



KWAZULU-NATAL PROVINCE

EDUCATION
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2024

Stanmorephysics.com

MARKS: 150

TIME: 3 hours



This question paper consists of 13 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer **ALL** the questions in the ANSWER BOOK provided.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off your answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.



QUESTION 1

The Human Resource Department of a company in KwaZulu–Natal wants to create a model to be used in determining the monthly salaries of its employees. Twelve of their current employees were surveyed and the information is displayed in the table below:

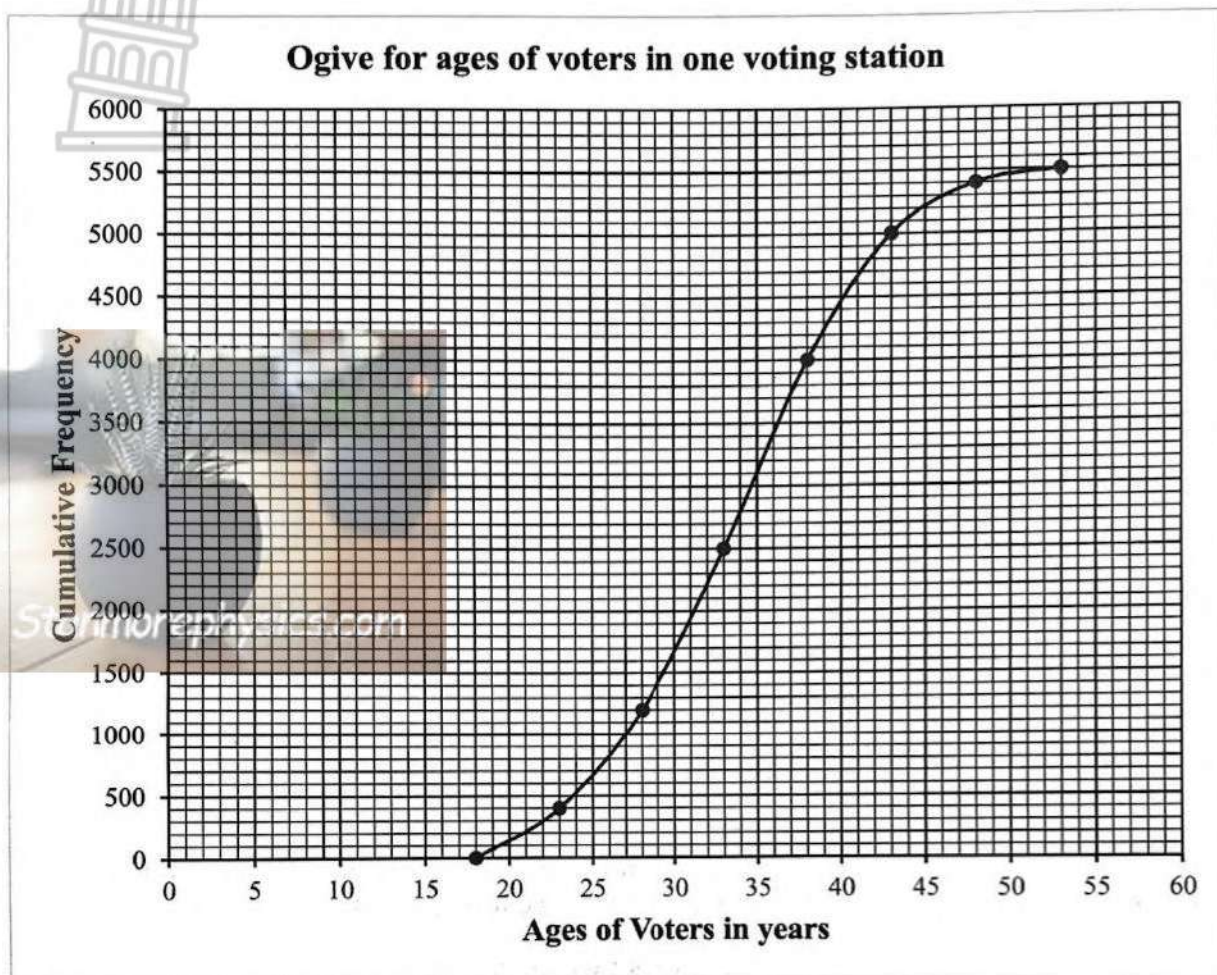
Employees' experience in number of years (x)	26	1	3	5	6	6	10	14	12	33	20	8
Salary in R1000s per month (y)	20	9	10,5	11	10	12	16	15	12	23	18	9

- 1.1 Calculate the
- 1.1.1 mean of the monthly salaries of these twelve employees. Round your answer off to the nearest rand. (2)
- 1.1.2 standard deviation of the monthly salaries of these twelve employees. Round your answer off to the nearest rand. (1)
- 1.2 How many of the twelve employees earn a monthly salary that is more than one standard deviation above the mean? (2)
- 1.3 Determine the equation of the least squares regression line for the data given in the table. (3)
- 1.4 Calculate the correlation coefficient between the experience in years and the monthly salary of an employee. (1)
- 1.5 Predict what the monthly salary will be of an employee who has been working for this company for 30 years. Round your answer off to the nearest rand. (2)
- 1.6 Is the prediction that is made in question 1.5 likely to be reliable? Give a reason for your answer. (2)

[13]

QUESTION 2

The cumulative frequency graph (ogive) drawn below shows the ages of the people who voted in the Local Government elections at one voting station. Use the graph to answer the questions that follow.



- 2.1 How many people voted at this voting station? (1)
- 2.2 Determine the interquartile range of the ages of the voters. (3)
- 2.3 What percentage of the voters was 25 years or younger? (2)

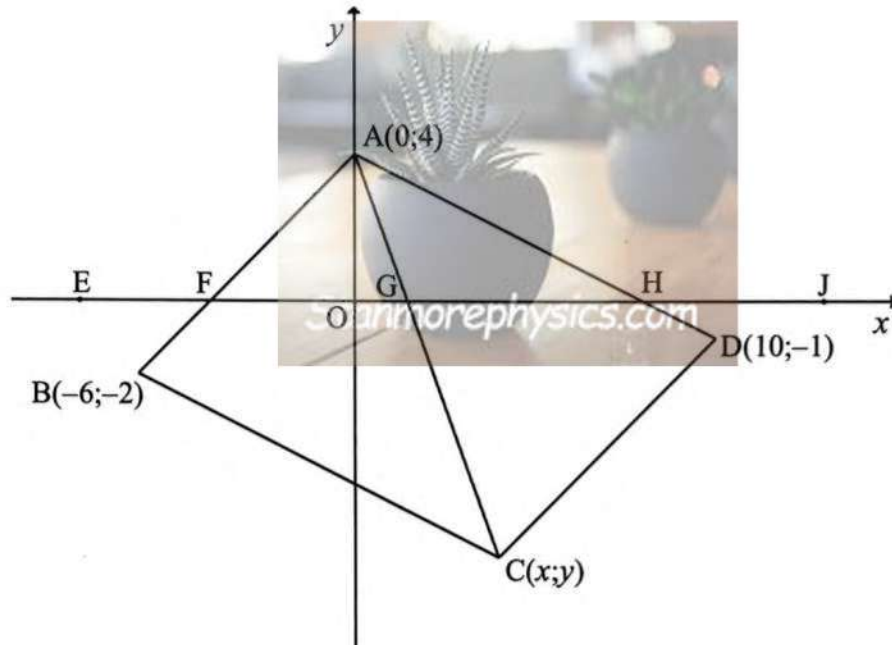
[6]

QUESTION 3

ABCD is a parallelogram with $A(0;4)$, $B(-6;-2)$, $C(x;y)$ and $D(10;-1)$ as shown below.

AC is drawn. F, G and H are the x -intercepts of AB, AC and AD respectively.

E is a point on the x -axis to the left of F and J a point on the x -axis to the right of H.



- 3.1 Determine the gradient of AB. (2)
 - 3.2 Determine the equation of CD. (3)
 - 3.3 Determine the coordinates of M, the midpoint of AC. (3)
 - 3.4 Determine the coordinates of C. (2)
 - 3.5 Determine the size of $\hat{BCD}$. (6)
- [16]**

QUESTION 4

4.1

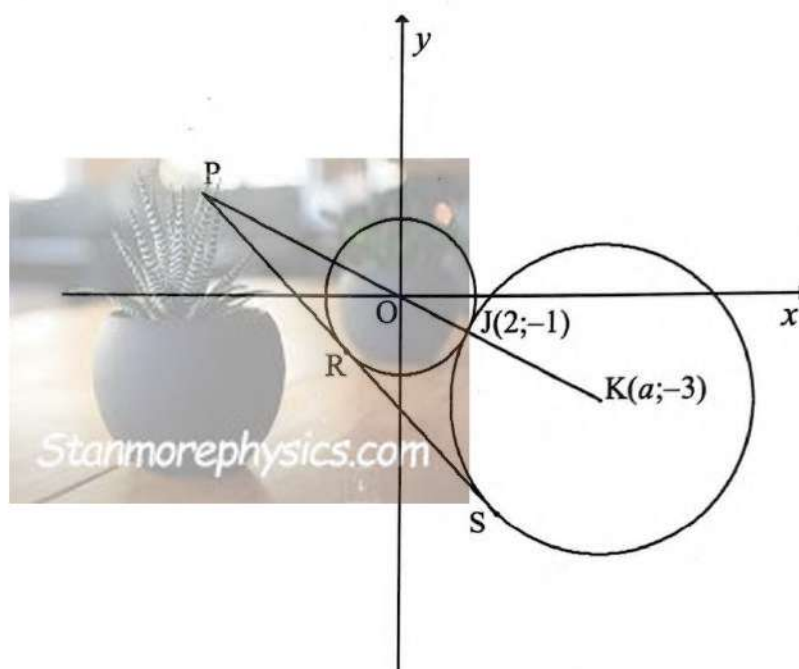
The diagram below shows two circles touching at $J(2;-1)$.

The smaller circle has its centre at the origin and the bigger circle has centre $K(a;-3)$.

The length of the radius of the bigger circle is TWICE the length of the radius of the smaller circle.

SR is a tangent to both circles, touching the bigger circle at S and the smaller circle at R.

KO and SR are both produced to intersect in point P.



4.1.1 Calculate the length of the radius of the smaller circle. (2)

4.1.2 Show that $a = 6$. (3)

4.1.3 Determine the equation of the bigger circle. (2)

4.1.4 Does the point $(10; -4)$ lie outside, inside or on the bigger circle? (3)

4.1.5 Calculate the length of PS. (5)

4.2

The length of the diameter of the circle with equation $x^2 - 4x + y^2 + 5y = -d$ is 24. Determine:

4.2.1 the coordinates of the centre of the circle. (4)

4.2.2 the value of d . (3)

[22]

QUESTION 5

5.1 If $\tan 58^\circ = n$, determine the following in terms of n without using a calculator.

5.1.1 $\sin 58^\circ$ (3)

5.1.2 $\sin 296^\circ$ (4)

5.1.3 $\cos 2^\circ$ (3)

5.2 Given the following identity:

$$\frac{1 - \cos 2x}{\sin 2x} = \tan x$$

5.2.1 Prove the identity. (3)

5.2.2 Use the identity to determine the value of $\tan 15^\circ$ in its simplest form. (3)
No calculator may be used.

5.3 Simplify to a single trigonometric ratio:

$$\sin(360^\circ + x) \cdot \cos(90^\circ + x) - \frac{\sin x}{\cos(-x) \cdot \tan(360^\circ - x)}$$
 (6)

5.4 Determine the general solution of: $\cos 2x - \frac{1}{3} = \frac{1}{3} \sin x$ (6)

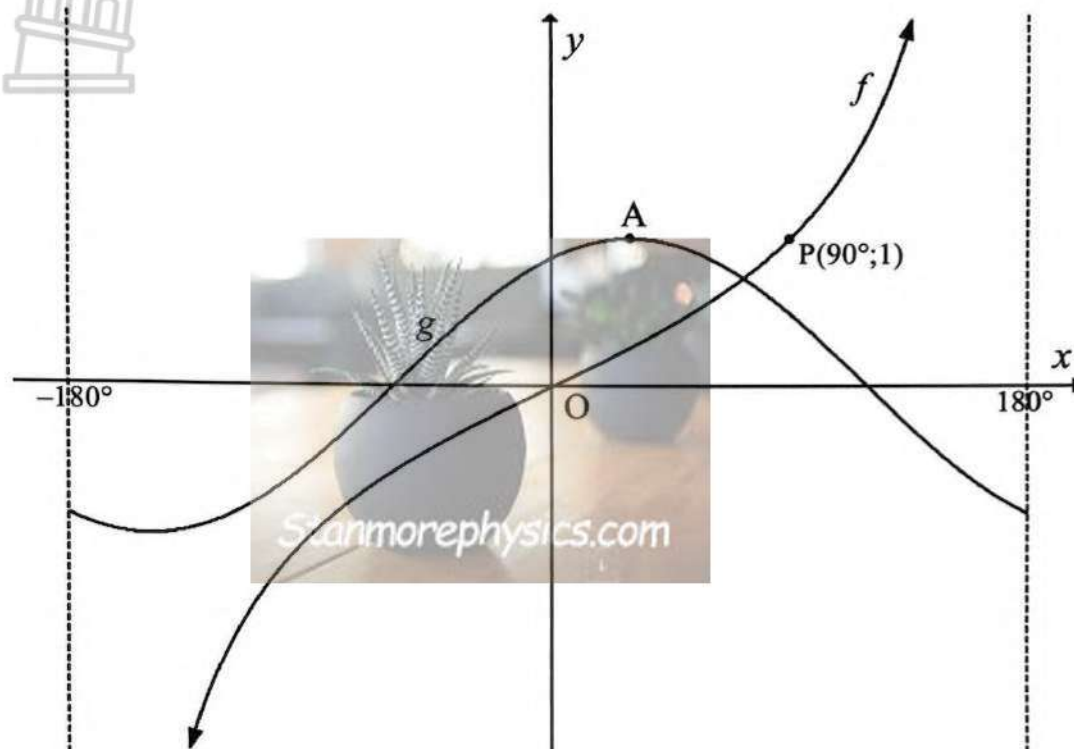
5.5 For which values of k will $\sin(2x + 30^\circ) + k = 3$ have no solution? (5)

[33]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30^\circ)$ are drawn on the same system of axes for $-180^\circ \leq x \leq 180^\circ$.

The point $P(90^\circ; 1)$ lies on f .



Use the diagram to answer the following questions:

- 6.1 Determine the value of b . (1)
- 6.2 Write down the period of g . (1)
- 6.3 Write down the coordinates of A , a turning point of g . (2)
- 6.4 Write down the equation(s) of the asymptote(s) of $y = \tan b(x + 20^\circ)$ for $x \in [-180^\circ; 180^\circ]$. (1)
- 6.5 Determine the range of h if $h(x) = 2g(x) - 1$. (2)

[7]

QUESTION 7

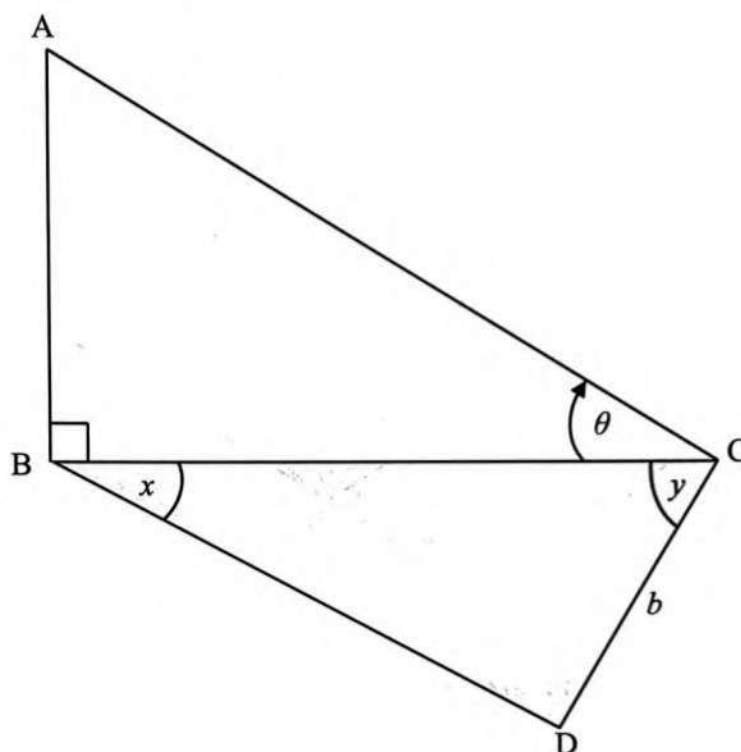
In the diagram, B, C and D lie in the same horizontal plane.

$BD = 2CD$.

$\hat{C}BD = x$, $\hat{B}CD = y$ and $CD = b$ meters.

AB is a vertical tower.

The angle of elevation of A from C is θ .



7.1 Show that $\sin y = 2 \sin x$. (2)

7.2 Prove that $AB = b \tan \theta \sqrt{5 + 4 \cos(x + y)}$ (7)

7.3 Hence, determine the height of the tower, rounded off to two decimal places, if:
 $b = 54,8$ metres, $x = 31^\circ$, $\theta = 42,6^\circ$ and $y = 75,84^\circ$. (2)

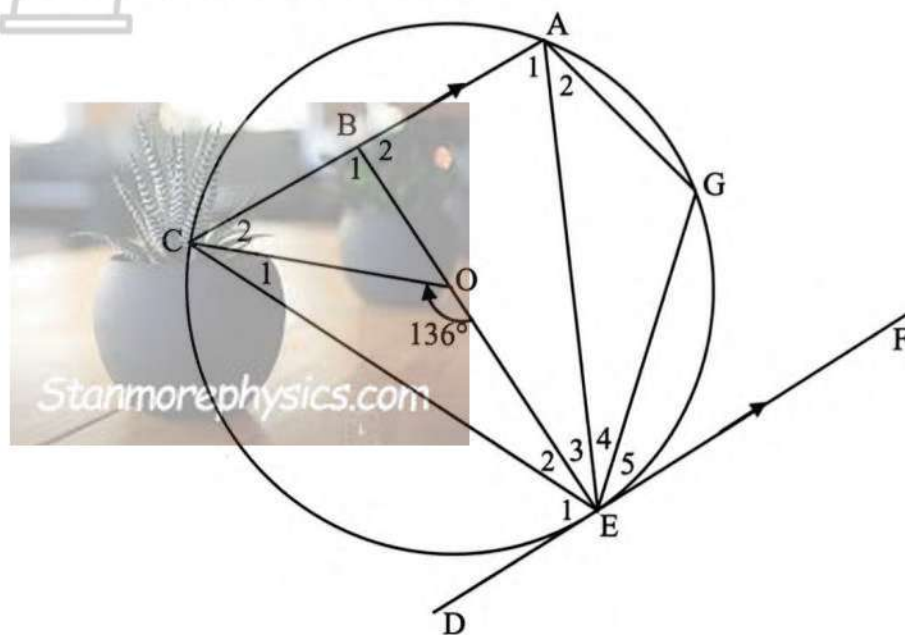
[11]

GIVE REASONS FOR YOUR STATEMENTS IN QUESTIONS 8, 9 AND 10.

QUESTION 8

In the diagram, A, C, E and G are points on the circumference of the circle with centre O.

$\angle COE = 136^\circ$. DEF is a tangent to the circle at E, with $DF \parallel CA$. BOE is a straight line, with B a point on AC. AE is drawn. $AC = 14$ units.



8.1 Calculate, with reasons, the size of each of the following:

8.1.1 $\hat{A}_1$ (2)

8.1.2 $\hat{E}_1$ (2)

8.1.3 $\angle BCE$ (2)

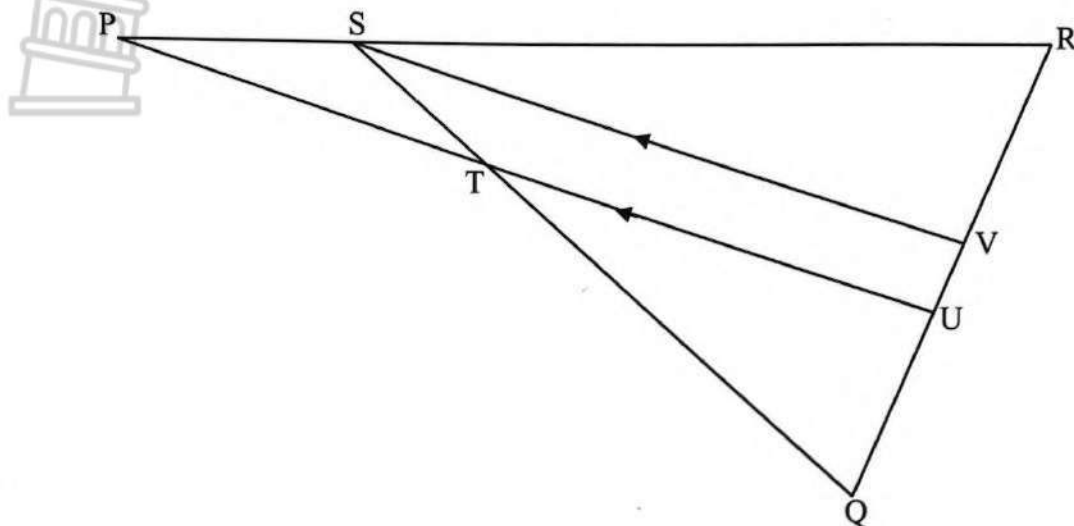
8.1.4 $\hat{G}$ (2)

8.2 Calculate, with reasons, the length of AB. (5)

[13]

QUESTION 9

In the diagram, $\triangle QRS$ is a triangle with RS produced to P . U and V are points on QR such that $PU \parallel SV$. PU intersects QS in T . $\frac{QU}{UR} = \frac{2}{3}$ and $\frac{QT}{TS} = \frac{5}{2}$.



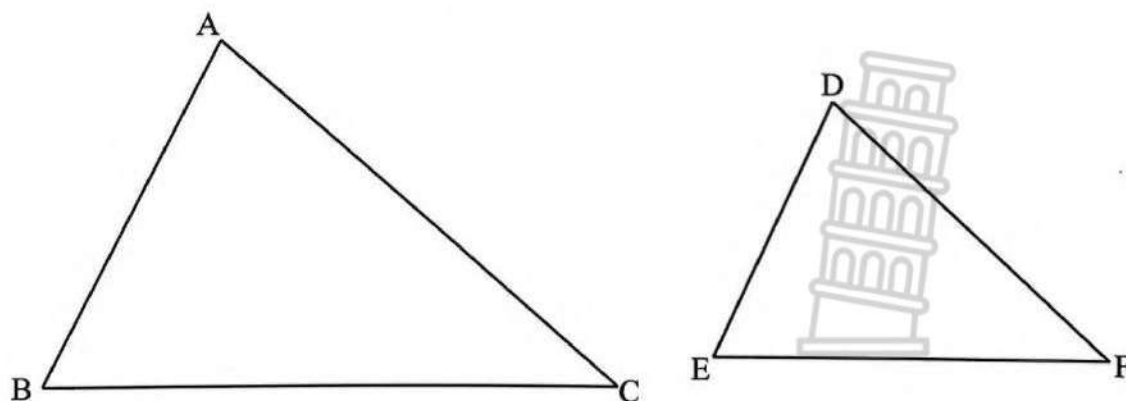
Calculate, giving reasons, the value of $\frac{PS}{SR}$.

(6)

[6]

QUESTION 10

10.1 In the diagram below, $\triangle ABC$ and $\triangle DEF$ are drawn with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.



Use the diagram in the ANSWER BOOK to prove the theorem which states that if two triangles are equiangular, then the corresponding sides are in proportion, i.e.

$$\frac{AB}{DE} = \frac{AC}{DF}.$$

(6)



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MARKING GUIDELINES

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These marking guidelines consist of 14 pages.

- If a candidate answered a QUESTION TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking guidelines.
- Assuming values/answer in order to solve a problem is unacceptable.

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason.)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
S/R	Award a mark if the statement AND reason are both correct.

QUESTION 1

Penalise only once for incorrect rounding in Question 1.

1.1.1	Mean = $\frac{165\,500}{12}$ = R13 792 Also accept: 13,79 thousand rand Answer only: Full marks If answer is given as 13,79 instead of R13 792, penalise in 1.1.1, but don't penalise again for this mistake in 1.1.2, 1.2 and 1.5.	✓ A 165 500 in numerator ✓ CA answer (2)
1.1.2	Standard deviation = R4 404	✓ A answer (1)
1.2	R13 792 + R4 404 = R18 196 2 employees earn a salary more than one standard deviation above the mean. Answer only: Full marks	✓ CA R18 196 ✓ CA 2 employees (2)
1.3	a = 8,45 b = 0,45 $\hat{y} = 0,45x + 8,45$ Answer only: Full marks	✓ A correct a value ✓ A correct b value ✓ CA answer (3)
1.4	r = 0,94	✓ A answer (1)
1.5	$\hat{y} = 0,45(30) + 8,45$ $\hat{y} = 21,95$ ∴ R21 950 OR R21 804 (calculator)	✓ CA substitution ✓ CA answer (2) OR ✓✓ CA CA (2)

1.6	Yes. $r = 0,94$ implies a strong correlation between employee experience and monthly salary and therefore a prediction would be reliable. OR Yes. $r = 0,94$, which is close to 1, and therefore implies a strong correlation between employee experience and monthly salary and therefore a prediction would be reliable.	✓CA answer ✓CA justification (2) OR ✓CA answer ✓CA justification (2)
		[13]

QUESTION 2

2.1	5500	✓A answer (1)
2.2	$Q_1 = 29$ (accept 28 – 29) $Q_3 = 39$ (accept 38 – 39) $IQR = 10$ (accept 9 – 11)	✓A value of Q_1 ✓A value of Q_3 ✓CA answer (3)
2.3	$\frac{650}{5500}$ (accept 620 – 700) $= 11,82\%$ (accept 11,27% – 12,73%)	✓A numerator in range 620 to 700 ✓CA answer (2)
		[6]

QUESTION 3

3.1	$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-2 - 4}{-6 - 0}$ $= 1$	✓A substitution ✓CA answer (2)
3.2	$m_{CD} = m_{AB} = 1$ $y = mx + c$ Substitute $(10 ; -1)$ and $m_{CD} = 1$: $-1 = 1(10) + c$ $c = -11$ $y = 1x - 11$	✓CA $m_{CD} = 1$ ✓CA substitution of gradient and point ✓CA answer (3)

3.3	Midpoint of AC is the same as the midpoint of BD [diagonals of parm. bisect each other] $\therefore$ Midpoint of AC $= M\left(\frac{-6+10}{2}; \frac{-2-1}{2}\right)$ $= M\left(2; \frac{-3}{2}\right)$ OR $C(4; -7)$ $\therefore$ Midpoint of AC $= M\left(\frac{0+4}{2}; \frac{4-7}{2}\right)$ $= M\left(2; \frac{-3}{2}\right)$ Answer only: Full marks	✓ A midpoint of BD ✓ CA x coordinate ✓ CA y-coordinate (3) OR ✓ A coordinates of C ✓ CA x coordinate ✓ CA y-coordinate (3)
3.4	$C(4; -7)$	✓ CA x coordinate ✓ CA y-coordinate (2)
3.5	$m_{AB} = 1$ $\tan \hat{AFG} = 1$ $\hat{AFG} = 45^\circ$ $m_{AD} = \frac{-1-4}{10-0}$ $= -\frac{1}{2}$ $\tan \hat{AHJ} = -\frac{1}{2}$ $\hat{AHJ} = 153, 43^\circ$ $\hat{BAD} = 153, 43^\circ - 45^\circ$ [exterior $\angle$ of $\triangle HAF$] $= 108, 43^\circ$ $\therefore \hat{BCD} = 108, 43^\circ$ [opp $\angle$ s of a parm.]	✓ CA $\tan \hat{AFG} = 1$ ✓ CA $\hat{AFG} = 45^\circ$ ✓ A $m_{AD} = -\frac{1}{2}$ ✓ CA $\hat{AHJ} = 153, 43^\circ$ ✓ CA $\hat{BAD} = 108, 43^\circ$ ✓ CA $\hat{BCD} = 108, 43^\circ$ (6)

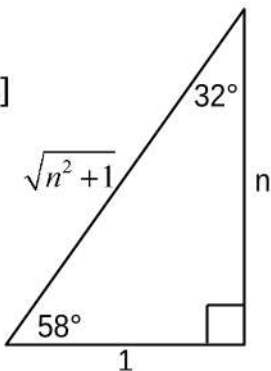
	OR $CD = \sqrt{(10 - 4)^2 + (-1 + 7)^2} = 6\sqrt{2}$ $BC = \sqrt{(-6 - 4)^2 + (-2 + 7)^2} = 5\sqrt{5}$ $BD = \sqrt{(-6 - 10)^2 + (-2 + 1)^2} = \sqrt{257}$ $BD^2 = BC^2 + CD^2 - 2 \cdot BC \cdot CD \cdot \cos \hat{BCD}$ $(\sqrt{257})^2 = (5\sqrt{5})^2 + (6\sqrt{2})^2 - 2 \cdot (5\sqrt{5}) \cdot (6\sqrt{2}) \cdot \cos \hat{BCD}$ $\therefore \cos \hat{BCD} = \frac{(5\sqrt{5})^2 + (6\sqrt{2})^2 - (\sqrt{257})^2}{2 \cdot (5\sqrt{5}) \cdot (6\sqrt{2})}$ $\hat{BCD} = 108,43^\circ$	OR ✓ CA length of CD ✓ CA length of BC ✓ A length of BD ✓ A use of cosine rule ✓ CA substitution into cosine rule ✓ CA answer (6)
		[16]

QUESTION 4

4.1.1	$r^2 = OJ^2 = 2^2 + (-1)^2$ $r = \sqrt{5}$	✓ A substitution ✓ A length of OJ (2)
4.1.2	$OK = OJ + JK = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$ $(3\sqrt{5})^2 = (a - 0)^2 + (-3 - 0)^2$ $45 = a^2 + 9$ $a^2 = 36$ $a = -6 \text{ or } a = 6$ N/A OR $OJ = \sqrt{5}$ $\therefore JK = 2\sqrt{5}$ $(2\sqrt{5})^2 = (a - 2)^2 + (-3 + 1)^2$ $20 = a^2 - 4a + 4 + 4$ $a^2 - 4a - 12 = 0$ $(a - 6)(a + 2) = 0$ $a = 6 \text{ or } a = -2$ N/A	✓ A length of OK ✓ A substitution ✓ A a^2 subject of formula (3) OR ✓ A length of JK ✓ A substitution ✓ A standard form (3)
4.1.3	$(x - 6)^2 + (y + 3)^2 = 20$	✓ A $(x - 6)^2 + (y + 3)^2$ ✓ CA = 20 (2)

4.1.4	Substitute (10; -4): $(10-6)^2 + (-4+3)^2$ $= 17$ $17 < 20,$ ∴ the point lies inside the circle	✓ CA substitution ✓ CA 17 < 20 ✓ CA conclusion (3)
4.1.5	$KO = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$ In $\triangle POR$ and $\triangle PKS$: $\hat{P} = \hat{P}$ [common] $\hat{PRO} = \hat{PSK}$ [= 90°; tangent ⊥ radius] $\hat{POR} = \hat{PKS}$ [remaining ∠s] $\triangle POR \parallel \triangle PKS$ [∠∠∠] $\frac{PO}{PK} = \frac{OR}{KS}$ [∥ ∆s] $= \frac{OR}{2OR} = \frac{1}{2}$ ∴ $PO = \frac{1}{2} PK$ $PO = OK = 3\sqrt{5}$ $PK = 2(3\sqrt{5}) = 6\sqrt{5}$ $\hat{PSK} = 90^\circ$ [radius ⊥ tangent] $PS^2 = PK^2 - KS^2$ [Theorem of Pythagoras] $= (6\sqrt{5})^2 - (2\sqrt{5})^2$ $= 160$ ∴ $PS = \sqrt{160} = 4\sqrt{10}$	✓ CA length of KO ✓ A $PO = \frac{1}{2} PK$ ✓ CA length of PK ✓ CA substitution in Theorem of Pythagoras ✓ CA answer (5)
4.2.1	$x^2 - 4x + 4 + y^2 + 5y + \frac{25}{4} = -d + 4 + \frac{25}{4}$ $(x-2)^2 + \left(y + \frac{5}{2}\right)^2 = -d + \frac{41}{4}$ Centre $\left(2; -\frac{5}{2}\right)$ Answer only: Full marks	✓ A completing the square ✓ A $(x-2)^2 + \left(y + \frac{5}{2}\right)^2$ ✓ CA x coordinate ✓ CA y coordinate (4)
4.2.2	diameter = 24 units, ∴ radius = 12 units $-d + \frac{41}{4} = 144$ $d = -\frac{535}{4}$	✓ A radius = 12 units ✓ CA equating ✓ CA answer (3)
		[22]

QUESTION 5

5.1.1	$\tan 58^\circ = n$  $r^2 = x^2 + y^2$ [Theorem of Pythagoras] $r^2 = 1^2 + n^2$ $r = \sqrt{n^2 + 1}$ $\therefore \sin 58^\circ = \frac{n}{\sqrt{1+n^2}}$	✓ A subst. in Theorem of Pythagoras ✓ A $r = \sqrt{n^2 + 1}$ ✓ CA answer (3)
5.1.2	$\sin 296^\circ$ $= -\sin 64^\circ$ $= -\sin 2(32^\circ)$ $= -2\sin 32^\circ \cos 32^\circ$ $= -2\cos 58^\circ \sin 58^\circ$ $= -2\left(\frac{1}{\sqrt{1+n^2}}\right)\left(\frac{n}{\sqrt{1+n^2}}\right)$ $= \frac{-2n}{1+n^2}$	✓ A $-\sin 64^\circ$ ✓ CA expansion ✓ CA co-functions ✓ CA answer (4)
5.1.3	$\cos 2^\circ$ $= \cos(60^\circ - 58^\circ)$ $= \cos 60^\circ \cos 58^\circ + \sin 60^\circ \sin 58^\circ$ $= \frac{1}{2} \times \frac{1}{\sqrt{1+n^2}} + \frac{\sqrt{3}}{2} \times \frac{n}{\sqrt{1+n^2}}$ $= \frac{1+\sqrt{3}n}{2\sqrt{1+n^2}}$ Stanmorephysics.com OR <del> $\cos 2^\circ$ $= \cos(32^\circ - 30^\circ)$ $= \cos 32^\circ \cos 30^\circ + \sin 32^\circ \sin 30^\circ$ $= \frac{n}{\sqrt{1+n^2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{1+n^2}} \times \frac{1}{2}$ $= \frac{\sqrt{3}n+1}{2\sqrt{1+n^2}}$ </del>	✓ A $\cos(60^\circ - 58^\circ)$ ✓ A expansion ✓ CA answer (3) OR ✓ A $\cos(32^\circ - 30^\circ)$ ✓ A expansion ✓ CA answer (3)

5.2.1	LHS $= \frac{1 - (1 - 2\sin^2 x)}{2\sin x \cos x}$ $= \frac{2\sin^2 x}{2\sin x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= \text{RHS}$ OR LHS $= \frac{\sin^2 x + \cos^2 x - (2\cos^2 x - 1)}{2\sin x \cos x}$ $= \frac{\sin^2 x - \cos^2 x + 1}{2\sin x \cos x}$ $= \frac{\sin^2 x - \cos^2 x + \sin^2 x + \cos^2 x}{2\sin x \cos x}$ $= \frac{2\sin^2 x}{2\sin x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= \text{RHS}$ OR LHS $= \frac{\sin^2 x + \cos^2 x - (\cos^2 x - \sin^2 x)}{2\sin x \cos x}$ $= \frac{2\sin^2 x}{2\sin x \cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= \text{RHS}$	✓ A $1 - 2\sin^2 x$ ✓ A $2\sin x \cos x$ ✓ A simplification (3) OR ✓ A $2\cos^2 x - 1$ ✓ A $2\sin x \cos x$ ✓ A simplification (3) OR ✓ A $\cos^2 x - \sin^2 x$ ✓ A $2\sin x \cos x$ ✓ A simplification (3)
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GRADE 12
Marking Guidelines

5.2.2	$\tan 15^\circ$ $= \frac{1 - \cos 2(15^\circ)}{\sin 2(15^\circ)}$ $= \frac{1 - \cos 30^\circ}{\sin 30^\circ}$ $= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $= \left(1 - \frac{\sqrt{3}}{2}\right) \times \frac{2}{1}$ $= 2 - \sqrt{3}$	$\checkmark A \frac{1 - \cos 2(15^\circ)}{\sin 2(15^\circ)}$ $\checkmark A \text{ substitution of special angle values}$ $\checkmark CA \text{ answer} \quad (3)$
5.3	$\sin(360^\circ + x) \cdot \cos(90^\circ + x) - \frac{\sin x}{\cos(-x) \cdot \tan(360^\circ - x)}$ $= \sin x \cdot (-\sin x) - \frac{\sin x}{\cos x \cdot (-\tan x)}$ $= -\sin^2 x + 1$ $= \cos^2 x$	$\checkmark A \sin x \quad \checkmark A -\sin x$ $\checkmark A \cos x \quad \checkmark A -\tan x$ $\checkmark CA 1$ $\checkmark CA \text{ answer} \quad (6)$
5.4	$\cos 2x - \frac{1}{3} = \frac{1}{3} \sin x$ $1 - 2\sin^2 x - \frac{1}{3} = \frac{1}{3} \sin x$ $3 - 6\sin^2 x - 1 = \sin x$ $6\sin^2 x + \sin x - 2 = 0$ $(3\sin x + 2)(2\sin x - 1) = 0$ $\sin x = -\frac{2}{3}$ $\therefore x = 221,81^\circ + k \cdot 360^\circ \text{ or } x = 318,19^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$ $\text{or } \sin x = \frac{1}{2}$ $\therefore x = 30^\circ + k \cdot 360^\circ \text{ or } x = 150^\circ + k \cdot 360^\circ, k \in \mathbb{Z}$	$\checkmark A 1 - 2\sin^2 x$ $\checkmark A \text{ standard form}$ $\checkmark CA \text{ factors}$ $\checkmark CA x = 221,81^\circ + k \cdot 360^\circ$ $\text{or } x = 318,19^\circ + k \cdot 360^\circ$ $\checkmark CA x = 30^\circ + k \cdot 360^\circ \text{ or } x = 150^\circ + k \cdot 360^\circ$ $\checkmark A k \in \mathbb{Z} \quad (6)$
5.5	$\sin(2x + 30^\circ) + k = 3$ $\sin(2x + 30^\circ) = 3 - k$ $\sin(2x + 30^\circ) < -1 \text{ or } \sin(2x + 30^\circ) > 1$ $3 - k < -1 \quad \text{or} \quad 3 - k > 1$ $k > 4 \quad \text{or} \quad k < 2$	$\checkmark A \sin(2x + 30^\circ) = 3 - k$ $\checkmark A \sin(2x + 30^\circ) < -1 \text{ or } \sin(2x + 30^\circ) > 1$ $\checkmark CA 3 - k < -1 \text{ or } 3 - k > 1$ $\checkmark CA k > 4$ $\checkmark CA k < 2 \quad (5)$
		[33]

QUESTION 6

6.1	$b = \frac{1}{2}$	✓ A answer (1)
6.2	period = 360°	✓ A answer (1)
6.3	A($30^\circ; 1$)	✓ A 30° ✓ A 1 (2)
6.4	$x = 160^\circ$	✓ A answer (1)
6.5	$-3 \leq y \leq 1$ OR $y \in [-3; 1]$	✓✓ AA OR ✓✓ AA (2)
		[7]

QUESTION 7

7.1	$\frac{\sin y}{2b} = \frac{\sin x}{b}$ $\sin y = \frac{2b \sin x}{b}$ $\sin y = 2 \sin x$ OR $b \sin y = 2b \sin x$	✓ A substitution in sine rule ✓ A $\sin y = \frac{2b \sin x}{b}$ OR $b \sin y = 2b \sin x$ (2)
7.2	$\frac{AB}{BC} = \tan \theta$ $\therefore AB = BC \cdot \tan \theta$ $\hat{D} = 180^\circ - (x + y)$ $BC^2 = BD^2 + CD^2 - 2BD \cdot CD \cos \hat{D}$ $BC^2 = (2b)^2 + b^2 + 2(2b)(b) \cos [180^\circ - (x + y)]$ $BC^2 = (2b)^2 + b^2 + 2(2b)(b) \cos (x + y)$ $BC^2 = 5b^2 + 4b^2 \cos (x + y)$ $BC^2 = b^2 (5 + 4 \cos (x + y))$ $BC = b \sqrt{5 + 4 \cos (x + y)}$ $\therefore AB = b \tan \theta \sqrt{5 + 4 \cos (x + y)}$	✓ A $\frac{AB}{BC} = \tan \theta$ ✓ A $AB = BC \cdot \tan \theta$ ✓ A $\hat{D} = 180^\circ - (x + y)$ ✓ A substitution in cosine rule ✓ A $+ \cos (x + y)$ ✓ A simplification ✓ A taking square root on LHS and RHS (7)
7.3	$AB = 54,8 \tan 42,6^\circ \sqrt{5 + 4 \cos (31^\circ + 75,84^\circ)}$ $AB = 98,76 \text{ metres}$	✓ A substitution ✓ A answer (2)
		[11]

QUESTION 8

8.1.1	$\hat{A}_1 = \frac{1}{2}(\hat{C}\hat{O}\hat{E})$ $= 68^\circ$	[$\angle$ at centre = $2 \times \angle$ at circumference]	$\checkmark$ R $\checkmark$ A answer (2)
8.1.2	$\hat{E}_1 = \hat{A}_1$ $= 68^\circ$	[tan chord theorem]	$\checkmark$ R $\checkmark$ CA answer (2)
8.1.3	$\hat{B}\hat{C}\hat{E} = \hat{E}_1$ $= 68^\circ$	[alt $\angle$ s; $DF \parallel CA$]	$\checkmark$ R $\checkmark$ CA answer (2)
8.1.4	$\hat{G} = 180^\circ - \hat{B}\hat{C}\hat{E}$ $= 112^\circ$	[opp. $\angle$ s of cyclic quad]	$\checkmark$ R $\checkmark$ CA answer (2)
8.2	$\hat{B}\hat{E}\hat{D} = 90^\circ$ $= \hat{B}_1$ $\therefore AB = \frac{1}{2}AC$ $= 7$ units	[radius $\perp$ tangent] [co-interior $\angle$ s ; $DF \parallel CA$] [line from centre $\perp$ to chord]	$\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ R $\checkmark$ A answer (5)
	OR $\hat{B}\hat{E}\hat{D} = 90^\circ$ $\therefore \hat{E}_2 = 22^\circ$ $\therefore \hat{B}_1 = 180^\circ - (\hat{B}\hat{C}\hat{E} + \hat{E}_2)$ $= 180^\circ - (68^\circ + 22^\circ)$ $= 90^\circ$ $\therefore AB = \frac{1}{2}AC$ $= 7$ units	[radius $\perp$ tangent] [sum of $\angle$ s of $\triangle BCE$] [line from centre $\perp$ to chord]	OR $\checkmark$ S $\checkmark$ R $\checkmark$ S/R $\checkmark$ R $\checkmark$ A answer (5)
			[13]

QUESTION 9

9.	In $\triangle QVS$: $\frac{QU}{UV} = \frac{QT}{TS}$ $= \frac{5}{2}$ $= \frac{5k}{2k}$ $\therefore 5k = 2x$; or: $x = \frac{5}{2}k$. And: $3x = \frac{15}{2}k$ In $\triangle UPR$: $\frac{PS}{PR} = \frac{UV}{UR}$ $= \frac{2k}{\frac{15}{2}k}$ $= \frac{4}{15}$ $\therefore \frac{PS}{SR} = \frac{4}{11}$	[prop. theorem; $UT \parallel VS$] OR [line $\parallel$ one side of $\triangle$] $\checkmark S \checkmark R$ $\checkmark x \text{ i.t.o. } k$ [prop. theorem; $UP \parallel VS$] OR [line $\parallel$ one side of $\triangle$] $\checkmark S$ $\checkmark S$ $\checkmark \text{answer}$
		(6) [6]

QUESTION 10

10.1	
	Construct $AP = DE$ and $AQ = DF$ In $\triangle APQ$ and $\triangle DEF$: 1. $AP = DE$ 2. $AQ = DF$ 3. $\hat{A} = \hat{D}$ $\therefore \triangle APQ \equiv \triangle DEF$ $\therefore \hat{P}_1 = \hat{E}$ But: $\hat{B} = \hat{E}$ $\therefore \hat{P}_1 = \hat{B}$ $\therefore PQ \parallel BC$ $\therefore \frac{AP}{AB} = \frac{AQ}{AC}$ $\therefore \frac{DE}{AB} = \frac{DF}{AC}$ [from construction] [from construction] [given] [SAS] [from $\equiv \Delta s$] [given] [corresponding $\angle s$ are $\equiv$] [prop. theorem; $PQ \parallel BC$] [$DE = AP$; $DF = AQ$] ✓ construction ✓ $\triangle APQ \equiv \triangle DEF$ ✓ $\hat{P}_1 = \hat{E}$ ✓ S ✓ R ✓ S/R
10.2.1	In $\triangle MKL$ and $\triangle MNP$: 1. $\frac{MK}{MN} = \frac{40}{16} = 2,5$ 2. $\frac{KL}{NP} = \frac{25}{10} = 2,5$ 3. $\frac{ML}{MP} = \frac{30}{12} = 2,5$ $\therefore \triangle MKL \parallel \triangle MNP$ [sides of Δs in proportion] ✓ S ✓ S ✓ S ✓ R

10.2.2	$\hat{NPM} = \hat{L}$ $\therefore$ KLNP is a cyclic quadrilateral OR $\hat{PNM} = \hat{K}$ $\therefore$ KLNP is a cyclic quadrilateral	[from Δs] [converse: ext. $\angle$ of cyclic quadrilateral] OR [ext. $\angle$ of quad = int. opp. $\angle$] [from Δs] [converse: ext. $\angle$ of cyclic quadrilateral] OR [ext. $\angle$ of quad = int. opp. $\angle$]	$\checkmark$S $\checkmark$R $\checkmark$R (3) OR $\checkmark$S $\checkmark$R $\checkmark$R (3)
10.3.1	In ΔBCE and ΔADE: 1. $\hat{E}_1 = \hat{E}_3$ 2. $\hat{C}_1 = \hat{D}_2$ 3. $\hat{B} = \hat{A}$ $\therefore \Delta BCE \parallel \Delta ADE$ $\therefore \frac{BC}{CE} = \frac{AD}{DE}$ $\therefore BC = \frac{AD \cdot CE}{DE}$	[vertically opp. $\angle$s] [$\angle$s in the same segment] [sum of $\angle$s of Δs] [$\angle \angle \angle$] [from Δs]	$\checkmark$ selecting triangles $\checkmark$S $\checkmark$S/R $\checkmark \hat{B} = \hat{A}$ OR [$\angle \angle \angle$] $\checkmark$R $\checkmark$S/R (5)
10.3.2	In ΔADE and ΔBDC: 1. $\hat{D}_2 = \hat{D}_1$ 2. $\hat{A} = \hat{B}$ 3. $\hat{E}_3 = \hat{BCD}$ $\therefore \Delta ADE \parallel \Delta BDC$ $\therefore \frac{AD}{BD} = \frac{DE}{CD}$ $\therefore AD \cdot CD = DE \cdot BD$ $= DE \cdot (DE + BE)$ $= DE^2 + DE \cdot BE$	[given] [$\angle$s in the same segment] [sum of $\angle$s of Δs] [$\angle \angle \angle$] [from Δs]	$\checkmark$ selecting triangles $\checkmark$S/R $\checkmark$R $\checkmark$S $\checkmark$ substitute DE + BE (5)
			[23]

TOTAL:

150