 x = \frac{1}{2}$ $\sin x = \pm \frac{1}{\sqrt{2}}$	✓ $1 - 2\sin^2 x$ ✓ $\frac{10}{2^{2\sin^2 x}}$ ✓ standard form/standaardvorm ✓ factors/faktore ✓ $2^{2\sin^2 x} \neq -5$ ✓ $\sin^2 x = \frac{1}{2}$ (6)
		[30]

QUESTION/VRAAG 7



7.1	$\tan 30^\circ = \frac{h}{BC}$ $BC = \frac{h}{\tan 30^\circ}$ $= \frac{h}{\frac{1}{\sqrt{3}}} = \sqrt{3}h$	✓ correct trig ratio/ <i>korrekte trig verhouding</i> ✓ BC as the subject/ <i>as die onderwerp</i>	(2)
7.2	$\hat{DBC} = 2\beta$	✓ answer/antwoord	(1)
7.3	$\frac{DC}{\sin B} = \frac{BC}{\sin D}$ $\frac{DC}{\sin 2\beta} = \frac{BC}{\sin(90^\circ - \beta)}$ $DC \cdot \cos \beta = BC \cdot 2 \sin \beta \cos \beta$ $DC = \frac{\sqrt{3}h \cdot 2 \sin \beta \cos \beta}{\cos \beta}$ $= 2\sqrt{3}h \sin \beta$ $= \sqrt{12}h \sin \beta$	✓ correct use of sine-rule/ <i>korrekte gebruik van sinus-reël</i> ✓ correct substitution into the formula/ <i>korrekte vervanging in die formule</i> ✓ $\sin 2\beta = 2 \sin \beta \cos \beta$ ✓ co-ratio/ko-verhouding ✓ $2\sqrt{3}h \sin \beta$	(5)
			[8]

GEOMETRY/MEETKUNDE

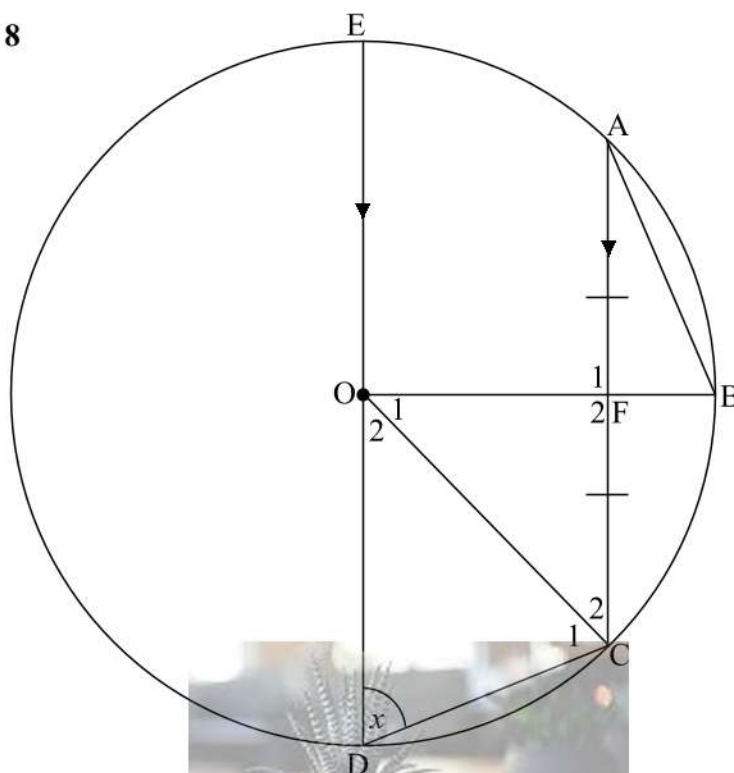
Please read carefully through the following table before marking **QUESTION 8–10** /
 Lees asseblief sorgvuldig deur die volgende tabel alvorens **VRAE 8–10** nagesien word.



	The order in which the candidate answers a geometry question must follow logically/ <i>Die volgorde waarin 'n kandidaat 'n meetkundevraag beantwoord moet logies volg.</i> Example/Voorbeeld Given/Gegee $AB \parallel CD$ and/en $\hat{EFD} = 115^\circ$ The candidate first needs to calculate x BEFORE he/she can calculate y/Die kandidaat moet eerste vir x bereken VOORDAT hy/sy vir y kan bereken.
S	A mark for a correct statement (A statement mark is independent of a reason) <i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct) <i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if the statement AND reason are both correct (Both MUST be correct to get one mark) <i>Ken 'n punt toe as die bewering EN rede beide korrek is (Beide MOET korrek wees om een punt te kry)</i>

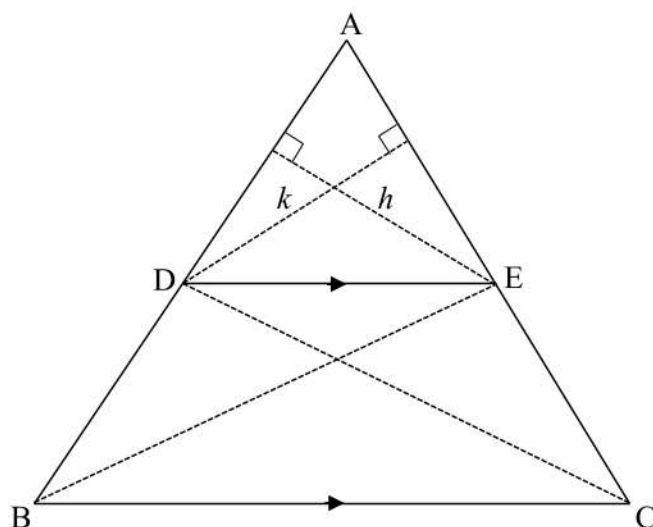


QUESTION/VRAAG 8



8.1	$\hat{F}_2 = 90^\circ$ [line from centre to midpt.chord/midpt.sirkel, midpt.koord]	✓S✓R (2)
8.2.1	$\hat{C}_1 = x$ [$\angle$ s opp. = radii/ $\angle$ e teenoor = radiusse] $\hat{O}_2 = 180^\circ - 2x$ [sum of $\angle$ s of / som van $\angle$ e van $\triangle DOC$]	✓S ✓S (2)
8.2.2	$\hat{DOF} = 90^\circ$ [co-int. $\angle$ s / ko-binne $\angle$ e; $AC \parallel ED$] $\hat{O}_1 = 90^\circ - (180^\circ - 2x) = 2x - 90^\circ$ $\hat{CAB} = x - 45^\circ$ [$\angle$ at centre = $2 \times \angle$ at circ. / midpts $\angle = 2 \times$ omtreks $\angle$] $\hat{B} + x - 45^\circ + 90^\circ = 180^\circ$ [sum of $\angle$ s of / som van $\angle$ e van $\triangle FAB$] $\hat{B} = 135^\circ - x$	✓S/R ✓S ✓S✓R ✓S/R ✓S (6) [10]

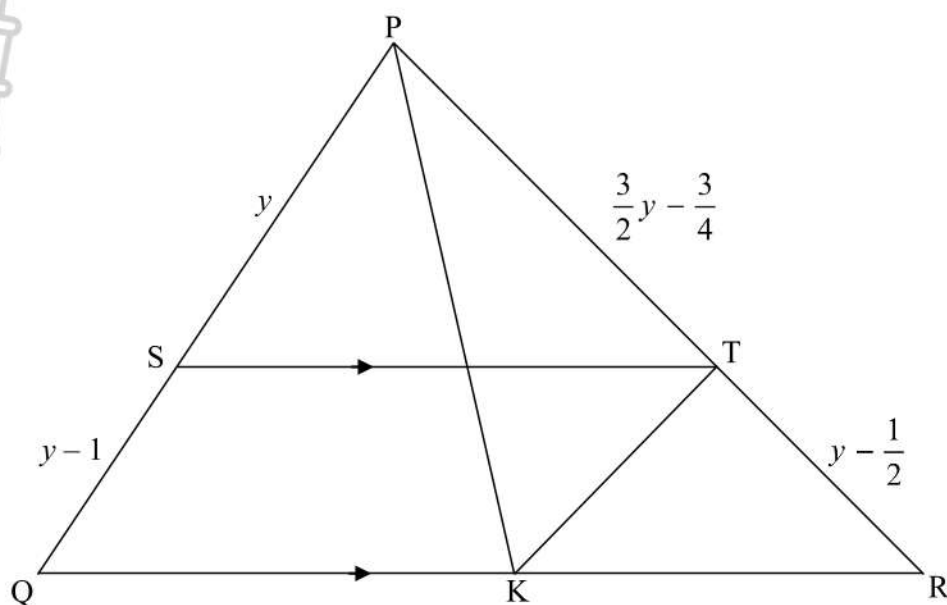
QUESTION/VRAAG 9



9.1	Construction: draw altitudes h and k; join B with E and D with C/ Konstruksie: trek hoogtelyne h en k; verbind B met E en D met C.	✓construction/ konstruksie
	$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\frac{1}{2} AD \times h}{\frac{1}{2} DB \times h} = \frac{AD}{DB}$	✓S
	$\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2} AE \times k}{\frac{1}{2} EC \times k} = \frac{AE}{EC}$	✓S
	area $\triangle DEB$ = area $\triangle DEC$ [same base; same ; <i>dieselfde basisse; dieselfde </i>]	✓S/R
	$\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle DEB} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$	✓S
	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$	
		(5)

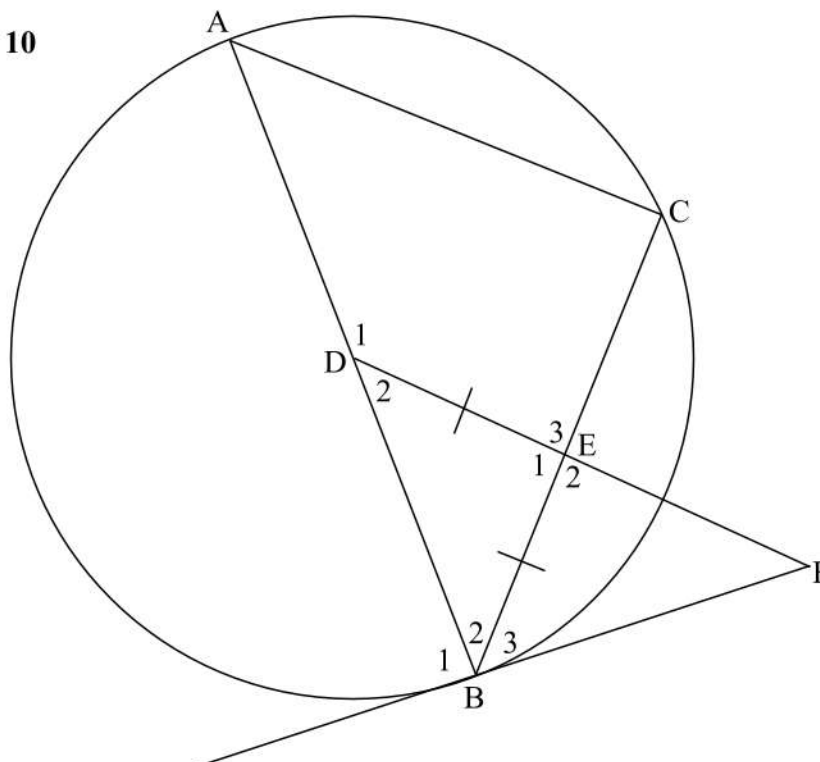


9.2



9.2.1	$\frac{y}{y-1} = \frac{\frac{3}{2}y - \frac{3}{4}}{y - \frac{1}{2}} \text{ [prop.th / eweredigheidsst.; ST QR]}$ $y\left(y - \frac{1}{2}\right) = (y-1)\left(\frac{3}{2}y - \frac{3}{4}\right)$ $y^2 - \frac{1}{2}y = \frac{3}{2}y^2 - \frac{9}{4}y + \frac{3}{4}$ $4y^2 - 2y = 6y^2 - 9y + 3$ $0 = 2y^2 - 7y + 3$ $0 = (y-3)(2y-1)$ $y = 3 \text{ or / of } y \neq \frac{1}{2}$	✓S/R ✓simplification/ vereenvoudiging ✓standard form/ standaardvorm ✓factors/faktore ✓only/slegs $y = 3$ (5)
9.2.2	$\hat{PST} = \hat{Q} \text{ [corresp } \angle\text{s/ooreenk. } \angle\text{e; ST QR]}$ $\hat{PST} = \hat{PKT} = \hat{Q}$ P, S, K and/en T are / is consyclic/konsiklies [converse $\angle\text{s in same segment / omgekeerde } \angle\text{e in dies. segment}]$ $\therefore$ PSKT is a / is 'n cyclic quadrilateral / koordevierhoek	✓S/R ✓R (2)
		[12]

QUESTION/VRAAG 10



10.1.1	$\hat{C} = 90^\circ$ [$\angle$ in semi circle./ $\angle$ in halfsirkel] $\hat{A}BF = 90^\circ$ [radius $\perp$ tangent / raaklyn] $\therefore \hat{C} = \hat{A}BF = 90^\circ$	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R (4)
10.1.2	In $\triangle DBF$ and / en $\triangle BCA$ $\hat{D}_2 = \hat{B}_2$ [$\angle$ s opp. equal sides/ $\angle$ e teenoor gelyke sye] $\hat{D}BF = \hat{C}$ [from / vanuit 10.1.1] $\triangle DFB \parallel \triangle BAC$ [$\angle \angle \angle$] $\frac{DF}{BA} = \frac{DB}{BC}$ [from / vanuit $\parallel \Delta$ s] $\therefore DF \cdot BC = AB \cdot BD$ OR / OF In $\triangle DBF$ and / en $\triangle BCA$ $\hat{D}_2 = \hat{B}_2$ [$\angle$ s opp. equal sides/ $\angle$ e teenoor gelyke sye] $\hat{D}BF = \hat{C}$ [from / vanuit 10.1.1] $\hat{F} = \hat{A}$ [sum of $\angle$ s of $\triangle$ /binne $\angle$ e van $\triangle$] $\triangle DFB \parallel \triangle BAC$ [$\angle \angle \angle$] $\frac{DF}{BA} = \frac{DB}{BC}$ [from / vanuit $\parallel \Delta$ s] $\therefore DF \cdot BC = AB \cdot BD$	$\checkmark$ S/R $\checkmark$ S $\checkmark$ R $\checkmark$ S/R (4) $\checkmark$ S/R $\checkmark$ S $\checkmark$ S $\checkmark$ S/R (4)



10.1.3	$\hat{B}_3 = \hat{A}$ [tan chord theorem / raaklyn – koordst.] $\hat{B}_3 = \hat{F} = \hat{A}$ $BE = EF$ [sides opp. = $\angle$ s/sye teenoor = $\angle e$] $\therefore BE = EF = DE$ $\therefore E$ is the midpt. of circle through D, B and F / E is die midpt. van sirkel deur D, B en F	$\checkmark S \checkmark R$ $\checkmark S/R$ $\checkmark S$	(4)
10.2	$\frac{AC}{BF} = \frac{AB}{DF}$ [from / vanuit $\parallel \Delta$ s] $= \frac{2AD}{2EF}$ $= \frac{AD}{EF}$ $\frac{1}{AC^3} \times \frac{AC}{BF} = \frac{1}{AC^3} \times \frac{AD}{EF}$ $\frac{1}{AC^2} = \frac{AD \cdot BF}{EF \cdot AC^3}$ $\frac{1}{AB^2 - BC^2} = \frac{AD \cdot BF}{EF \cdot AC^3}$ $-\frac{1}{BC^2 - AB^2} = \frac{AD \cdot BF}{EF \cdot AC^3}$ $\frac{1}{BC^2 - AB^2} = -\frac{AD \cdot BF}{EF \cdot AC^3}$	$\checkmark S$ $\checkmark S$ $\checkmark \times \frac{1}{AC^3}$ both sides/beide kante $\checkmark S$ $\checkmark$ Pythagoras $\checkmark - \frac{1}{BC^2 - AB^2}$	(6)
			[18]

TOTAL/TOTAAL: 150

