



DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS
LEFAPHA LA THUTO
ISEBE LEZEMFUNDO

**PROVINSIALE VOORBEREIDENDE EKSAMEN/
PROVINCIAL PREPARATORY EXAMINATION**

GRAAD/GRADE 12

WISKUNDE/MATHEMATICS

VRAESTEL/PAPER 2

Stanmorephysics.com
SEPTEMBER 2024

PUNTE/MARKS: 150

TYD/TIME: 3 uur/hours

**Hierdie vraestel bestaan uit 12 bladsye, 1 inligtingsblad
en 'n antwoordeboek van 23 bladsye./
This question paper consists of 12 pages, 1 information sheet
and an answer book of 23 pages.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.



QUESTION 1

Twenty five rugby players were asked about the number of times they visited the gymnasium during the December festive season. The responses were as follows:

Number of gym visits	Number of players
0	1
x	1
$x + 5$	3
10	1
13	6
14	3
15	2
18	4
20	1
22	3

- 1.1 The mean of gym visits is 13,96. Use calculations to show that $x = 4$. (2)
- 1.2 Calculate the standard deviation of the data. (2)
- 1.3 How many players visited the gym more than one standard deviation above the mean? (2)
- 1.4 Determine the interquartile range of the data. (3)
- 1.5 During January, each player increased his number of gym visits with k visits. What impact will this have on the:
 - 1.5.1 standard deviation? (1)
 - 1.5.2 interquartile range? (1)

[11]

QUESTION 2

A restaurant situated on the beach front wants to predict how many cups of coffee will be sold on a given day, depending on the temperature. They collected the following data over 11 days:

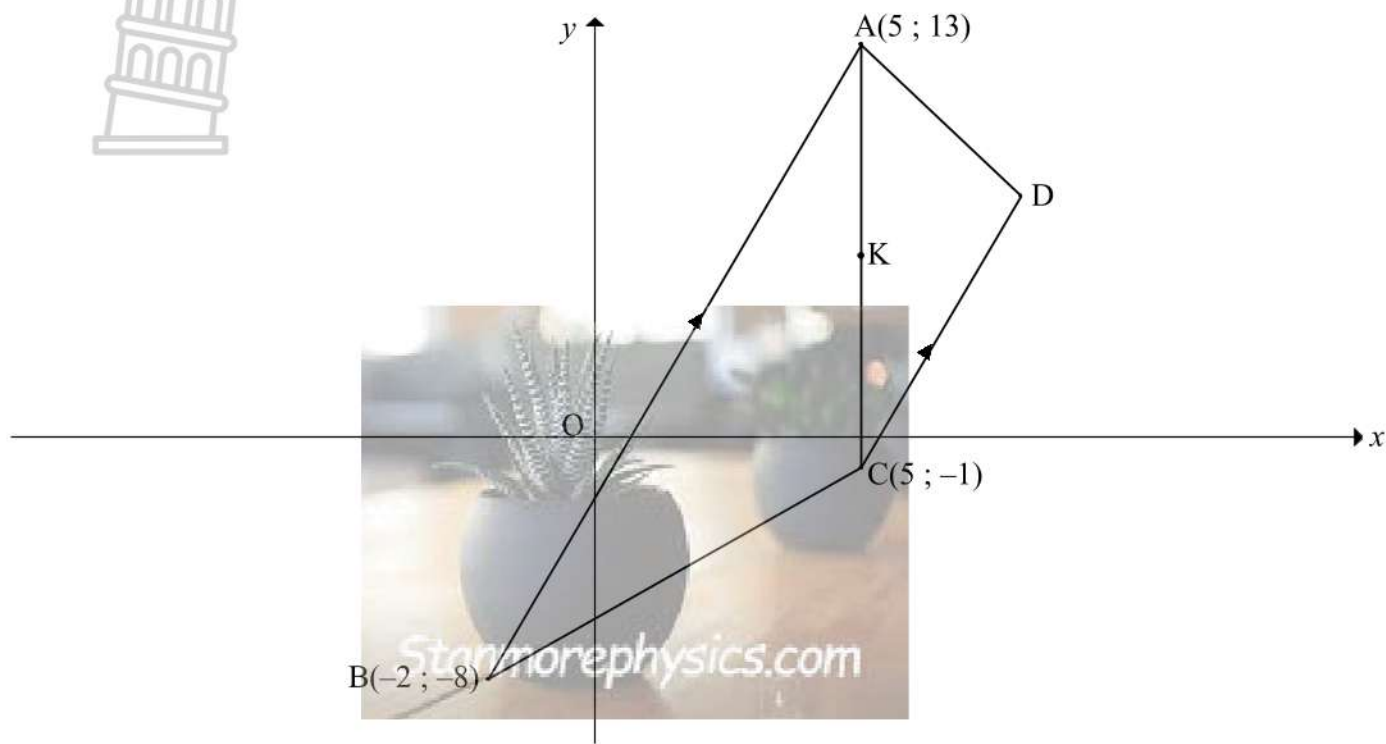
Temperature in °C (x)	19	21	16	23	26	25	28	19	27	22	35
Cups of coffee sold (y)	381	370	400	161	173	184	165	398	182	310	65

- 2.1 Determine the equation of the least squares regression line for this data. (3)
- 2.2 Write down the correlation coefficient. (1)
- 2.3 Hence, comment on the:
- 2.3.1 trend of the data (1)
- 2.3.2 strength of the relationship between the sales and the temperature of the day (1)
- 2.4 Predict the number of cups of coffee the restaurant will most probably sell if the day temperature is 30°C. (2)
- [8]**



QUESTION 3

ABCD is a trapezium with vertices $A(5; 13)$, $B(-2; -8)$, $C(5; -1)$ and D .
K is the midpoint of AC and $BA \parallel CD$.

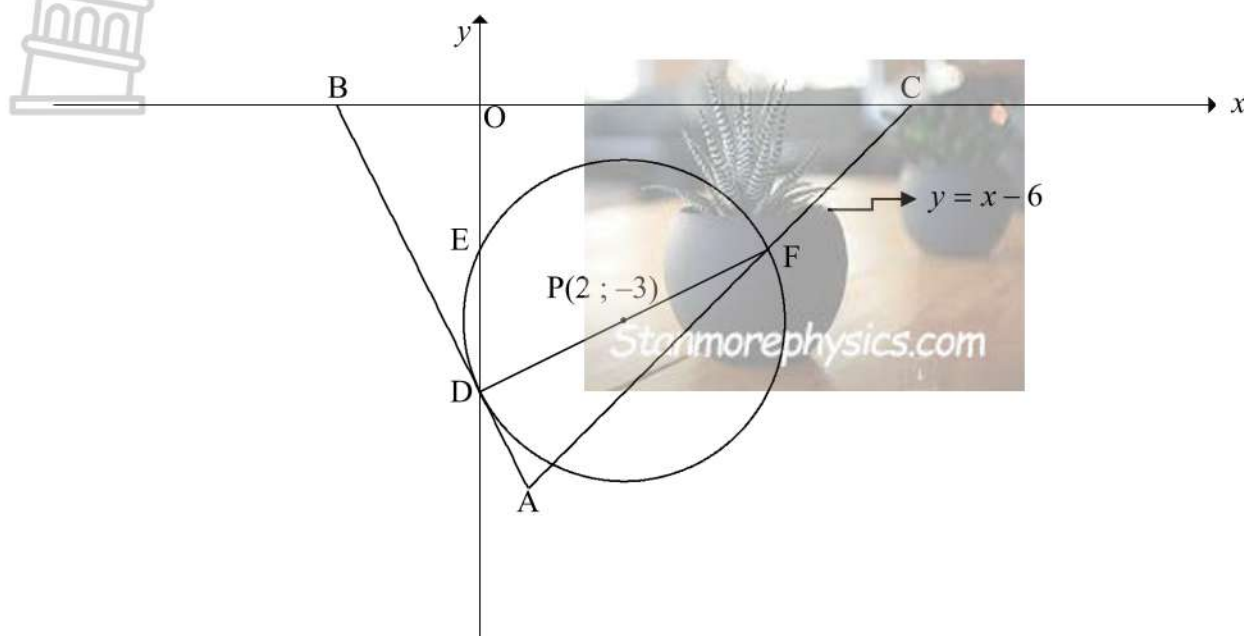


- 3.1 Write down the coordinates of K. (2)
- 3.2 Calculate:
 - 3.2.1 The gradient of AB (2)
 - 3.2.2 p , if $D(p; p)$ (2)
- 3.3 ABED is a parallelogram with E a point in the fourth quadrant. Calculate the coordinates of E. (2)
- 3.4 Calculate the area of obtuse $\triangle ABC$. (3)
- 3.5 $\triangle ABC$ is reflected in the line $x = 5$ to form $\triangle AFC$.
 - 3.5.1 Calculate the perimeter of $\triangle ABF$. (5)
 - 3.5.2 Determine the equation of a circle, centred at O, the origin, passing through point F. (2)

[18]

QUESTION 4

In the diagram, $P(2 ; -3)$ is the centre of the circle with equation $x^2 - 4x + y^2 + 6y + 8 = 0$, passing through D, E and F. D and E are y -intercepts of the circle. The equation of AC is $y = x - 6$. AB is a tangent to the circle at D. B and C are points on the x -axis.



- 4.1 Calculate the coordinates of:
 - 4.1.1 D and E (3)
 - 4.1.2 F (2)
- 4.2 Write the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$. (3)
- 4.3 Determine the equation of AB in the form $y = mx + c$. (4)
- 4.4 Calculate the:
 - 4.4.1 size of $\hat{BDE}$ (3)
 - 4.4.2 coordinates of A (4)
- 4.5 Circle P is translated such that the new centre has coordinates $(x + k ; -3)$. Determine all possible values of k for which the translated circle will never intersect the y -axis. (4)

[23]

QUESTION 5

5.1 Simplify the following expression **without using a calculator**.

$$\sin(90^\circ - x) \cdot \tan(360^\circ - x) - 2\sin(180^\circ + x) \quad (5)$$

5.2 Calculate the value of the following expression **without using a calculator**.

$$\sin(23^\circ + x) \cos(7^\circ - x) + \cos(23^\circ + x) \sin(7^\circ - x) \quad (3)$$

5.3 If $\sin 2\theta = \frac{-4\sqrt{2}}{9}$, and $2\theta \in [90^\circ; 270^\circ]$, determine the value of the following, **without using a calculator**:

$$5.3.1 \quad \cos 2\theta \quad (3)$$

$$5.3.2 \quad \sin \theta \quad (4)$$

5.4 Determine the general solution of $2\sin \alpha \cos \alpha = 2\cos^2 \alpha - 2\sin^2 \alpha$ (7)

5.5 Consider:
$$\frac{\cos 2x + \cos^2 x + 3\sin^2 x}{2 - 2\sin^2 x} = \frac{1}{\cos^2 x}$$

5.5.1 Prove the identity. (4)

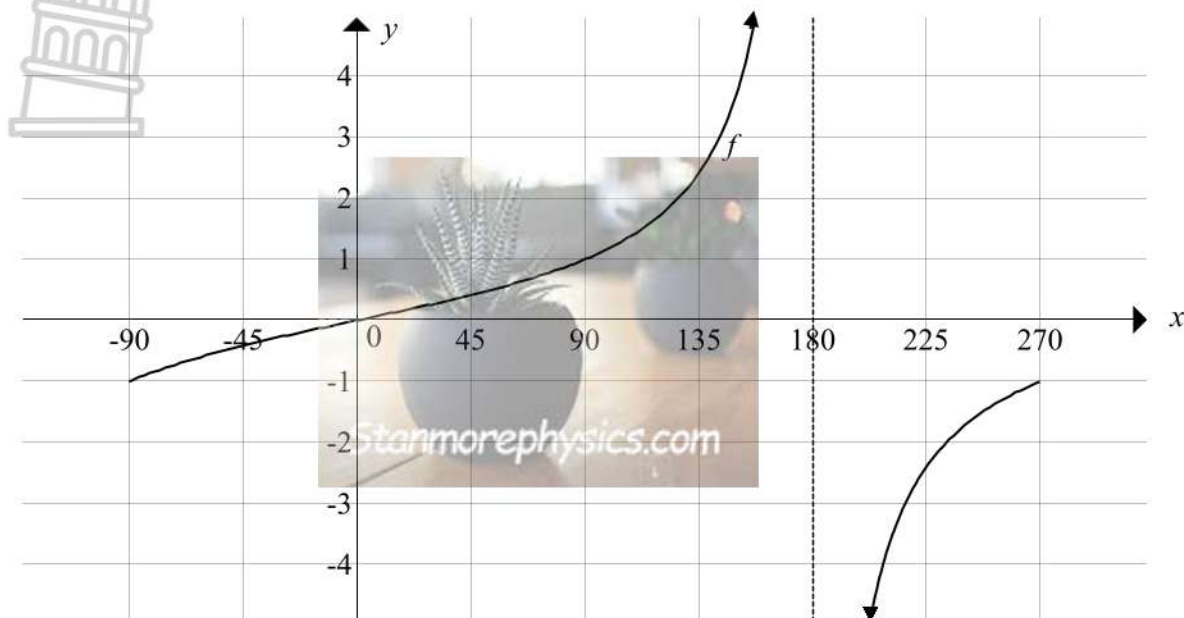
5.5.2 For which value(s) of x , $x \in [0^\circ; 360^\circ]$, will the identity be undefined? (2)

[28]



QUESTION 6

The graph of $f(x) = \tan \frac{1}{2}x$ for the interval $x \in [-90^\circ; 270^\circ]$, is drawn below.

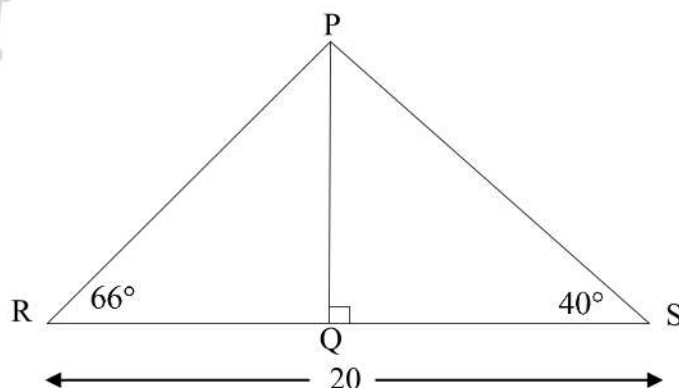


- 6.1 Write down the period of f . (1)
- 6.2 Write down the equation of the asymptote of f in the interval $x \in [-90^\circ; 270^\circ]$. (1)
- 6.3 On the grid provided in the ANSWER BOOK, draw the graph of $g(x) = 2\cos x$ for the interval $x \in [-90^\circ; 270^\circ]$. Clearly show all intercepts with the axes, turning points and end points. (3)
- 6.4 Give 2 values of x , in the interval $x \in [-90^\circ; 270^\circ]$, for which $g(x) - f(x) = 1$. (1)
- 6.5 Write down the range of $g(x) - 3$ in the interval $x \in [-90^\circ; 270^\circ]$. (2)
- 6.6 Determine the maximum value of $[5 - 2\sin(90^\circ - x)]^2$ for $x \in R$. (3)

[11]

QUESTION 7

- 7.1 In the diagram below, $\triangle PRS$ is drawn with $RS = 20$ units, $\hat{R} = 66^\circ$ and $\hat{S} = 40^\circ$. Q is a point on RS such that $PQ \perp RS$.

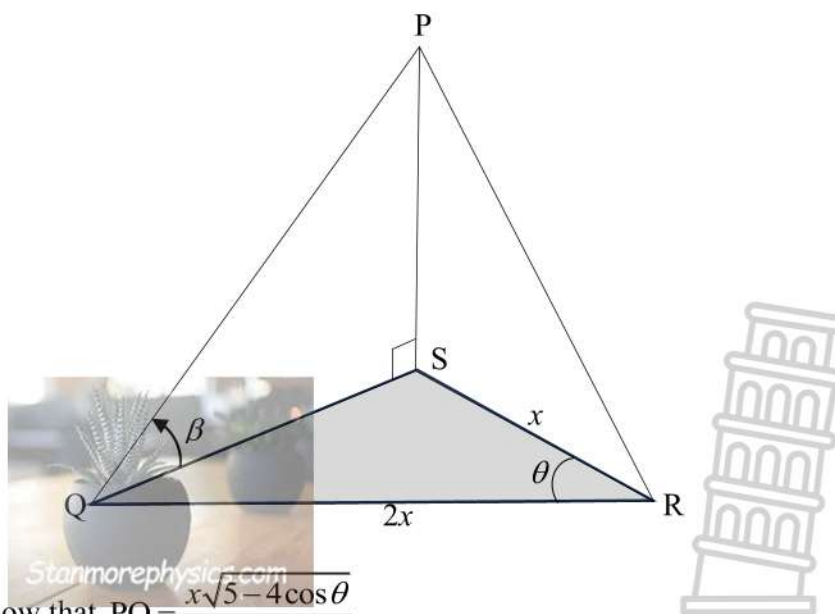


Calculate the length of:

7.1.1 PS (3)

7.1.2 PQ (2)

- 7.2 In the diagram, PS is a vertical flagpole. Q , R and S lie in the same horizontal plane. PQ and PR are two cables, anchored at Q and R . $\widehat{PQS} = \beta$ and $\widehat{QRS} = \theta$. $SR = x$ and $QR = 2x$.



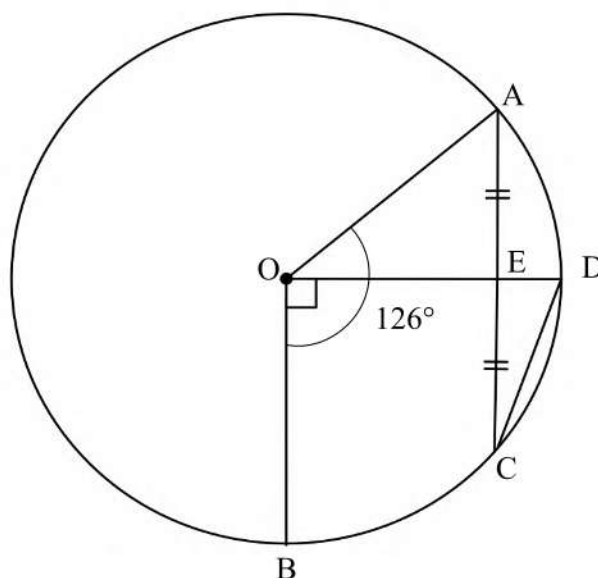
7.2.1 Show that $PQ = \frac{x\sqrt{5-4\cos\theta}}{\cos\beta}$ (4)

7.2.2 The area of $\triangle QRS$ is $57,36m^2$ and $\theta = 35^\circ$. Calculate the value of x . (2)

[11]

QUESTION 8

- 8.1 In the diagram, a circle with centre O is drawn. Radii OB, OD and OA are drawn such that $BO \perp OD$ and obtuse angle $\hat{AOB} = 126^\circ$. OD bisects chord AC at E and CD is drawn.

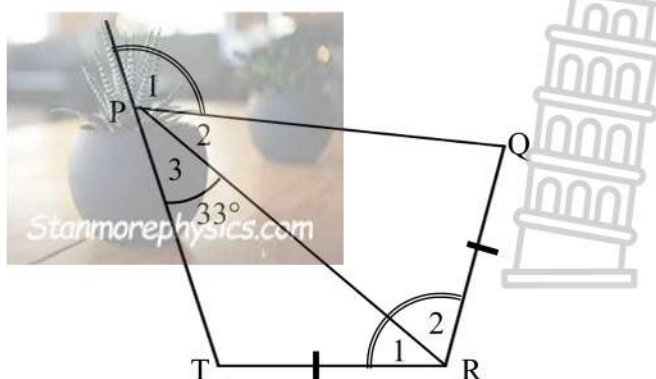


Calculate, giving reasons, the size of:

8.1.1 $\hat{OAC}$ (4)

8.1.2 $\hat{ACD}$ (2)

- 8.2 In the diagram below, quadrilateral PQRT is drawn with $TR = RQ$. TP is produced such that $\hat{P}_1 = \hat{TRQ}$. PR is drawn. $\hat{P}_3 = 33^\circ$ and $\hat{R}_2 = 66^\circ$.

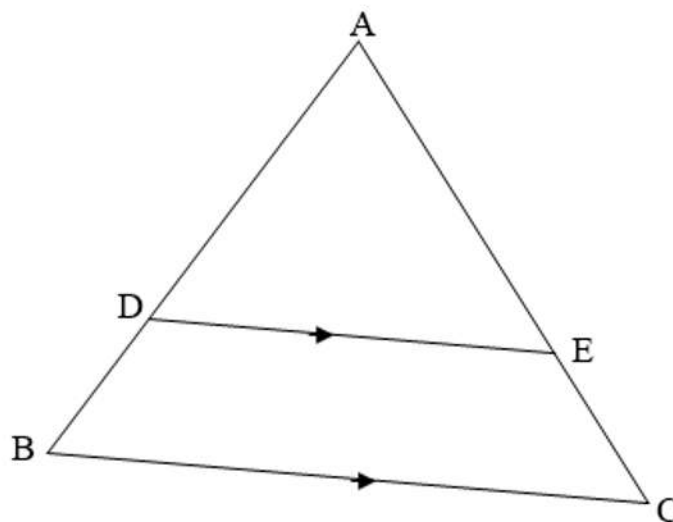


Calculate, giving reasons, the size of $\hat{T}$. (6)

[12]

QUESTION 9

- 9.1 In the diagram, $\triangle ABC$ is drawn. D and E are points on AB and AC respectively such that $DE \parallel BC$.

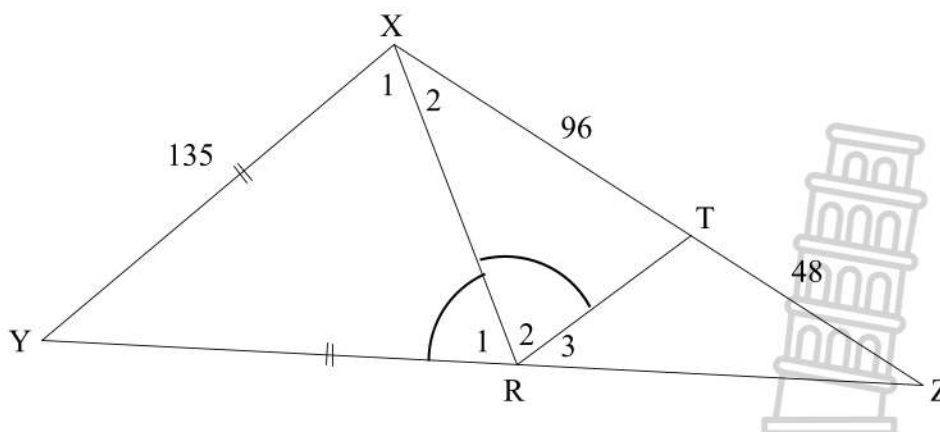


Use the diagram in the ANSWER BOOK to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, i.e.

prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

(5)

- 9.2 In the diagram, $\triangle XYZ$ is drawn. R and T are points on YZ and XZ respectively and XR and RT are drawn. $XY = 135$ units, $XT = 96$ units and $TZ = 48$ units. $\hat{R}_1 = \hat{R}_2$ and $XY = YR$.



- 9.2.1 Prove, giving reasons, that $RT \parallel YX$.

(3)

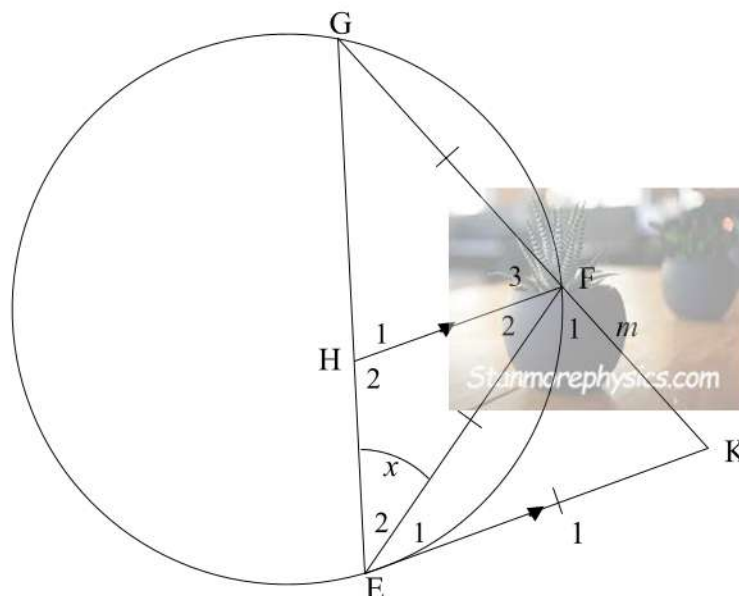
- 9.2.2 Calculate, giving reasons, the length of RZ.

(3)

[11]

QUESTION 10

In the diagram, E, F and G are points on the circle. KE is a tangent to the circle at E. KFG is a straight line. H is a point on chord GE such that $HF \parallel EK$.
 $KE = EF = FG$, $KF = m$ units and $KE = 1$ unit. $\hat{E}_2 = x$.



- 10.1 Name, giving reasons, THREE other angles each equal to x . (4)
- 10.2 Express $\hat{K}$ in terms of x . (1)
- 10.3 Calculate, with reasons, the size of x . (3)
- 10.4 Prove that:
- 10.4.1 $\triangle KGE \parallel \triangle KEF$ (3)
- 10.4.2 $m^2 + m - 1 = 0$ (2)
- 10.5 If it is given that $GH = 2$ units, calculate the length of HE , rounded off to 2 decimal places. (4)

[17]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = (1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



NAME OF LEARNER/*NAAM VAN LEERDER*

MATHEMATICS P2/*WISKUNDE V2*

GRADE 12/*GRAAD 12*

SEPTEMBER 2024

SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

QUESTION/ <i>VRAAG</i>	MARK/ <i>PUNT</i>	MOD 1	MOD 2
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
TOTAL/ <i>TOTAAL</i>			

This answer book consists of 23 pages./*Hierdie antwoordeboek bestaan uit 23 bladsye.*

QUESTION 1/VRAAG 1



Number of gym visits/ <i>Aantal gimnasiumbesoeke</i>	Number of players/ <i>Aantal spelers</i>
0	1
x	1
$x + 5$	3
10	1
13	6
14	3
15	2
18	4
20	1
22	3

	Solution/ <i>Oplossing</i>	Mark/ <i>Punt</i>
1.1		(2)
1.2		(2)
1.3		(2)

	Solution/<i>Oplossing</i>	Mark/ <i>Punt</i>
1.4		(3)
1.5.1		(1)
1.5.2		(1)

[11]



QUESTION 2/VRAAG 2

Temperature in °C/ Temperatuur in °C (x)	19	21	16	23	26	25	28	19	27	22	35
Cups of coffee sold/Koppies koffie verkoop (y)	381	370	400	161	173	184	165	398	182	310	65

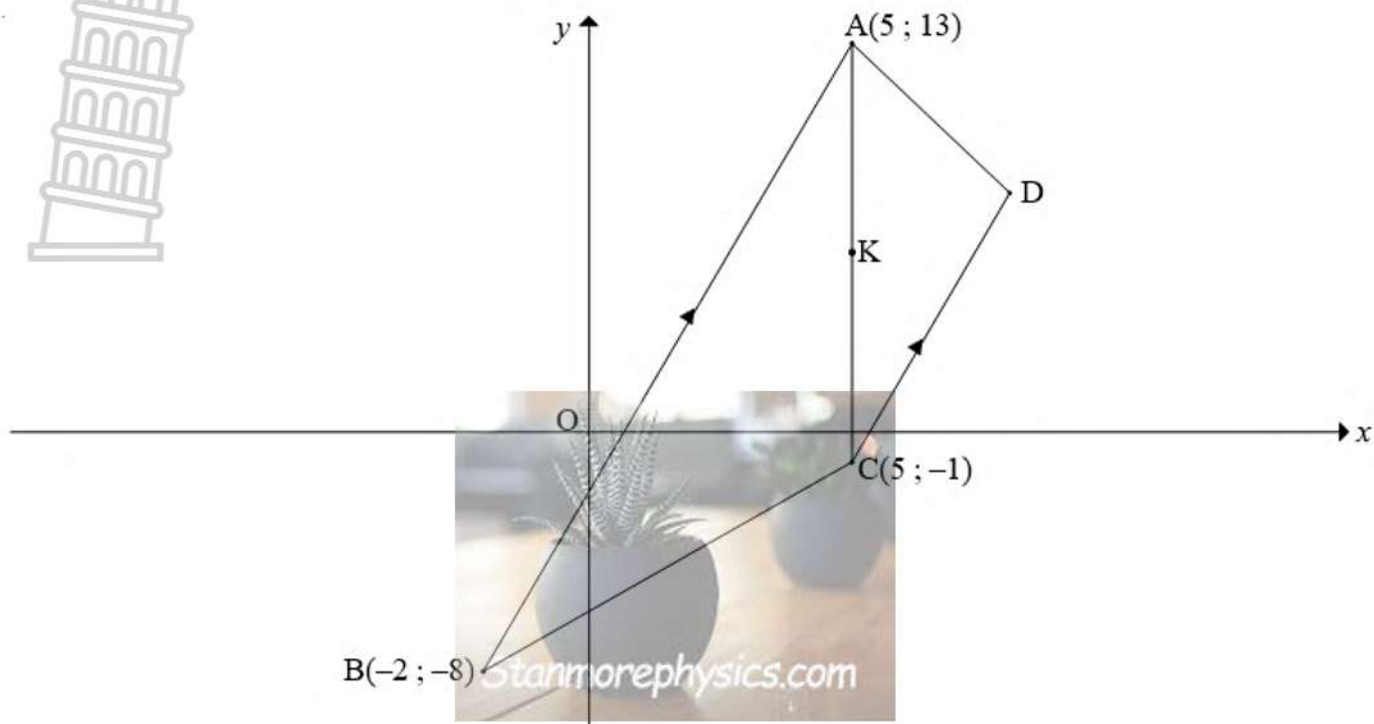
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	Solution/Oplossing	Mark/ Punt
2.1		(3)
2.2		(1)
2.3.1		(1)
2.3.2		(1)

	Solution/<i>Oplossing</i>	Mark/ <i>Punt</i>
2.4		(2) [8]



QUESTION 3/VRAAG 3

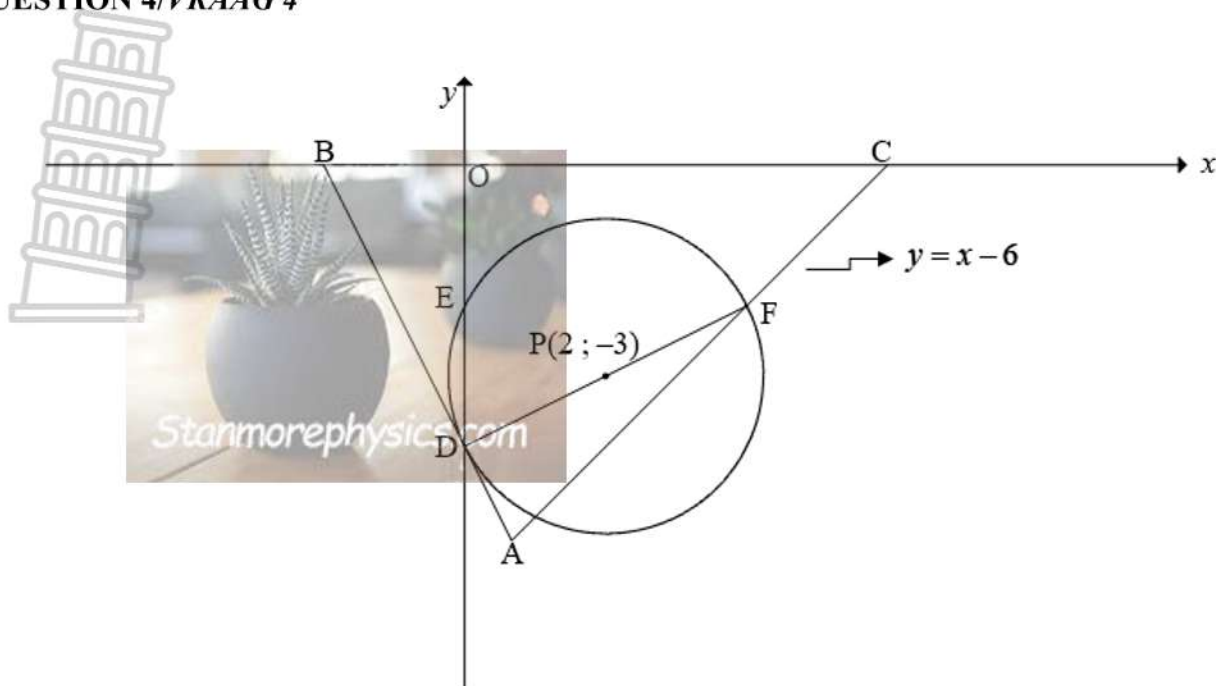


	Solution/Oplossing	Mark/ Punt
3.1		(2)
3.2.1		(2)
3.2.2		(2)

	Solution/ <i>Oplossing</i>	Mark/ <i>Punt</i>
3.3		
3.4		(2)
3.5.1		(3)
3.5.2		(5)
		(2)

[18]

QUESTION 4/VRAAG 4



	Solution/Oplissing	Mark/ Punt
4.1.1		(3)
4.1.2		(2)
4.2		(3)

	Solution/<i>Oplossing</i>	Mark/ <i>Punt</i>
4.3		
		(4)
4.4.1		
		(3)
4.4.2		
		
		(4)
4.5		
		(4)

[23]

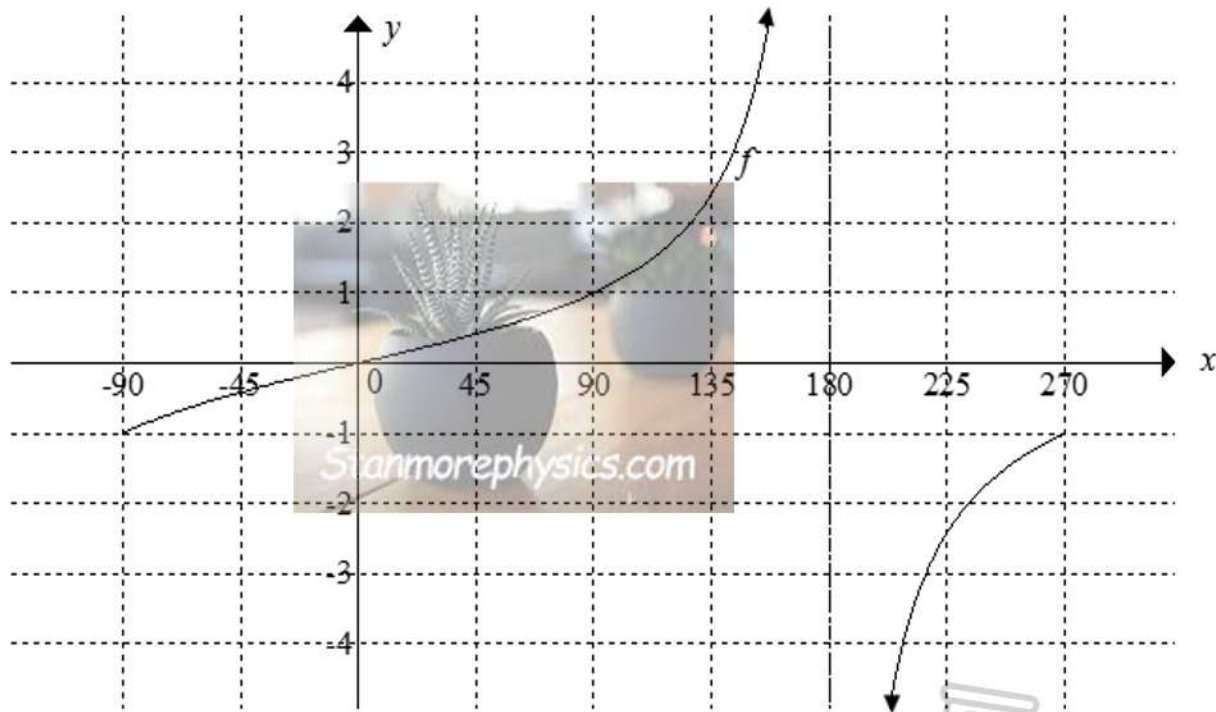
QUESTION 5/VRAAG 5

	Solution/Oplissing	Mark/ Punt
5.1		(5)
5.2		(3)
5.3.1		(3)
5.3.2		(4)

	Solution/ <i>Oplossing</i>	Mark/ <i>Punt</i>
5.4		
5.5.1	 <i>Stanmorephysics.com</i>	(7)
5.5.2		(4)
		(2)

[28]

QUESTION 6/VRAAG 6

Solution/Oplissing		Mark/ Punt
6.1		(1)
6.2		(1)
6.3		(3)
		
6.4		(1)
6.5		(2)

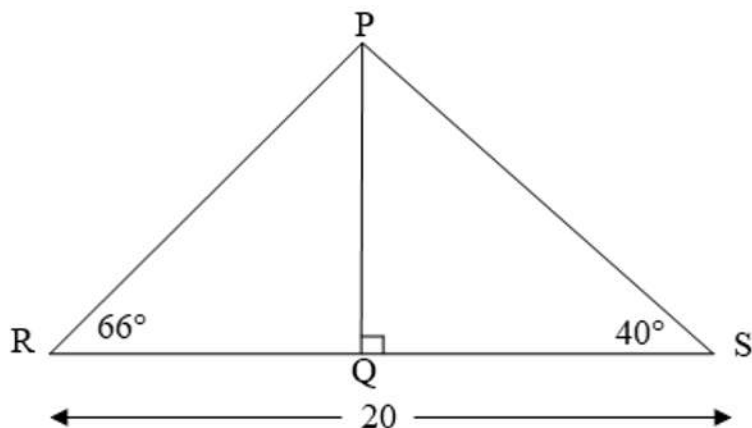
	Solution/<i>Oplossing</i>	Mark/ <i>Punt</i>
6.6		
		(3)

[11]



QUESTION 7 / VRAAG 7

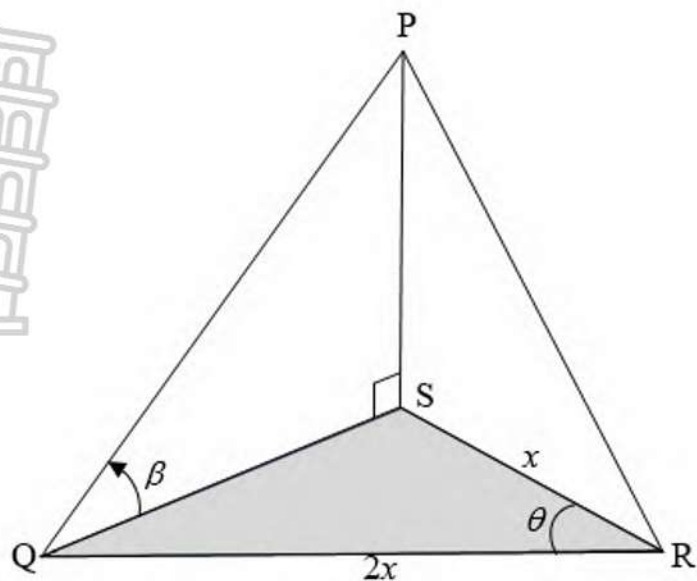
7.1



	Solution/ <i>Oplossing</i>	Mark/ <i>Punt</i>
7.1.1		(3)
7.1.2		(2)



7.2

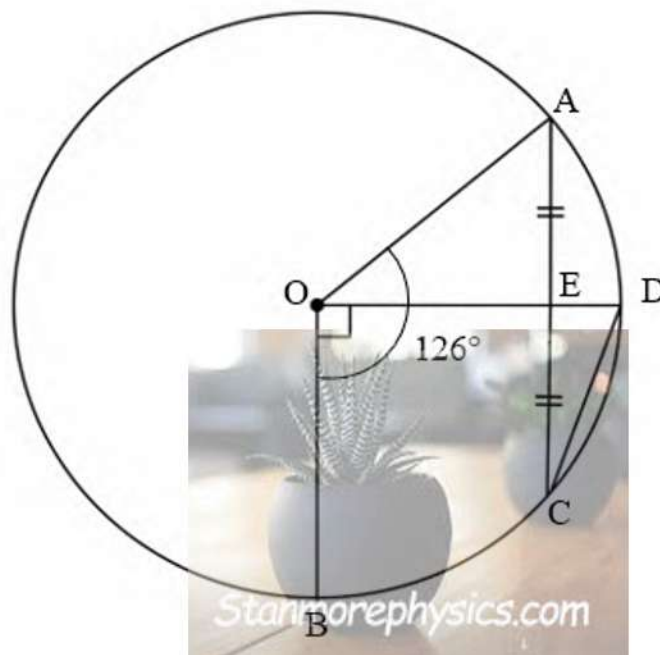


	Solution/Oplissing	Mark/ Punt
7.2.1		(4)
7.2.2		(2)

[11]

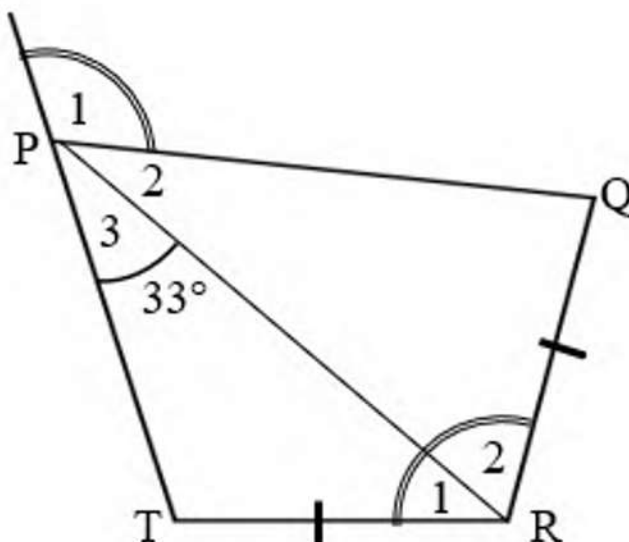
QUESTION 8/VRAAG 8

8.1



	Solution/Oplossing	Mark/ Punt
8.1.1		(4)
8.1.2		(2)

8.2

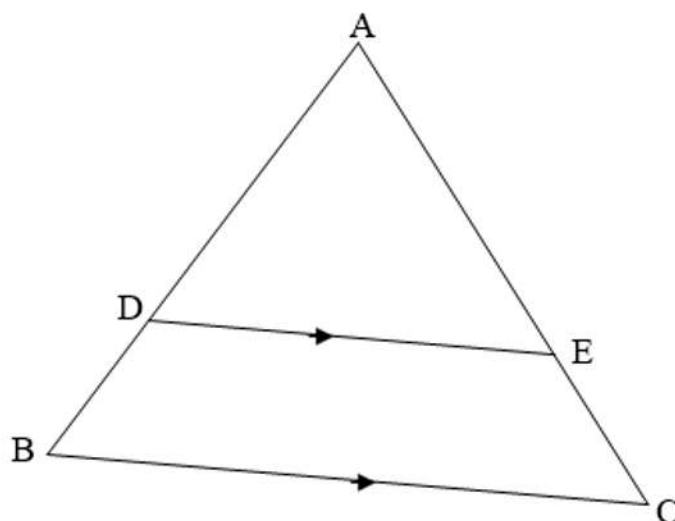
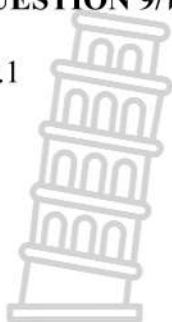


	Solution/Oplissing	Mark/ Punt
8.2		
		(6)

[12]

QUESTION 9/VRAAG 9

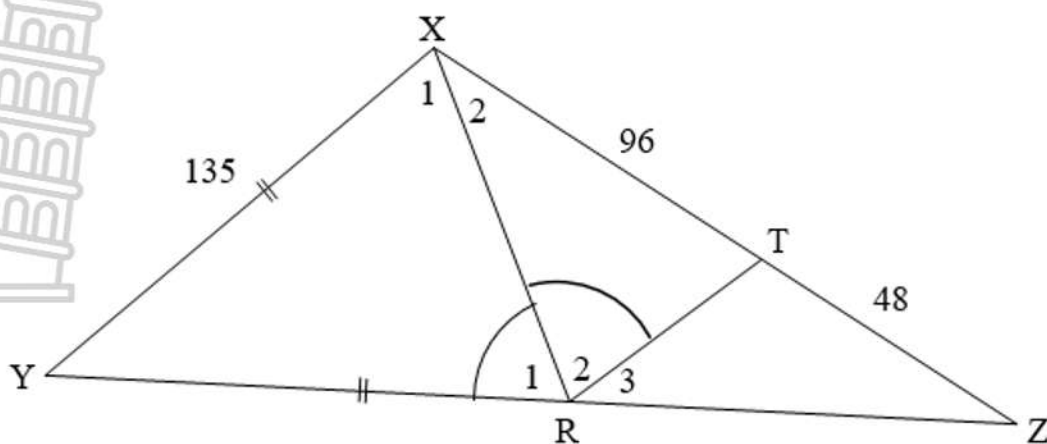
9.1



	Solution/Oplissing	Mark/ Punt
9.1		
		(5)



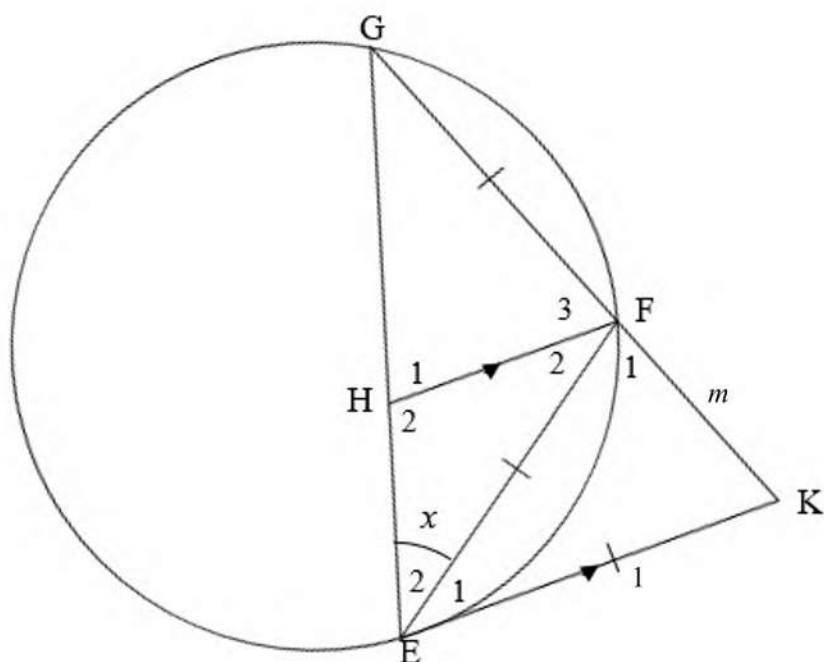
9.2



	Solution/Oplissing	Mark/ Punt
9.2.1		(3)
9.2.2		(3)

[11]

QUESTION 10/VRAAG 10



	Solution/Oplissing	Mark/ Punt
10.1		(4)
10.2		(1)
10.3		(3)



	Solution/<i>Oplossing</i>	Mark/ <i>Punt</i>
10.4.1		
		(3)
10.4.2		
		(2)
10.5		
		(4)
		[17]

TOTAL/TOTAAL: 150







DEPARTMENT OF EDUCATION
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**PROVINCIAL PREPARATORY EXAMINATION/
PROVINSIALE VOORBEREIDENDE EKSAMEN**

GRADE/GRAAD 12

MATHEMATICS/WISKUNDE

PAPER/VRAESTEL 2

SEPTEMBER 2024

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MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 19 pages./
Hierdie nasienriglyne bestaan uit 19 bladsye.**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values to solve a problem is NOT acceptable.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
S/R	Award a mark if statement AND reason are both correct

QUESTION/VRAAG 1

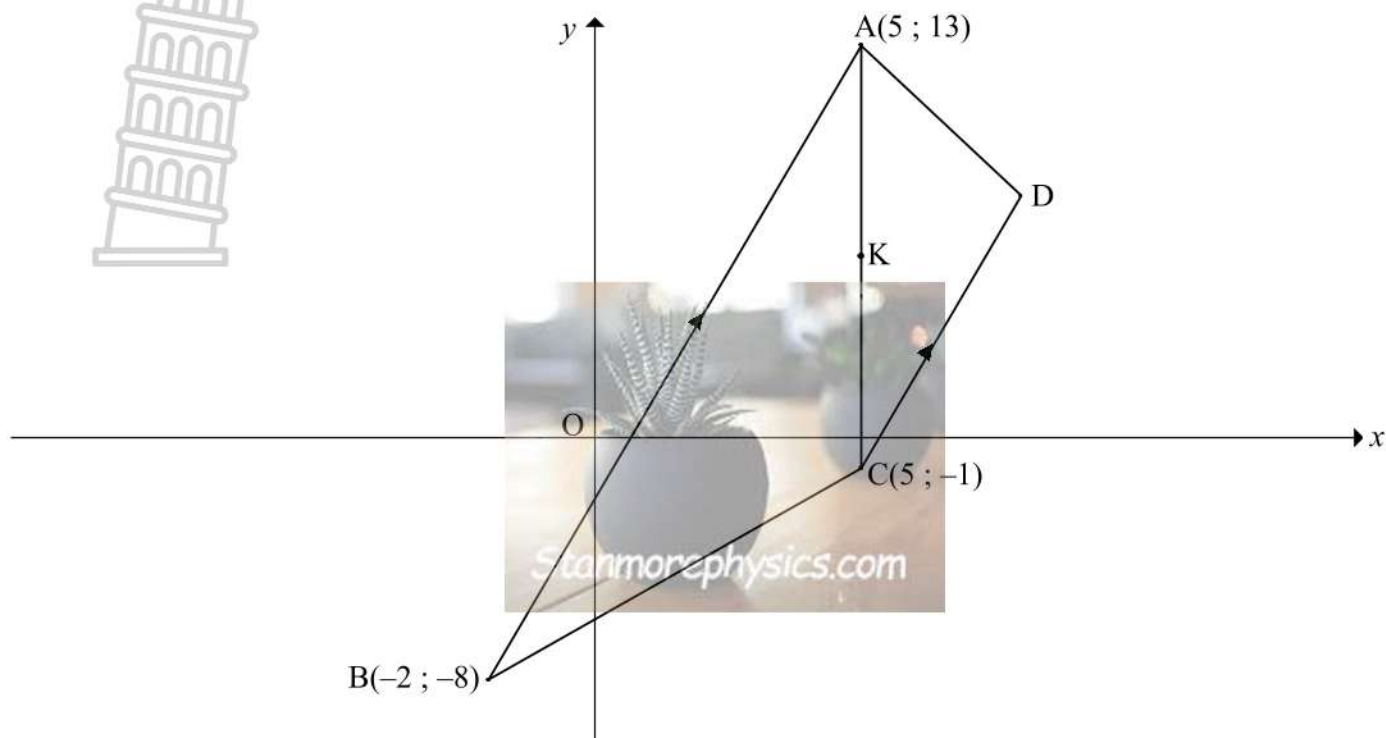
1.1	$13,96 = \frac{0 + x + 3(x + 5) + 10 + 13(6) + 14(3) + 15(2) + 18(4) + 20 + 22(3)}{25}$ $13,96 = \frac{4x + 333}{25}$ $349 - 333 = 4x$ $16 = 4x$ $4 = x$ If substitutes $x = 4$: only method mark	✓ method ✓ simplify (2)
1.2	$\sigma = 5,2419\dots$ $= 5,24$ Ignore rounding: accept 5,2 or 5,2419...	✓✓ 5,24 (2)
1.3	$13,96 + 5,24 = 19,2$ 4 players	✓ $\bar{x} + \sigma_x$ ✓ answer (2)
1.4	$Q_1 = T_6 + T_7$ $= \frac{10 + 13}{2}$ $= 11,5$ visits $Q_3 = T_{19} + T_{20}$ $= \frac{18 + 18}{2}$ $= 18$ visits IQR = $18 - 11,5 = 6,5$ visits	✓ Q1 ✓ Q3 ✓ Answer (3)
1.5.1	The standard deviation remains the same.	✓ Answer (1)
1.5.2	The IQR remains the same.	✓ Answer (1)
		[11]

QUESTION/VRAAG 2

2.1	$a = 746,06$ (746,064 ...) $b = -20,76$ (-20,7575 ...) $\hat{y} = 746,06 - 20,76x$	Ignore rounding AO FULL MARKS	$\checkmark a = 746,06$ $\checkmark b = -20,76$ $\checkmark$ equation (3)
2.2	$r = -0,91$ (-0,9132 ...)		$\checkmark r = -0,91$ (1)
2.3.1	As the temperature increases, the number of cups sold decreases. OR/OF As the temperature decreases, the number of cups sold increases.	Must be answered in context	$\checkmark$ answer (1)
2.3.2	There is a (very) <u>strong negative</u> linear correlation.		$\checkmark$ strong negative (1)
2.4	$\hat{y} = 746,06 - 20,76(30)$ $\hat{y} = 123,26$ cups $= 123$ cups OR/OF $\hat{y} = 123,34$ cups $= 123$ cups	Ignore rounding	$\checkmark$ substitution $\checkmark$ answer $\checkmark\checkmark$ $\hat{y} = 123,34$ cups (2)
			[8]



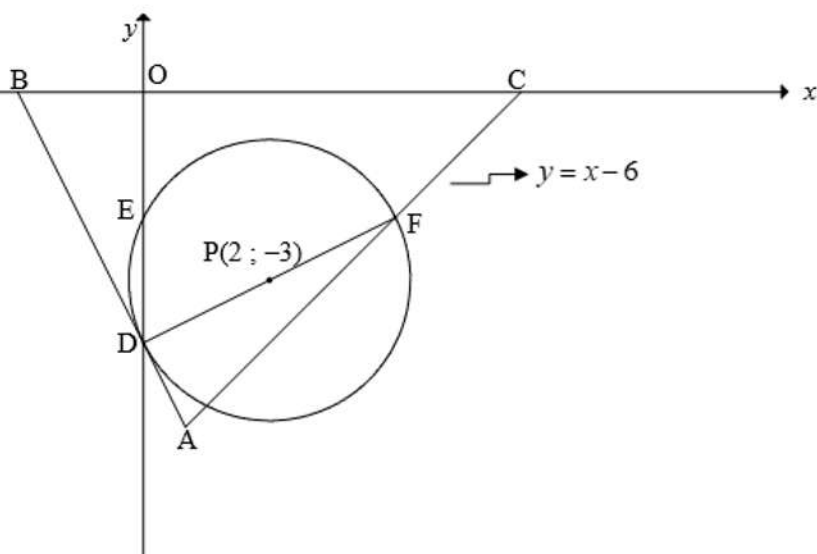
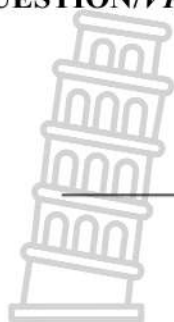
QUESTION/VRAAG 3



3.1	K(5 ; 6)	✓ x ✓ y (2)
3.2.1	$m_{AB} = \frac{13 - (-8)}{5 - (-2)}$ $= \frac{21}{7}$ $= 3$	✓ substitution A & B ✓ answer (2)
3.2.2	$m_{AB} = m_{CD} (AB \parallel CD)$ $3 = \frac{p+1}{p-5}$ $3p - 15 = p + 1$ $p = 8$	✓ substitute (p ; p) and C and equating to 3.2.1 ✓ answer (2)
3.3	D (8 ; 8) $A(x; y) \rightarrow D(x + 3 ; y - 5)$ $B(-2 ; -8) \rightarrow E(1 ; -13)$	✓ x ✓ y (2)

3.4	$\perp h : 5 - (-2) = 7 \text{ units}$ $\text{base: } 13 - (-1) = 14 \text{ units}$ $\text{Area} = \frac{1}{2}(14)(7)$ $= 49 \text{ units}^2$ OR $b^2 = a^2 + c^2 - ac \cdot \cos B$ $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$ $\cos B = \frac{98 + 490 - 14^2}{2 \cdot \sqrt{98} \cdot \sqrt{490}}$ $\hat{B} = 26,57^\circ$ $\text{Area} = \frac{1}{2} \times \sqrt{98} \times \sqrt{490} \times \sin 26,57^\circ$ $= 49,01 \text{ units}^2$	✓ height ✓ base ✓ area ✓ angle ✓ area formule ✓ area (3)
3.5.1	$F(12 ; -8)$ $d_{AB} = \sqrt{(5 - (-2))^2 + (13 - (-8))^2}$ $= \sqrt{490} \text{ or } 7\sqrt{10} \text{ or } 22,135...$ $\text{Perimeter} = 7\sqrt{10} + 7\sqrt{10} + 14$ $= 58,27$	✓ coordinates of F ✓ Substitution ✓ AB ✓ adding 3 sides ✓ Answer (5)
3.5.2	$r^2 = (-12)^2 + (-8)^2$ $r^2 = 208$ $x^2 + y^2 = 208$	✓ r^2 ✓ equation (2)
		[18]

QUESTION/VRAAG 4



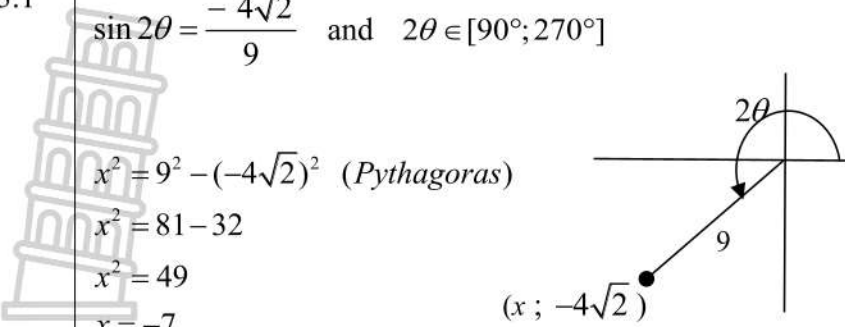
4.1.1	Put $x = 0$: $y^2 + 6y + 8 = 0$ $(y + 2)(y + 4) = 0$ $y = -2 \text{ or } y = -4$ $E(0; -2) \quad D(0; -4)$	✓ $y^2 + 6y + 8 = 0$ ✓ $-2 \text{ \& } -4$ ✓ E and D in coordinate form (3)
4.1.2	$D(0; -4) \quad P(2; -3)$ $2 = \frac{x_F + 0}{2} \therefore 4 = x_F$ $-3 = \frac{y_F - 4}{2} \therefore y_F = -2$ $F(4; -2)$ OR/OF $D \rightarrow P: (x + 2; y + 1)$ $P \rightarrow F: (2 + 2; -3 + 1)$ $F: (4; -2)$	✓ x ✓ y ✓ x ✓ y (2)

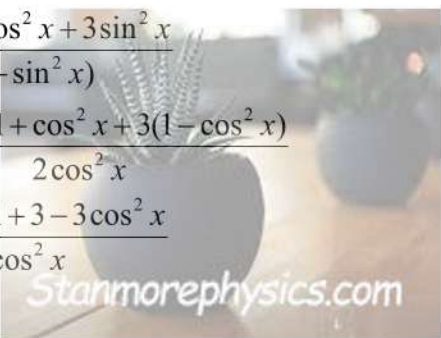
4.2	$r^2 = (2+0)^2 + (-3+4)^2$ $r^2 = 5$ $(x-2)^2 + (y+3)^2 = 5$ OR/OF $(x^2 - 4x + 4) + (y^2 + 6y + 9) = 4 + 9 - 8$ $(x-2)^2 + (y+3)^2 = 5$	✓ substitution ✓ r^2 ✓ LHS ✓ complete the square. ✓ LHS ✓ r^2 (3)
4.3	$m_{DP} = \frac{-4 - (-3)}{0 - 2}$ $= \frac{1}{2}$ $m_{AB} = -2$ (rad $\perp$ tan) $y = -2x - 4$	✓ substitution of P & D ✓ m_{DP} ✓ m_{AB} ✓ equation (4)
4.4.1	$\tan \theta = m_{AB}$ $= -2$ $\hat{D}BO = 63,43^\circ$ $\hat{B}DE = 90^\circ - 63,43^\circ$ $= 26,57^\circ$ OR $\tan \theta = -2$ Inclination AB: $180^\circ - 63,43^\circ = 116,57^\circ$ $\hat{B}DE = 116,57^\circ - 90^\circ$ $= 26,57^\circ$ OR $\tan \hat{B}DE = \frac{1}{2}$ $\hat{B}DE = 26,57^\circ$	✓ $\tan \theta = -2$ ✓ $\hat{D}BO$ ✓ answer ✓ $\tan \theta = -2$ ✓ inclination angle ✓ answer ✓ $\tan \hat{B}DE$ ✓ $\frac{1}{2}$ ✓ answer (3)

4.4.2	$-2x - 4 = x - 6$ $2 = 3x$ $\frac{2}{3} = x$ $y = \frac{2}{3} - 6$ $y = -\frac{16}{3}$ or $-5\frac{1}{3}$ $A\left(\frac{2}{3}; -\frac{16}{3}\right)$	✓ equating ✓ x ✓ substitution ✓ y (4)
4.5	centre: $(2; -3)$ and $r = \sqrt{5}$ moving centre $(x+k; -3)$ $2 + \sqrt{5} = 4,24$ (x -value on the right) $2 - \sqrt{5} = -0,24$ (x -value on the left) $k > \sqrt{5} - 2$ OR $k < -(\sqrt{5} + 2)$ $k > 0,24$ $k < -2 - \sqrt{5}$ $k < -4,24$	✓ x value ✓ x value ✓ $k > 0,24$ ✓ $k < -4,24$ (4)
		[23]

QUESTION/VRAAG 5

5.1	$\sin(90^\circ - x) \cdot \tan(360^\circ - x) - 2 \sin(180^\circ + x)$ $= \cos x (-\tan x) - 2(-\sin x)$ $= \frac{-\cos x \cdot \sin x}{\cos x} + 2 \sin x$ $= \sin x$	✓ $\cos x$ ✓ $-\tan x$ ✓ $-\sin x$ ✓ tan identity ✓ answer (5)
5.2	$\sin(23^\circ + x) \cos(7^\circ - x) + \cos(23^\circ + x) \sin(7^\circ - x)$ $= \sin(23^\circ + x + 7^\circ - x)$ $= \sin 30^\circ$ $= \frac{1}{2}$	✓ compound angle ✓ $\sin 30^\circ$ ✓ answer (3)

5.3.1	 $\sin 2\theta = \frac{-4\sqrt{2}}{9} \quad \text{and} \quad 2\theta \in [90^\circ; 270^\circ]$ $x^2 = 9^2 - (-4\sqrt{2})^2 \quad (\text{Pythagoras})$ $x^2 = 81 - 32$ $x^2 = 49$ $x = -7$ $\therefore \cos 2\theta = \frac{-7}{9}$	✓ using Pythagoras ✓ $x = -7$ ✓ answer (3)
5.3.2	$\cos 2\theta = 1 - 2\sin^2 \theta$ $-\frac{7}{9} = 1 - 2\sin^2 \theta$ $2\sin^2 \theta = 1 + \frac{7}{9}$ $\sin^2 \theta = \frac{8}{9}$ $\sin \theta = \frac{\sqrt{8}}{3} \quad [\theta \in (45^\circ; 135^\circ)]$	✓ identity ✓ substitute value of $\cos 2\theta$ ✓ simplification ✓ answer in + surd form (4)
5.4	$\sin 2\alpha = 2(\cos^2 \alpha - \sin^2 \alpha)$ $= 2\cos 2\alpha$ $\tan 2\alpha = 2$ $2\alpha = 63,43^\circ + 180^\circ k$ $\alpha = 31,72^\circ + 90^\circ k; k \in \mathbb{Z}$ OR $\sin 2\alpha = 2(\cos^2 \alpha - \sin^2 \alpha)$ $= 2\cos 2\alpha$ $\tan 2\alpha = 2$ $2\alpha = 63,43^\circ + 360^\circ k \text{ or } 243,43^\circ + 360^\circ k$ $\alpha = 31,72^\circ + 180^\circ k; k \in \mathbb{Z}$ or $\alpha = 121,72^\circ + 180^\circ k; k \in \mathbb{Z}$	✓ factorise ✓ double identity ✓ double identity ✓ $\tan 2\alpha = 2$ ✓ solution 2α ✓ $31,72^\circ$ ✓ general solution period ✓ factorise ✓ double identity ✓ double identity ✓ $\tan 2\alpha = 2$ ✓ solution 2α ✓ $31,72^\circ$ & $121,72^\circ$ ✓ general solution period (7)

5.5.1	 $ \begin{aligned} LHS &= \frac{\cos 2x + \cos^2 x + 3 \sin^2 x}{2(1 - \sin^2 x)} \\ &= \frac{2 \cos^2 x - 1 + \cos^2 x + 3(1 - \cos^2 x)}{2 \cos^2 x} \\ &= \frac{3 \cos^2 x - 1 + 3 - 3 \cos^2 x}{2 \cos^2 x} \\ &= \frac{2}{2 \cos^2 x} \\ &= \frac{1}{\cos^2 x} = RHS \end{aligned} $	✓ compound identity ✓ identity numerator ✓ identity denominator ✓ simplification (4)
5.5.2	$\cos^2 x = 0$ $x = 90^\circ + 180^\circ k, k \in \mathbb{Z}$ $\therefore x = 90^\circ \text{ or } x = 270^\circ$	✓ 90° ✓ 270° (2)
		[28]

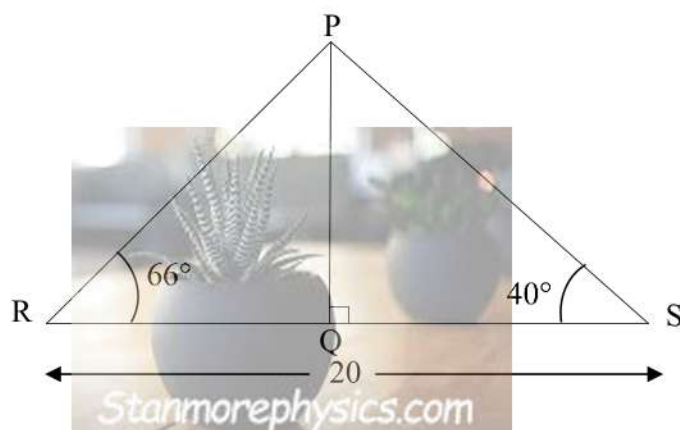


QUESTION/VRAAG 6

6.1	360°	$\checkmark 360^\circ$ (1)
6.2	$x = 180^\circ$	$\checkmark x = 180^\circ$ (1)
6.3		$\checkmark x$ intercepts $\checkmark y$ intercept $\checkmark (180^\circ; -2)$ (3)
6.4	$x = -90^\circ; 270^\circ; 45^\circ; 225^\circ$	$\checkmark$ Answer (any 2) (1)
6.5	$-5 \leq y \leq -1$ OR/OR $y \in [-5; -1]$	$\checkmark$ critical values $\checkmark$ answer (2)
6.6	$[5 - 2\sin(90^\circ - x)]^2$ $= [5 - 2\cos x]^2$ $= [5 - (-2)]^2$ $= 49$ AO FULL MARKS	$\checkmark$ co-function $\checkmark$ minimum value $\checkmark$ answer (3)
		[11]

QUESTION/VRAAG 7

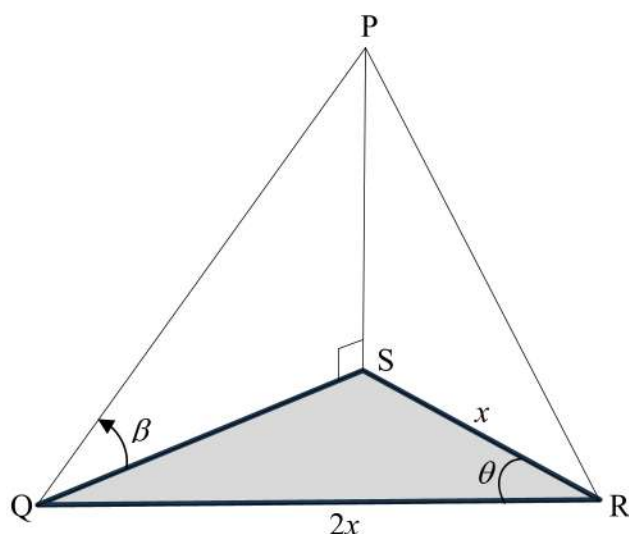
7.1



7.1.1	$\hat{RPS} = 74^\circ$ (Sum on the int $\angle$'s of Δ) / (Som v binne $\angle$ e v Δ) In ΔPRS : $\frac{PS}{\sin 66^\circ} = \frac{RS}{\sin 74^\circ}$ $PS = \frac{20 \cdot \sin 66^\circ}{\sin 74^\circ}$ $PS = 19,01 \text{ units}$	$\checkmark$ S $\checkmark$ substitution into sine rule $\checkmark$ answer (3)
7.1.2	In ΔPQS : $\frac{PQ}{PS} = \sin 40^\circ$ $PQ = 19,01(\sin 40^\circ)$ $PQ = 12,22 \text{ units}$	$\checkmark$ ratio $\checkmark$ answer (2)



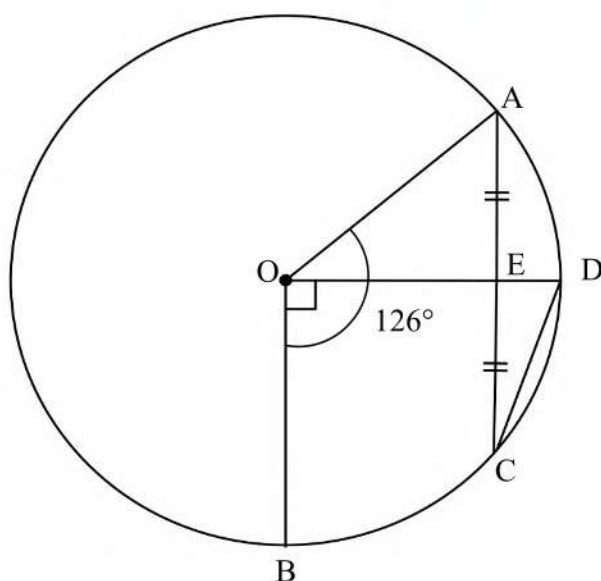
7.2



7.2.1	In $\triangle QRS$: $QS^2 = x^2 + (2x)^2 - 2(x)(2x)\cos\theta$ $QS^2 = x^2 + 4x^2 - 4x^2\cos\theta$ $QS^2 = 5x^2 - 4x^2\cos\theta$ $QS = \sqrt{x^2(5 - 4\cos\theta)}$ $QS = x\sqrt{5 - 4\cos\theta}$ In $\triangle PQS$: $\cos\beta = \frac{QS}{PQ}$ $PQ = \frac{QS}{\cos\beta}$ $PQ = \frac{x\sqrt{5 - 4\cos\theta}}{\cos\beta}$	✓ substitution into cosine rule ✓ simplification ✓ correct ratio ✓ substitution of QS and PQ as subject (4)
7.2.2	$57,36 = \frac{1}{2}(x)(2x)\sin 35^\circ$ $100,004\dots = x^2$ $10 = x$	✓ substitution into area rule ✓ answer (2)
		[11]

QUESTION/VRAAG 8

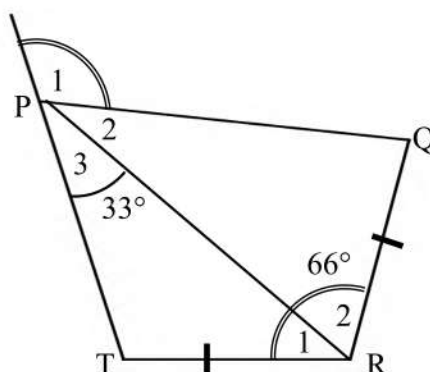
8.1



8.1.1	$\hat{AOE} = 126^\circ - 90^\circ$ $= 36^\circ$ $\hat{OEC} = 90^\circ$ (line from centre to midpoint of chord / <i>lyn van midpt v sirkel na middelpunt v koord</i>) $\hat{OAC} = 180^\circ - 90^\circ - 36^\circ$ ($\angle$ s of Δ) $= 54^\circ$	$\checkmark$ S $\checkmark$ S $\checkmark$ R $\checkmark$ S (4)
8.1.2	$\hat{ACD} = 18^\circ$ (Angle at the centre = $2 \times$ angle at circumference / <i>Middelpts hoek = $2 \times$ omtrekshoek</i>)	$\checkmark$ S $\checkmark$ R (2)



8.2

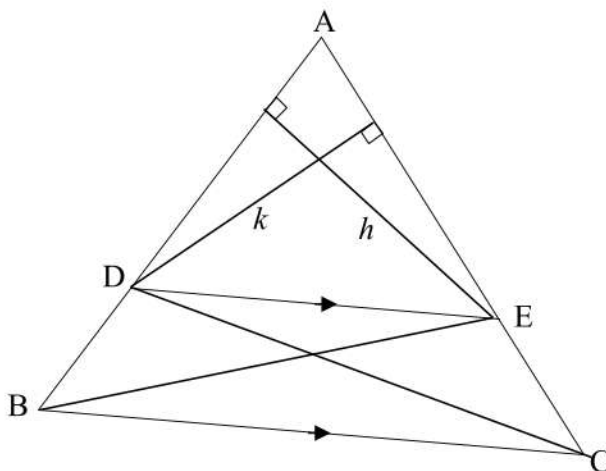


$\hat{P}_1 = \hat{R}$ (given/gegee) $\therefore PQRT = \text{cyclic quadrilateral/koordevh}$ (ext. $\angle = \text{opp interior } \angle /$ <i>buite $\angle = \text{teenoorst binne } \angle$)</i> OR (Converse ext $\angle$ cyclic quad) $\hat{P}_2 = \hat{P}_3 = 33^\circ$ (= chords; = $\angle s / = \text{koorde; } = \angle e$) $\hat{Q} = 180^\circ - (66^\circ + 33^\circ)$ ($\angle s$ of Δ) $= 81^\circ$ $\hat{T} = 99^\circ$ (ext $\angle$ of $\Delta PTR / \text{buite } \angle \text{ van } \Delta PTR$ OR/OF opp $\angle s$ cyc quad / <i>teenoorst $\angle e$ van koordevh</i>)	$\checkmark S$ PQRT = cyclic quad $\checkmark R$ $\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark S/R$
	(6)
	[12]



QUESTION/VRAAG 9

9.1

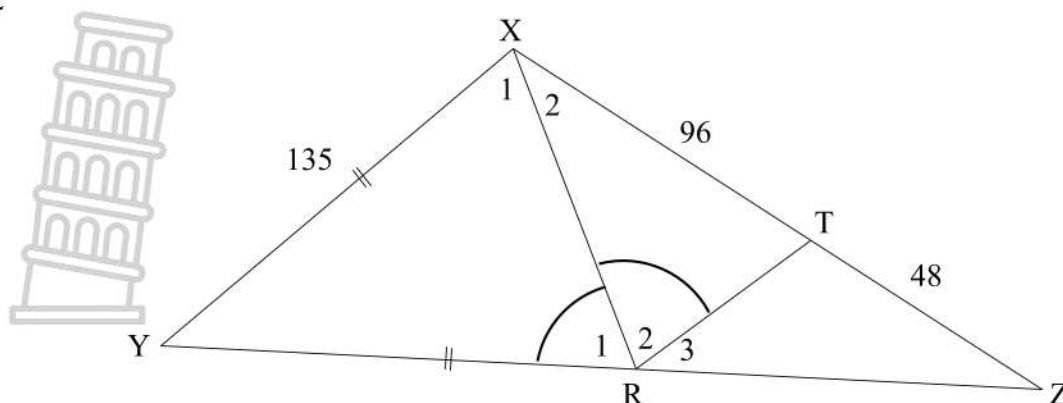


NOTE: No construction (0 /5)

9.1	Constr: Draw DC and BE. Draw Perpendicular height (h) from E on AD and perpendicular height (k) from D on AE. <i>Verbind DC en BE. Trek loodregte hoogte (h) van E op AD en loodregte hoogte (k) van D op AE</i> PROOF/BEWYS: $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\frac{1}{2} AD \cdot h}{\frac{1}{2} DB \cdot h} = \frac{AD}{DB} \quad (\text{same base and height})$ <i>(Selfde basis en loodregte hoogte)</i> $\frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE} = \frac{\frac{1}{2} AE \cdot k}{\frac{1}{2} EC \cdot k} = \frac{AE}{EC} \quad (\text{same base and height})$ Area $\triangle BDE$ = Area $\triangle CDE$ (Same base, same height/ Dieselfde basis en hoogte) $\therefore \frac{\text{Area } \triangle ADE}{\text{Area } \triangle BDE} = \frac{\text{Area } \triangle ADE}{\text{Area } \triangle CDE}$ $\therefore \frac{AD}{BD} = \frac{AE}{EC}$	✓ construction (On diagram or in words) ✓ both S ✓ R ✓ S /R ✓ S (5)
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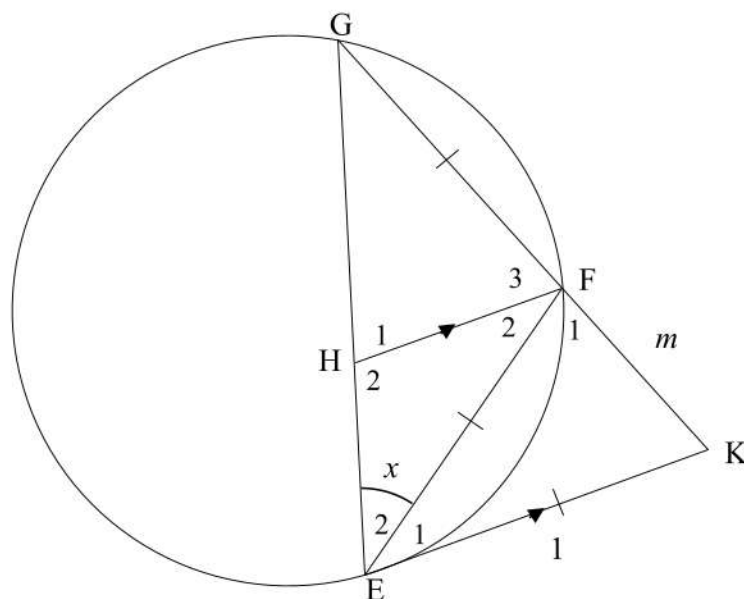
9.2



9.2.1	$\hat{R}_1 = \hat{X}_1$ ($\angle$ s opp = sides/ $\angle$ e teenoor = sye) $\therefore \hat{R}_2 = \hat{X}_1$ $\therefore RT \parallel YX$ (Alt $\angle$ s = /Verwisselende $\angle$ 'e =)	✓S/R ✓S ✓R (3)
9.2.2	$\frac{ZR}{RY} = \frac{ZT}{TX}$ (line $\parallel$ one side Δ /lyn $\parallel$ een sy Δ) OR (Proportionality th, $RT \parallel YX$) $\frac{RZ}{135} = \frac{48}{96}$ $RZ = 67,5cm$	✓S/R ✓S ✓answer (3)
		[11]



QUESTION/VRAAG 10



10.1	$\hat{G} = x$ ($\angle$ s opp = sides/ $\angle$ e teenoor = sye) $\hat{E}_1 = x$ (Tan-chord Th / Raaklyn-koord st) $\hat{F}_2 = x$ (Alt $\angle$'s, HF $\parallel$ EK / Verw $\angle$ e, HF $\parallel$ EK)	✓ S/R ✓ S ✓ R ✓ S/R (4)
10.2	$\hat{K} = 180 - 3x$ OR $\hat{K} = 2x$ OR $\hat{K} = 90 - \frac{1}{2}x$	✓ S (1)
10.3	$\hat{F}_1 = 180^\circ - 3x$ ($\angle$ s opp = sides/ $\angle$ e teenoor = sye) $x + 180^\circ - 3x + 180^\circ - 3x = 180^\circ$ ($\angle$ s of Δ) $-5x = -180^\circ$ $x = 36^\circ$ OR/OF $\hat{F}_1 = 180^\circ - 3x$ ($\angle$ s opp = sides/ $\angle$ e teenoor = sye) $180^\circ - 3x = 2x$ (ext $\angle$ of Δ) $-5x = -180^\circ$ $x = 36^\circ$	✓ S ✓ S/R ✓ Answer (3) ✓ S ✓ S/R ✓ Answer (3)



10.4.1	In $\triangle KEF$ and $\triangle KGE$: 1. $\hat{K} = \hat{K}$ (common) 2. $\hat{KEF} = \hat{G} = x$ (proven/bewys) $\therefore \triangle KGE \parallel \triangle KEF$ ($\angle; \angle; \angle$) OR/OF In $\triangle KEF$ and $\triangle KGE$: 1. $\hat{K} = \hat{K}$ (common) 2. $\hat{KEF} = \hat{G} = x$ (proven/bewys) 3. $\hat{F} = \hat{G} = 2x$ (proven/bewys) $\therefore \triangle KGE \parallel \triangle KEF$ (AAA)	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.4.2	$\frac{KG}{KE} = \frac{KE}{KF}$ ($\triangle KGE \parallel \triangle KEF$) $\frac{1}{m} = \frac{m+1}{1}$ ($KE = GF$) $m^2 + m = 1$ $m^2 + m - 1 = 0$	✓ S ✓ S (2)
10.5	$\frac{GH}{HE} = \frac{GF}{FK}$ (line $\parallel$ side of $\triangle$/lyn $\parallel$ sy van $\triangle$) $\frac{2}{HE} = \frac{1}{m}$ $HE = 2m$ $m^2 + m - 1 = 0$ $m = \frac{-1 \pm \sqrt{5}}{2}$ $= 0,62$ $\therefore HE = 1,24$ units OR/OF $\frac{GH}{HE} = \frac{GF}{FK}$ (line $\parallel$ side of $\triangle$/lyn $\parallel$ sy van $\triangle$) $\frac{2}{HE} = \frac{1}{m}$ $HE = 2m$ In $\triangle FEK$: $m^2 = (1)^2 + (1)^2 - 2(1)(1)\cos 36^\circ$ $= 0,381...$ $m = 0,62$ $\therefore HE = 1,24$ units	✓ S/R ✓ $HE = 2m$ ✓ m ✓ Answer (4) ✓ S/R ✓ $HE = 2m$ ✓ m ✓ Answer (4)
		[17]

TOTAL/TOTAAL: 150