



**LIMPOPO**

PROVINCIAL GOVERNMENT  
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF  
**EDUCATION**

**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 12**

**MATHEMATICS P2**

**PRE – JUNE EXAMINATION 2024**

Stanmorephysics.com

**MARKS: 150**

**TIME: 3 Hours**

**This question paper consists of 8 pages and 1 information sheet.**

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 Questions.
2. Answer **ALL** the questions.
3. Number your answers correctly according to the numbering system used in this question paper.
4. Clearly show **ALL** calculations, diagrams and graphs that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
7. If necessary, answers should be rounded off to **TWO** decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.



### QUESTION 1

Fifty motorists were asked to record the number of kilometres travelled in one week.

The following table shows the results:

| Number of kilometres | Number of motorists | Cumulative frequency |
|----------------------|---------------------|----------------------|
| $10 < x \leq 20$     | 2                   |                      |
| $20 < x \leq 30$     | 7                   |                      |
| $30 < x \leq 40$     | 4                   |                      |
| $40 < x \leq 50$     | 13                  |                      |
| $50 < x \leq 60$     | 16                  |                      |
| $60 < x \leq 70$     | 8                   |                      |

- 1.1 Complete the Cumulative frequency column. (2)
- 1.2 Draw the cumulative frequency curve (ogive curve) (4)
- 1.3 Use your graph to estimate the median number of kilometres travelled per week. (2)
- 1.4 What percentage of motorists travelled more than 50km in one week? (2)

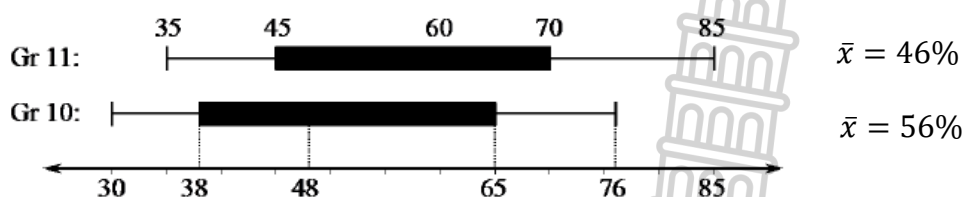
[10]

### QUESTION 2

- 2.1 The maximum daily temperatures in degrees Celsius for Polokwane for the first 10 days in July were recorded in the following table:

| Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 | Day 8 | Day 9 | Day 10 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| 24    | 25    | 22    | 28    | 27    | 21    | 18    | 17    | 24    | 25     |

- 2.1.1 Calculate the mean temperature for this data. (2)
- 2.1.2 Calculate the standard deviation from the mean for this data. (2)
- 2.1.3 How many days did the temperature lie outside one standard deviation of the mean? (4)
- 2.2 The box and whisker plots given below represent the Mathematics marks of the same 25 students at the end of their Grade 10 and 11 years at school.

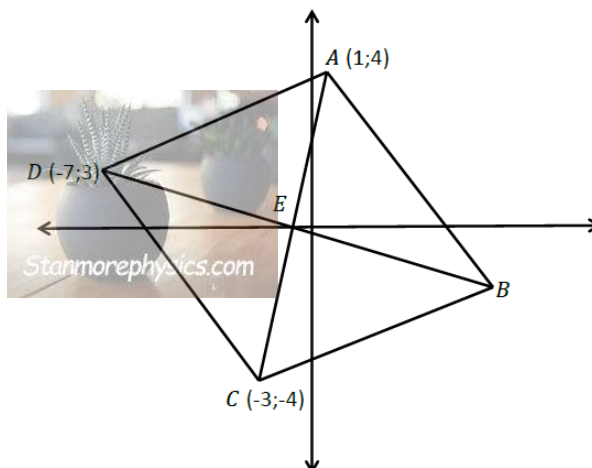


- 2.2.1 Calculate the inter-quartile range of each set of data. (4)
- 2.2.2 Comment on the way in which the distribution of the marks changes from the end of the Grade 10 year to the end of the Grade 11 year. (3)

[15]

### QUESTION 3

In the diagram below  $ABCD$  is a parallelogram.

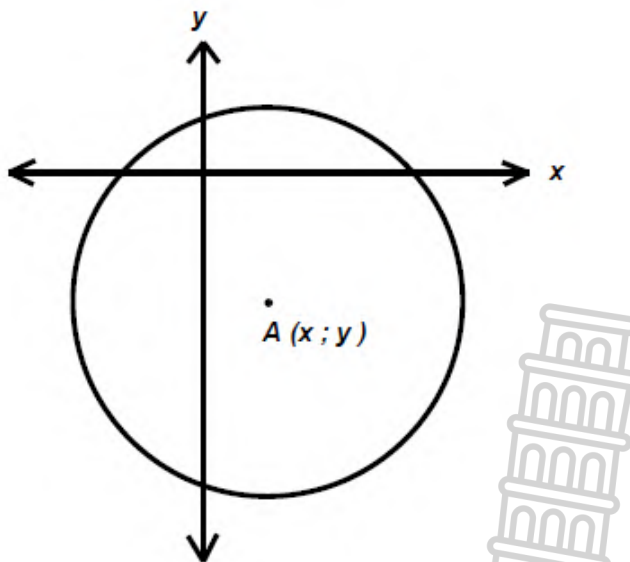


- 3.1 Give the co-ordinates of point  $B$ . (2)
- 3.2 Find  $E$ , the midpoint of  $AC$ . (3)
- 3.3 Show that  $AC \perp DB$ . (3)
- 3.4 Hence give a reason why  $ABCD$  is a rhombus. (1)
- 3.5 Find the size of angle  $ADC$ . (6)

[15]

### QUESTION 4

- 4.1 The equation of a circle is  $x^2 + y^2 - 2x + 4y - 4 = 0$ .



- 4.1.1 Determine the coordinates of  $A$ , the centre of the circle and the length of the radius,  $r$ . (5)
- 4.1.2 Calculate the value of  $p$  if  $N(1; p)$  with  $p > 0$  is a point on the circle. (1)
- 4.1.3 Determine the equation of the tangent to the circle at  $N$ . (2)
- 4.2 A second circle, centre  $B$ , with equation  $(x - 4)^2 + y^2 = k^2$  cuts the circle given in (4.1) twice. Determine the values of  $k$  for which point  $A$  will be inside the circle  $B$ . (6)

[14]

**QUESTION 5**

5.1 If  $\sin \alpha = \frac{-2}{3}$  and  $\cos \alpha > 0$ , calculate the values of the following without the use of a calculator:

5.1.1  $\tan \alpha$  (3)

5.1.2  $2\sin \alpha \cdot \cos \alpha$  (3)

5.1.3  $\sin^2 \alpha + \cos^2 \alpha$  (4)

5.2 Given that  $\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$ , derive the identity for  $\sin(\alpha - \beta)$  (4)

5.3 Determine, without the use of a calculator, the value of :  
 $\cos 35^\circ \cdot \sin 25^\circ - \cos(-205^\circ) \cdot \cos 55^\circ$  (5)

5.4 Consider the following identity:

$$\frac{2 \sin^3 x + \sin 2x \cos x}{\cos x} = 2 \tan x$$

5.4.1 Prove the identity. (4)

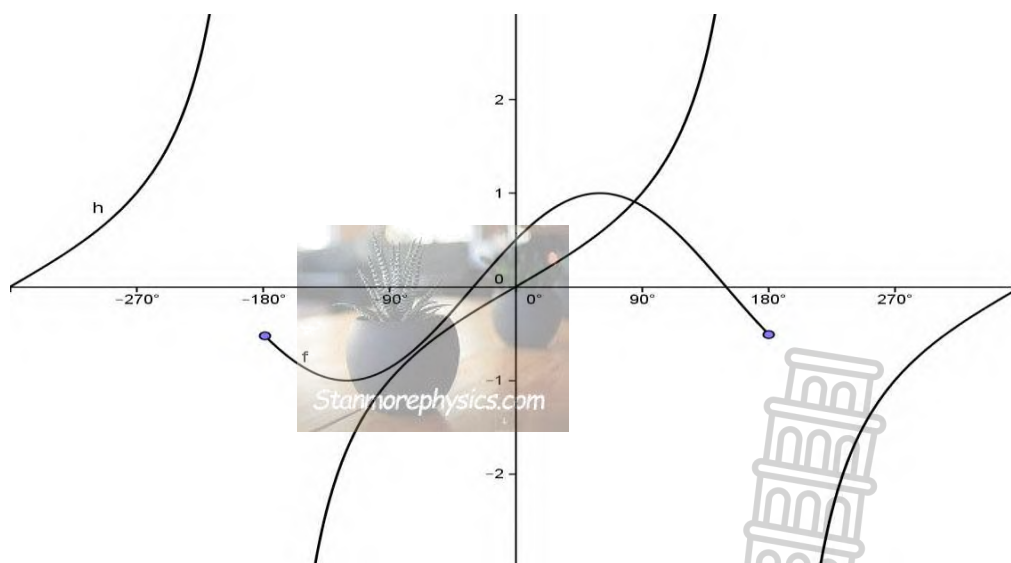
5.4.2 For which values of  $x$ ,  $x \in (-180^\circ; 180^\circ)$ , is this identity not valid? (2)

5.5 Determine the general solution for  $\cos 2x - \cos x = 2$  (7)

[32]

**QUESTION 6**

The diagram shows the graphs of  $f(x) = \sin(x + 30^\circ)$  and  $h(x) = \tan \frac{1}{2}x$ .



6.1 Write down the domain of  $f(x)$ . (2)

6.2 Write down the period of  $h(x)$ . (2)

6.3 Determine the equation of the function,  $g(x)$ , obtained when  $h(x)$  is translated  $45^\circ$  to the right and then 2 units downwards. (2)

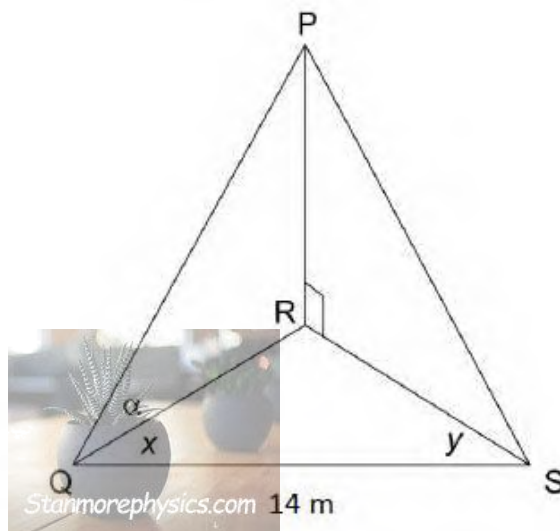
6.4 Determine a value of  $x$  for which  $f(x) - h(x) = \frac{1}{2}$ . (2)

6.5 Determine the distance between  $f(x)$  and  $h(x)$  if  $x = -135^\circ$ . (3)

[11]

### QUESTION 7

The diagram below shows a vertical tower PR, with points Q, R and S all on horizontal ground. The angle of elevation of P from Q is  $\alpha$ , the length QS = 14 m.  $\angle RQS = x$  and  $\angle RSQ = y$ .



7.1 Express PR in terms of QR and  $\alpha$  (2)

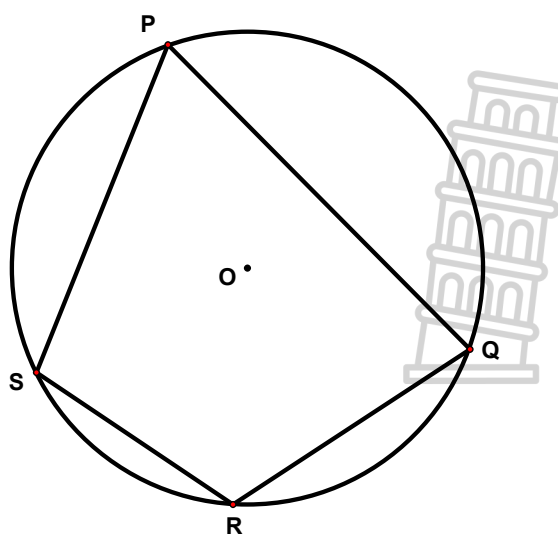
7.2 Show that  $QR = \frac{14 \sin y}{\cos y}$  (3)

7.3 If  $x = y$ , show that  $PR = \frac{7 \tan \alpha}{\cos y}$  (4)

[9]

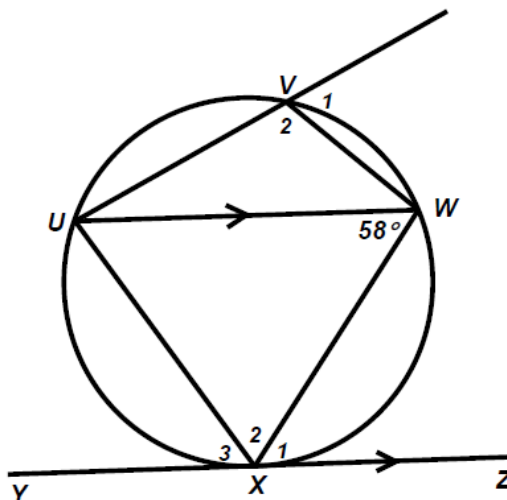
### QUESTION 8

8.1 In the diagram below, P, Q, R and S are points that lie on the circumference of the circle with centre O. Given below is the partially completed proof of the theorem that states that  $\hat{P} + \hat{R} = 180^\circ$ .



(7)

- 8.2 In the figure below, UVWX is a cyclic quadrilateral, with  $UW \parallel YZ$  and tangent YXZ touching the circle at X.  $\widehat{UWX} = 58^\circ$ .



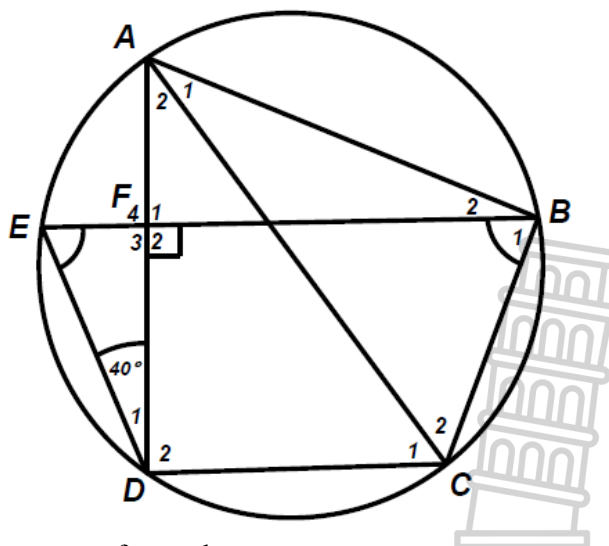
Determine the values of the following angles, showing all steps and reasons:

- |       |             |     |
|-------|-------------|-----|
| 8.2.1 | $\hat{X}_1$ | (2) |
| 8.2.2 | $\hat{X}_3$ | (2) |
| 8.2.3 | $\hat{X}_2$ | (2) |
| 8.2.4 | $\hat{V}_1$ | (2) |

[15]

### QUESTION 9

In the diagram below, AC is a chord of circle ABCDE.  $AFD \perp EFB$ ,  $\angle D1 = 40^\circ$  and  $\angle E = \angle B1$ .



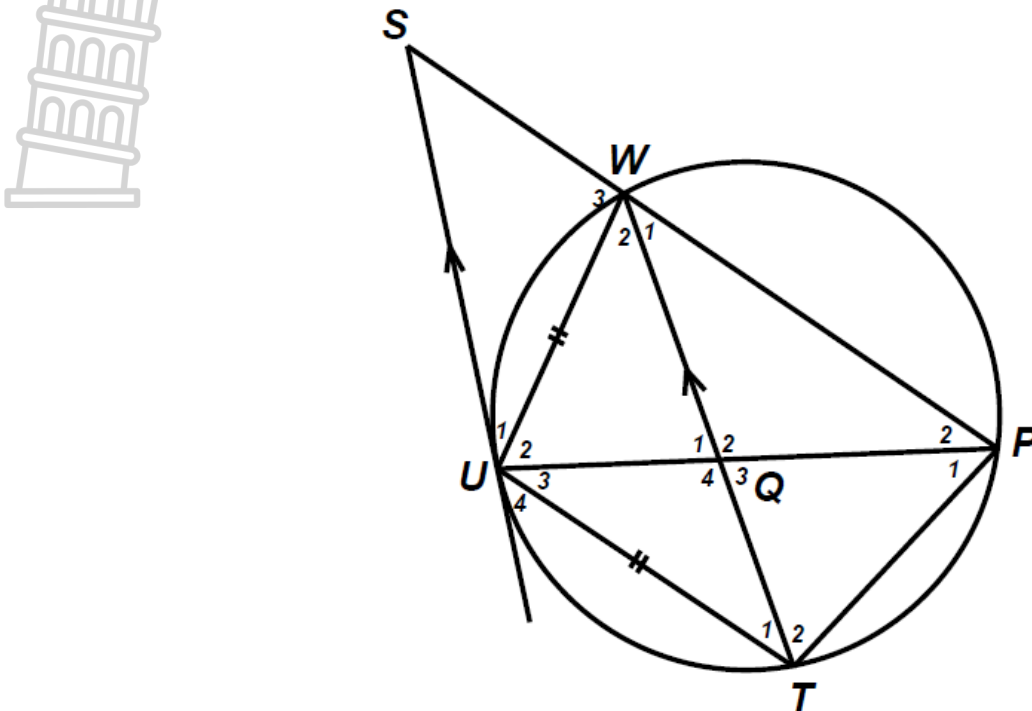
In the following questions, give a reason for each statement:

- |     |                                              |     |
|-----|----------------------------------------------|-----|
| 9.1 | Name THREE angles each equal to $50^\circ$ . | (3) |
| 9.2 | Calculate the size of $\angle DCB$ .         | (3) |
| 9.3 | Prove that $EB \parallel DC$ .               | (3) |
| 9.4 | Prove that AC is a diameter of the circle.   | (4) |

[13]

### QUESTION 10

In the diagram, WPTU is a cyclic quadrilateral with  $UW = UT$ . Chords WT and PU intersect at Q. PW extends to S such that  $SU \parallel WT$ .

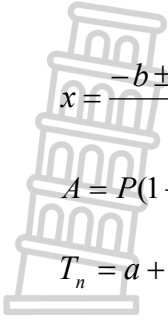


Prove that:

- 10.1 US is a tangent to circle PWUT at U. (6)
- 10.2  $\triangle SPU \sim \triangle SUW$  (4)
- 10.3  $SU^2 = SP \cdot SW$  (2)
- 10.4  $SU^2 \cdot QU = PU \cdot SW^2$  (4)

[16]

# INFORMATION SHEET



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



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DEPARTMENT OF  
**EDUCATION**

**VHEMBE WEST DISTRICT**

**GRADE 12**

**MATHEMATICS P2**

**PRE-MIDYEAR EXAMINATION 2024**

**MEMORANDUM**

Stanmorephysics.com

**MARKS: 140**



**This question paper consists of 21 pages including the cover page**

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**NOTA:**

- As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.

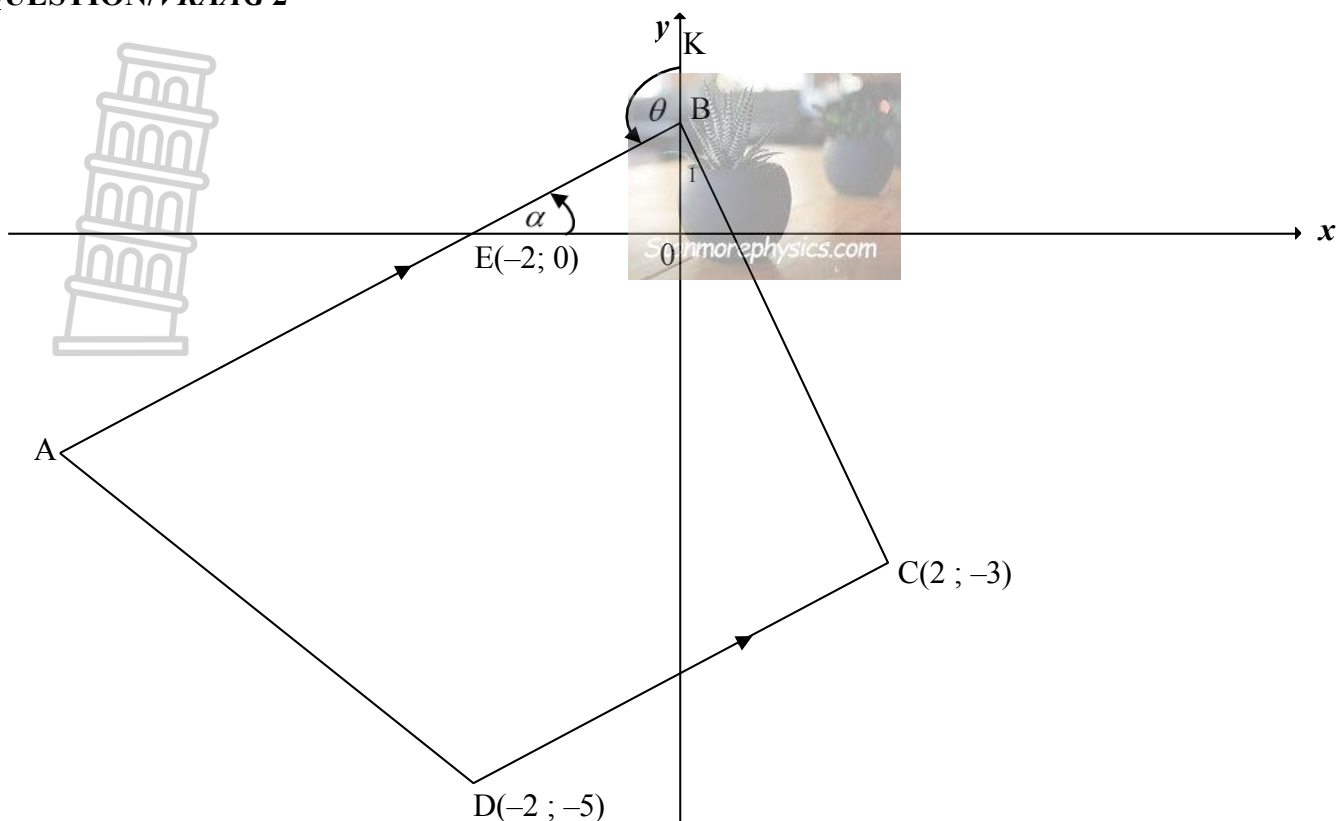
| GEOMETRY • MEETKUNDE |                                                                                                       |
|----------------------|-------------------------------------------------------------------------------------------------------|
| S                    | A mark for a correct statement<br>(A statement mark is independent of a reason)                       |
|                      | 'n Punt vir 'n korrekte bewering<br>( 'n Punt vir 'n bewering is onafhanklik van die rede)            |
| R                    | A mark for the correct reason<br>(A reason mark may only be awarded if the statement is correct)      |
|                      | 'n Punt vir 'n korrekte rede<br>( 'n Punt word slegs vir die rede toegeken as die bewering korrek is) |
| S/R                  | Award a mark if statement AND reason are both correct                                                 |
|                      | Ken 'n punt toe as die bewering EN rede beide korrek is                                               |

**QUESTION/VRAAG 1**

| 1.1                                | 45 children                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | ✓ answer<br>(1)                                                                                                     |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
|------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|-----------------------|-------------------------|----------------|---|---|-----------------|----|----|------------------|---|----|------------------|---|----|------------------|---|----|------------------|---|----|------------------|---|----|----------------------------------------------------------------------|
| 1.2                                | $\bar{x} = \frac{\sum fx}{n} = \frac{(4 \times 2) + (8 \times 10) + (12 \times 9) + (16 \times 7) + (20 \times 8) + (24 \times 7) + (28 \times 2)}{45}$ $\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15,38$ minutes<br>Answer only: full marks                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | ✓ 692<br>✓ answer<br>(2)                                                                                            |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| 1.3                                | <table border="1"> <thead> <tr> <th>Time taken (<math>t</math>)<br/>(in minutes)</th><th>Number of<br/>children</th><th>Cumulative<br/>frequency</th></tr> </thead> <tbody> <tr> <td><math>2 &lt; t \leq 6</math></td><td>2</td><td>2</td></tr> <tr> <td><math>6 &lt; t \leq 10</math></td><td>10</td><td>12</td></tr> <tr> <td><math>10 &lt; t \leq 14</math></td><td>9</td><td>21</td></tr> <tr> <td><math>14 &lt; t \leq 18</math></td><td>7</td><td>28</td></tr> <tr> <td><math>18 &lt; t \leq 22</math></td><td>8</td><td>36</td></tr> <tr> <td><math>22 &lt; t \leq 26</math></td><td>7</td><td>43</td></tr> <tr> <td><math>26 &lt; t \leq 30</math></td><td>2</td><td>45</td></tr> </tbody> </table> | Time taken ( $t$ )<br>(in minutes)                                                                                  | Number of<br>children | Cumulative<br>frequency | $2 < t \leq 6$ | 2 | 2 | $6 < t \leq 10$ | 10 | 12 | $10 < t \leq 14$ | 9 | 21 | $14 < t \leq 18$ | 7 | 28 | $18 < t \leq 22$ | 8 | 36 | $22 < t \leq 26$ | 7 | 43 | $26 < t \leq 30$ | 2 | 45 | ✓ first 4 cum freq<br>correct<br>✓ last 3 cum freq<br>correct<br>(2) |
| Time taken ( $t$ )<br>(in minutes) | Number of<br>children                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Cumulative<br>frequency                                                                                             |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $2 < t \leq 6$                     | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 2                                                                                                                   |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $6 < t \leq 10$                    | 10                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 12                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $10 < t \leq 14$                   | 9                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 21                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $14 < t \leq 18$                   | 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 28                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $18 < t \leq 22$                   | 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 36                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $22 < t \leq 26$                   | 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 43                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| $26 < t \leq 30$                   | 2                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 45                                                                                                                  |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| 1.4                                | <p><b>CUMULATIVE FREQUENCY GRAPH<br/>(OGIVE)</b></p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | ✓ plotting cum<br>freq at upper<br>limits correctly<br>(all points)<br>✓ shape (smooth)<br>✓ grounding (2;0)<br>(3) |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
| 1.5                                | On graph at the $y$ -value of 22,5 or 23<br>Median = $\pm 15$ minutes.<br>Answer only: full marks                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | graph<br>answer<br>(2)                                                                                              |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |
|                                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <b>[10]</b>                                                                                                         |                       |                         |                |   |   |                 |    |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                  |   |    |                                                                      |



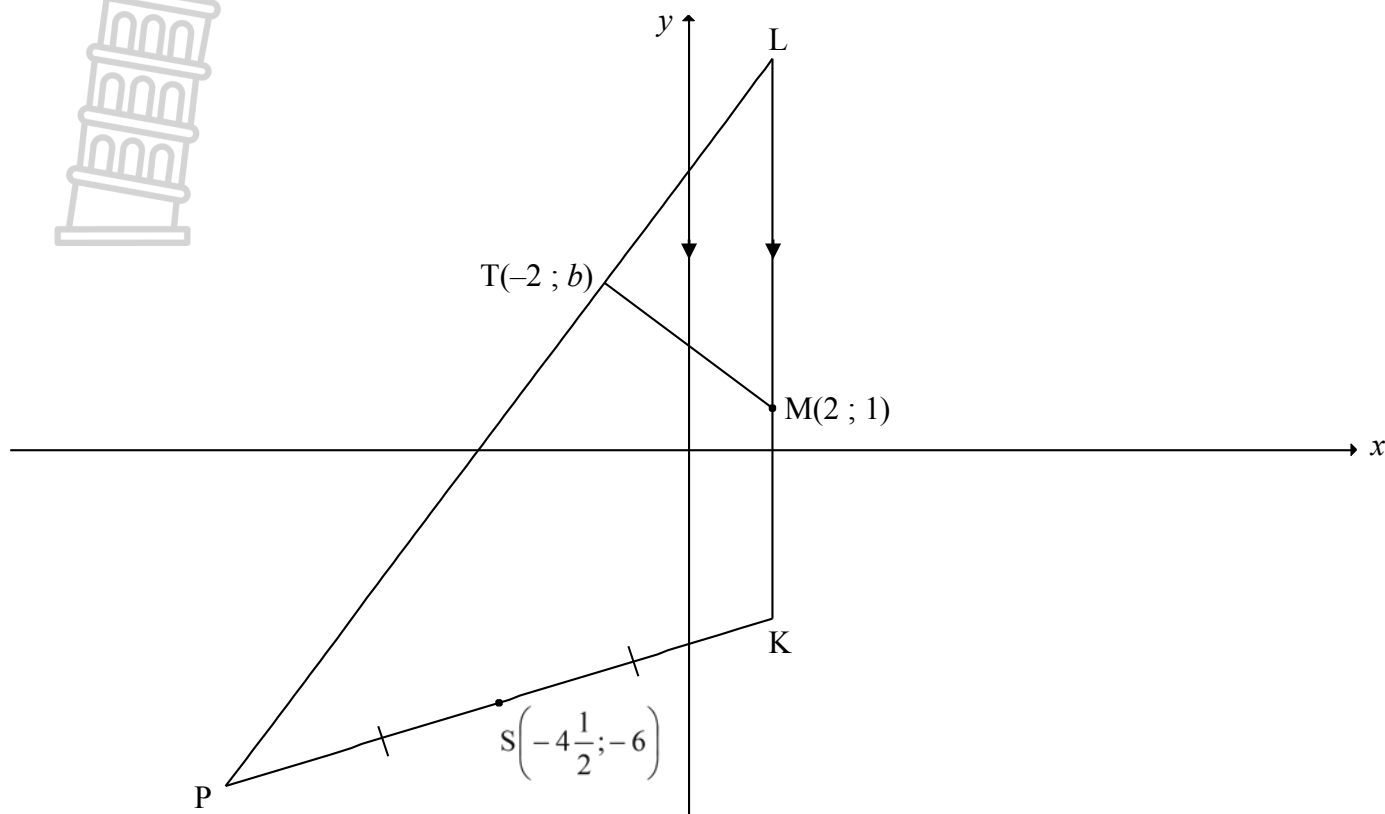
QUESTION/VRAAG 2



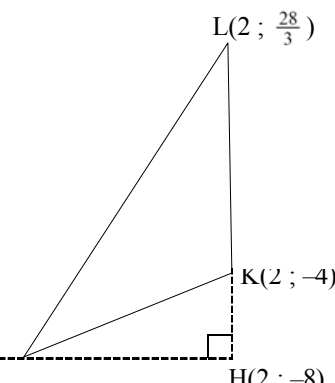
|       |                                                                                                                                                                                                                                    |                                                                                    |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| 2.1.1 | Midpoint of EC:<br>$= \left( \frac{-2+2}{2} ; \frac{0+(-3)}{2} \right) = \left( 0 ; \frac{-3}{2} \right)$                                                                                                                          | ☞ x value ☞ y value<br>(2)                                                         |
| 2.1.2 | $m_{DC} = \frac{-3-(-5)}{2-(-2)} \text{ OR } \frac{-5-(-3)}{-2-2}$ $= \frac{2}{4} = \frac{1}{2}$ <div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: full marks</div>                          | ☞ substitution<br>☞ answer<br>(2)                                                  |
| 2.1.3 | $m_{AB} = \frac{1}{2} \quad [AB \parallel DC]$ $y = \frac{1}{2}x + c$ $0 = \frac{1}{2}(-2) + c \quad \text{OR} \quad y - y_1 = \frac{1}{2}(x - x_1)$ $c = 1 \quad y - 0 = \frac{1}{2}(x - (-2))$ $\therefore y = \frac{1}{2}x + 1$ | ☞ $m_{AB} = \frac{1}{2}$<br>☞ substitution of (-2;0)<br>☞ equation<br>(3)          |
| 2.1.4 | $\tan \alpha = m_{AB} = \frac{1}{2}$ $\alpha = 26,57^\circ$ $\theta = 90^\circ + 26,57^\circ \quad [\text{ext } \angle \text{ of } \Delta]$ $= 116,57^\circ$                                                                       | ☞ $\tan \alpha = \frac{1}{2}$<br>☞ value of $\alpha$<br>☞ value of $\theta$<br>(3) |

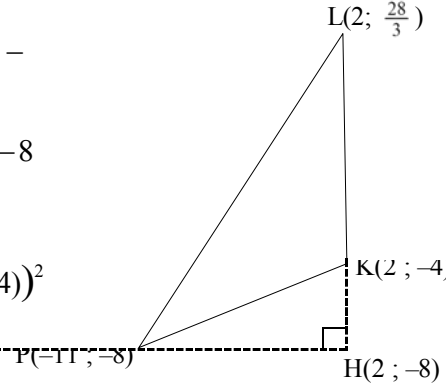
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                     |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2.2   | <p>B(0 ; 1)</p> $m_{BC} = \frac{1 - (-3)}{0 - 2} \quad \text{OR} \quad m_{BC} = \frac{(-3) - 1}{2 - 0}$ $= -2 \quad \quad \quad = -2$ $m_{AB} \times m_{BC} = \frac{1}{2} \times -2$ $= -1$ <p><math>\therefore AB \perp BC</math></p>                                                                                                                                                                                                                                                                                                            | <p>☞ coordinates of B</p> <p>☞ <math>m_{BC} = -2</math></p> <p>☞ product of gradients = -1</p> <p>(3)</p>                                                                                                                           |
| 2.3.1 | <p><math>\hat{A}BC = 90^\circ</math></p> <p><math>\therefore EC</math> is diameter [converse: <math>\angle</math> in semi circle]</p> <p><math>\therefore</math> centre of circle = <math>\left( 0 ; -\frac{3}{2} \right)</math></p>                                                                                                                                                                                                                                                                                                              | <p>☞ answer</p> <p>(1)</p>                                                                                                                                                                                                          |
| 2.3.2 | $(x-0)^2 + \left( y + \frac{3}{2} \right)^2 = r^2$ $(-2-0)^2 + \left( 0 + \frac{3}{2} \right)^2 = r^2 \quad \text{OR} \quad (2-0)^2 + \left( -3 - \left( -\frac{3}{2} \right) \right)^2 = r^2$ $\text{OR} \quad (0-0)^2 + \left( 1 - \left( -\frac{3}{2} \right) \right)^2 = r^2$ $\text{OR} \quad r = \frac{EC}{2} = \frac{\sqrt{(-2-2)^2 + (0-(-3))^2}}{2}$ $\text{OR} \quad r = 1 - \left( -\frac{3}{2} \right)$ $\therefore r^2 = \frac{25}{4} \quad \text{or} \quad r = \frac{5}{2}$ $x^2 + \left( y + \frac{3}{2} \right)^2 = \frac{25}{4}$ | <p>☞ substitution of centre</p> <p>☞ correct substitution of E(-1 ; 0), B(0 ; 1) or C(2 ; -3) to calculate <math>r^2</math> or <math>r</math></p> <p>☞ value of <math>r^2</math> or <math>r</math></p> <p>☞ equation</p> <p>(4)</p> |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | [18]                                                                                                                                                                                                                                |

QUESTION/VRAAG 3



|       |                                                                                                                                                                                                                    |                                                                                                                                  |                                                                                              |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|
| 3.1   | $(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $(b-1)^2 = 9$ $b-1 = \pm 3$ $\therefore b=4 \text{ or } b=-2$                                                                                                   | $(x-2)^2 + (y-1)^2 = 25$ $(-2-2)^2 + (b-1)^2 = 25$ $16 + b^2 - 2b + 1 = 25$ $b^2 - 2b - 8 = 0$ $\therefore b=4 \text{ or } b=-2$ | ☞ equation of the circle<br>☞ substitution of point T<br>☞ simplification<br>☞ answer<br>(4) |
| 3.2.1 | $K(2; 1-5)$ $\therefore K(2; -4)$                                                                                                                                                                                  | Answer only: full marks                                                                                                          | ☞ x value ☞ y value<br>(2)                                                                   |
| 3.2.2 | $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3} \quad [\text{radius} \perp \text{tangent}]$ $y = \frac{4}{3}x + c$ $4 = \frac{4}{3}(-2) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$ | $m_{MT} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3}$                                                                                   | ☞ substitution of $m_{PL}$ and the point T<br>☞ equation<br>(4)                              |

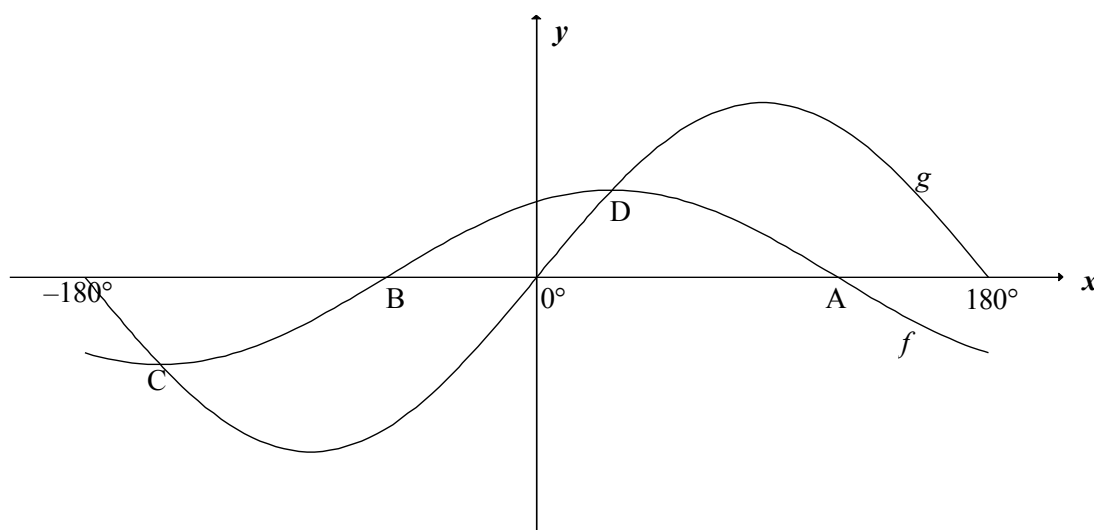
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|              | <p>OR</p> $m_{MT} = \frac{4-1}{-2-2} = -\frac{3}{4}$ $m_{PL} = \frac{4}{3} \quad [\text{radius} \perp \text{tangent}]$ $y - y_1 = \frac{4}{3}(x - x_1)$ $y - 4 = \frac{4}{3}(x + 2)$ $y = \frac{4}{3}x + \frac{20}{3}$ <p>OR</p> <p>P(-11 ; -8)</p> $m_{PL} = \frac{4 - (-8)}{-2 - (-11)}$ $= \frac{4}{3}$ $y = \frac{4}{3}x + c$ $-8 = \frac{4}{3}(-11) + c$ $c = \frac{20}{3}$ $y = \frac{4}{3}x + \frac{20}{3}$                                                                                                                                                                                                                                                                                                                             | <p>☞ <math>m_{MT}</math></p> <p>☞ <math>m_{PL} = \frac{4}{3}</math></p> <p>☞ substitution of <math>m_{PL}</math> and the point T</p> <p>☞ equation (4)</p> <p>☞ coordinates of P</p> <p>☞ <math>m_{PL} = \frac{4}{3}</math></p> <p>☞ substitution of <math>m_{PL}</math> and the point P or T</p> <p>☞ equation (4)</p> |
| <p>3.2.3</p> | <p><math>y = -\frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}</math></p> <p><math>(2 ; \frac{28}{3})</math> and K(2 ; -4): <math>LK = \frac{28}{3} - (-4) = \frac{40}{3}</math></p> <p><u>Coordinates of P:</u></p> $\frac{x+2}{-4} = \frac{1}{3} \quad \text{and} \quad \frac{y-4}{-6} = \frac{1}{3}$ <p><math>\therefore x = -11 \quad y = -8</math></p> <p><math>\therefore P(-11; -8)</math></p> <p><math>\perp</math> height (PH) = <math>2 - (-11) = 13</math></p> <p>Area <math>\Delta PKL = \frac{1}{2}(LK)(PH)</math></p> <p><math>= \frac{1}{2} \times \frac{40}{3} \times 13</math></p> <p><math>= \frac{260}{3}</math> OR 86,67 square units</p>  | <p>☞ <math>y = \frac{28}{3}</math></p> <p>☞ length of LK</p> <p>☞ <math>x_P</math> ☞ <math>y_P</math></p> <p>☞ length of <math>\perp</math> height</p> <p>☞ substitution into the area formula</p> <p>☞ answer (7)</p>                                                                                                  |

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                          |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>3.2.3</p> | <p><b>OR</b></p> $y = \frac{4}{3}(2) + \frac{20}{3} = \frac{28}{3}$ <p><math>(2; \frac{28}{3})</math> and <math>K(2; -4)</math>: <math>LK = \frac{28}{3} - (-4) = \frac{40}{3}</math></p> <p>Coordinates of P:</p> $\frac{x+2}{1} = - \frac{1}{1} \quad \frac{y-4}{1} = -$ <p><math>\therefore x = -11 \quad y = -8</math></p> <p><math>\therefore P(-11; -8)</math></p> $PK^2 = (-11-2)^2 + (-8-(-4))^2$ $\sqrt{185}$  <p><math>\tan \theta = \frac{4}{13} \therefore \theta = 17,1027\dots^\circ</math></p> <p><math>\therefore \hat{PKL} = 90^\circ + 17,1027\dots^\circ = 107,1^\circ</math></p> <p>Area <math>\Delta PKL = \frac{1}{2}(PK)(LK) \sin \hat{PKL}</math></p> $= \frac{1}{2}(\sqrt{185})\left(\frac{40}{3}\right) \sin 107,10^\circ$ $= 86,67 \text{ square units}$ | <p><math>y = \frac{28}{3}</math></p> <p>length of LK</p> <p><math>x_P \quad y_P</math></p> <p><math>\hat{PKL}</math></p> <p>substitution into the area rule</p> <p>answer</p> <p>(7)</p> |
| <p>3.3</p>   | <p>The centres of the two circles lie on the same vertical line <math>x = 2</math>. and the sum of the radii = 10</p> $n-1 = 10 \quad \text{or} \quad 1-n = 10$ $n=11 \quad \text{or} \quad n=-9$ <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: full marks</p> </div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | <p>correct method</p> <p>sum of radii = 10</p> <p><math>n=11 \quad n=-9</math></p> <p>(4)</p>                                                                                            |
|              | <p>[21]</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                          |

**QUESTION/VRAAG 4**

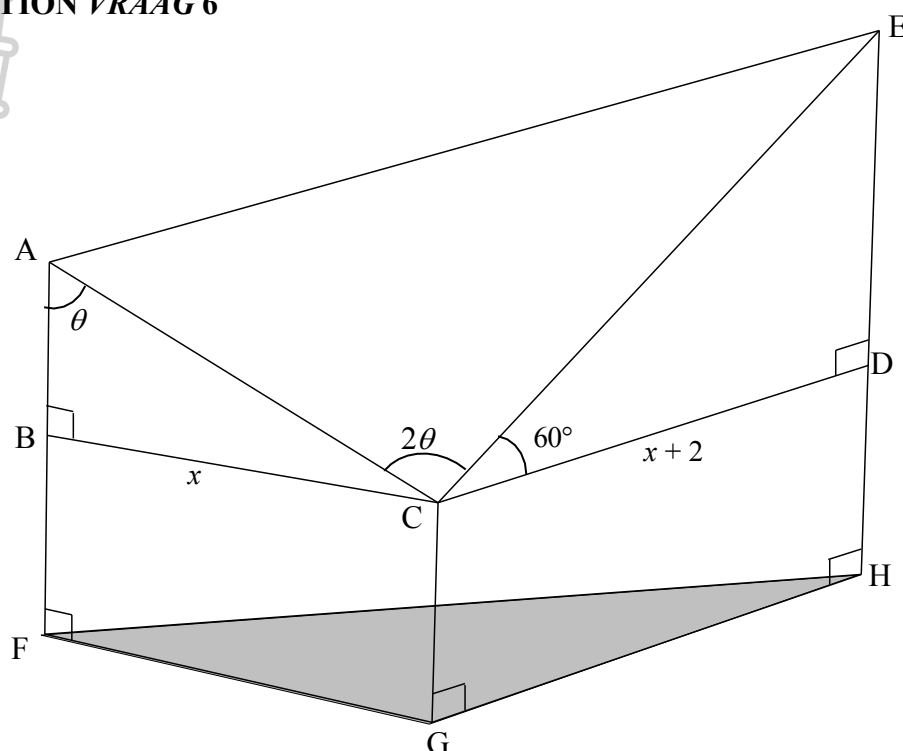
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|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4.1.1 | $\sin 191^\circ$<br>$= -\sin 11^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | $\Rightarrow -\sin 11^\circ$<br>(1)                                                                                                                                |
| 4.1.2 | $\cos 22^\circ$<br>$= \cos(2 \times 11^\circ)$<br>$= 1 - 2\sin^2 11^\circ$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | $\Rightarrow$ answer<br>(1)                                                                                                                                        |
| 4.2   | $\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$<br>$= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$<br>$= -\cos x + \sqrt{2}\left(\sin x \left(\frac{1}{\sqrt{2}}\right) + \cos x \left(\frac{1}{\sqrt{2}}\right)\right)$<br>$= -\cos x + \sin x + \cos x$<br>$= \sin x$<br><b>OR</b><br>$\cos(x - 180^\circ) + \sqrt{2} \sin(x + 45^\circ)$<br>$= -\cos x + \sqrt{2}(\sin x \cos 45^\circ + \cos x \sin 45^\circ)$<br>$= -\cos x + \sqrt{2}\left(\sin x \left(\frac{\sqrt{2}}{2}\right) + \cos x \left(\frac{\sqrt{2}}{2}\right)\right)$<br>$= -\cos x + \sin x + \cos x$<br>$= \sin x$ | $\Rightarrow -\cos x$ $\Rightarrow$ expansion<br>$\Rightarrow$ special angle ratios<br>$\Rightarrow$ simplification of last 2 terms<br>$\Rightarrow$ answer<br>(5) |
| 4.3   | $\sin P + \sin Q = \sin P + \cos P$<br>$(\sin P + \cos P)^2 = \left(\frac{7}{5}\right)^2$<br>$\sin^2 P + 2 \sin P \cos P + \cos^2 P = \frac{49}{25}$<br>$2 \sin P \cos P = \frac{49}{25} - 1$<br>$\sin 2P = \left(\frac{49}{25} - \frac{25}{25}\right)$<br>$= \frac{24}{25}$                                                                                                                                                                                                                                                                                                                                    | $\Rightarrow \sin Q = \cos P$<br>$\Rightarrow$ squaring<br>$\Rightarrow$ expansion<br>$\Rightarrow \sin^2 P + \cos^2 P = 1$<br>$\Rightarrow$ answer<br>(5)         |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <b>[12]</b>                                                                                                                                                        |

|     |                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                     |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.1 | $\cos(x - 30^\circ) = 2 \sin x$<br>$\cos x \cos 30^\circ + \sin x \sin 30^\circ = 2 \sin x$<br>$\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = 2 \sin x$<br>$\frac{\sqrt{3}}{2} \cos x = \frac{3}{2} \sin x$<br>$\tan x = \frac{\sqrt{3}}{3}$<br>$x = 30^\circ + k \cdot 180^\circ; \quad k \in \mathbb{Z}$<br><br><b>OR</b><br>$x = 30^\circ + k \cdot 360^\circ$ or $x = 210^\circ + k \cdot 360^\circ; \quad k \in \mathbb{Z}$ | <p>☞ expansion</p> <p>☞ special <math>\angle</math> s</p> <p>☞ simplification</p> <p>☞ equation in tan</p> <p>☞ <math>30^\circ</math></p> <p>☞ <math>k \cdot 180^\circ; k \in \mathbb{Z}</math></p> <p><b>OR</b></p> <p>☞ <math>30^\circ</math> and <math>210^\circ</math></p> <p>☞ <math>k \cdot 360^\circ; k \in \mathbb{Z}</math></p> <p>(6)</p> |
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|          |                                                                                                                                                                                                                                                                                                                                                                             |                                                              |
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| 5.2.1(a) | A(120° ; 0)                                                                                                                                                                                                                                                                                                                                                                 | ☞ answer (1)                                                 |
| 5.2.1(b) | C(-150° ; -1)                                                                                                                                                                                                                                                                                                                                                               | ☞ x value ☞ y value (2)                                      |
| 5.2.2(a) | $x \in (-90^\circ ; 30^\circ)$ OR $-90^\circ < x < 30^\circ$                                                                                                                                                                                                                                                                                                                | ☞ endpoints<br>☞ correct interval (2)                        |
| 5.2.2(b) | $x \in (-160^\circ ; 20^\circ)$ OR $-160^\circ < x < 20^\circ$                                                                                                                                                                                                                                                                                                              | ☞ endpoints<br>☞ correct interval (2)                        |
| 5.2.3    | $y = 2^{2 \sin x + 3}$<br>Range of $y = 2 \sin x$ : $y \in [-2 ; 2]$ <b>OR</b> $-2 \leq y \leq 2$<br>Range of $y = 2 \sin x + 3$ : $y \in [1 ; 5]$ <b>OR</b> $1 \leq y \leq 5$<br>Range: $y = 2^{2 \sin x + 3}$ : $y \in [2 ; 32]$ <b>OR</b> $2 \leq y \leq 32$<br><div style="border: 1px solid black; padding: 2px; display: inline-block;">Answer only: full marks</div> | <p>☞ 1 ☞ 5</p> <p>☞ 2 ☞ 32</p> <p>☞ correct interval (5)</p> |

/ QUESTION VRAAG 6



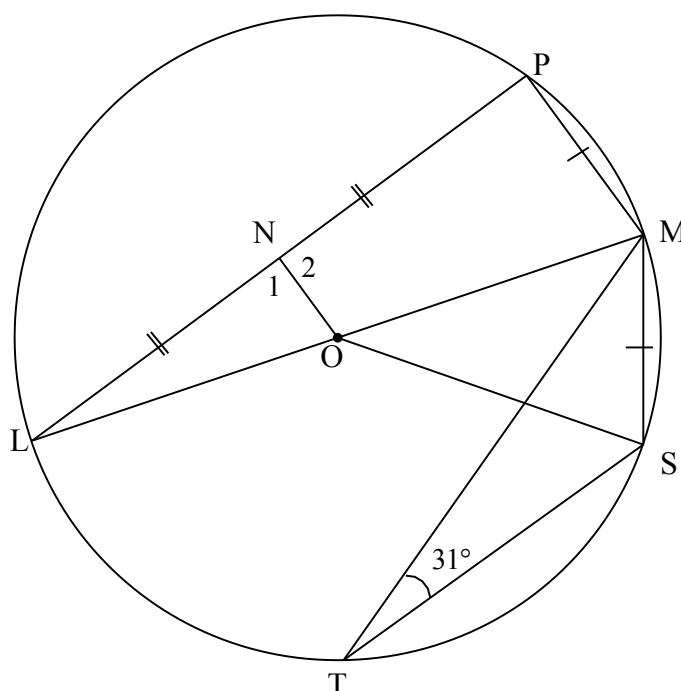
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| 6.1.1 | $\sin \theta = \frac{x}{AC}$ $AC = \frac{x}{\sin \theta}$ <p style="text-align: center;"><b>OR</b></p> $\frac{\sin \theta}{x} = \frac{\sin 90^\circ}{AC}$ $AC = \frac{x}{\sin \theta}$                                                                                                                                                                                                                                                | <p>↻ trig ratio</p> <p>↻ simplification</p> <p style="text-align: right;">(2)</p>                                                                                                     |
| 6.1.2 | $\cos 60^\circ = \frac{x+2}{CE}$ $CE = \frac{x+2}{\cos 60^\circ}$ $= \frac{x+2}{\frac{1}{2}} = 2(x+2)$ <p style="text-align: center;"><b>OR</b></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <math display="block">\frac{\sin 30^\circ}{x+2} = \frac{\sin 90^\circ}{CE}</math> <math display="block">CE = \frac{x+2}{\sin 30^\circ}</math> <math display="block">= 2(x+2)</math> </div> | <p>↻ trig ratio</p> <p>↻ making CE the subject</p> <p style="text-align: right;">(2)</p>                                                                                              |
| 6.2   | $\text{Area } \triangle ACE = \frac{1}{2} AC \cdot EC \cdot \sin \hat{A} C E$ $= \frac{1}{2} \left( \frac{x}{\sin \theta} \right) (2(x+2)) \sin 2\theta$ $= \frac{x(x+2) \times 2 \sin \theta \cos \theta}{\sin \theta}$ $= 2x(x+2) \cos \theta$                                                                                                                                                                                      | <p>↻ use area rule correctly</p> <p>↻ substitution of <math>\frac{x}{\sin \theta}</math></p> <p>↻ substitution of <math>\sin 2\theta</math></p> <p style="text-align: right;">(3)</p> |

|     |                                                                                                                                                                                                      |                                                                                                 |
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| 6.3 | $EC = 2(12 + 2) = 28$ $AE^2 = AC^2 + EC^2 - 2(AC)(EC)\cos\hat{A}CE$ $= \left( \frac{12}{\sin 55^\circ} \right)^2 + 28^2 - 2 \left( \frac{12}{\sin 55^\circ} \right)(28)\cos 110^\circ$ $AE = 35,77m$ | <p>☞ EC</p> <p>☞ use cosine rule correctly</p> <p>☞ substitution</p> <p>☞ answer</p> <p>(4)</p> |
|     |                                                                                                                                                                                                      | [11]                                                                                            |



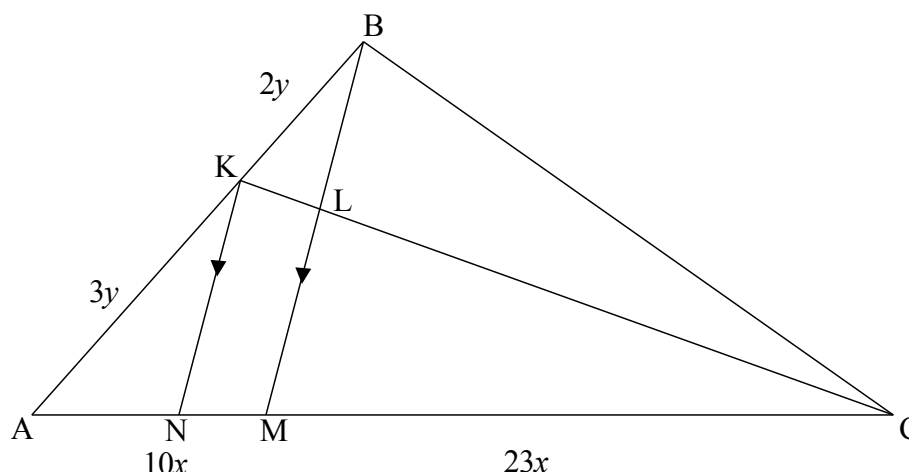
QUESTION/VRAAG 7

7.1



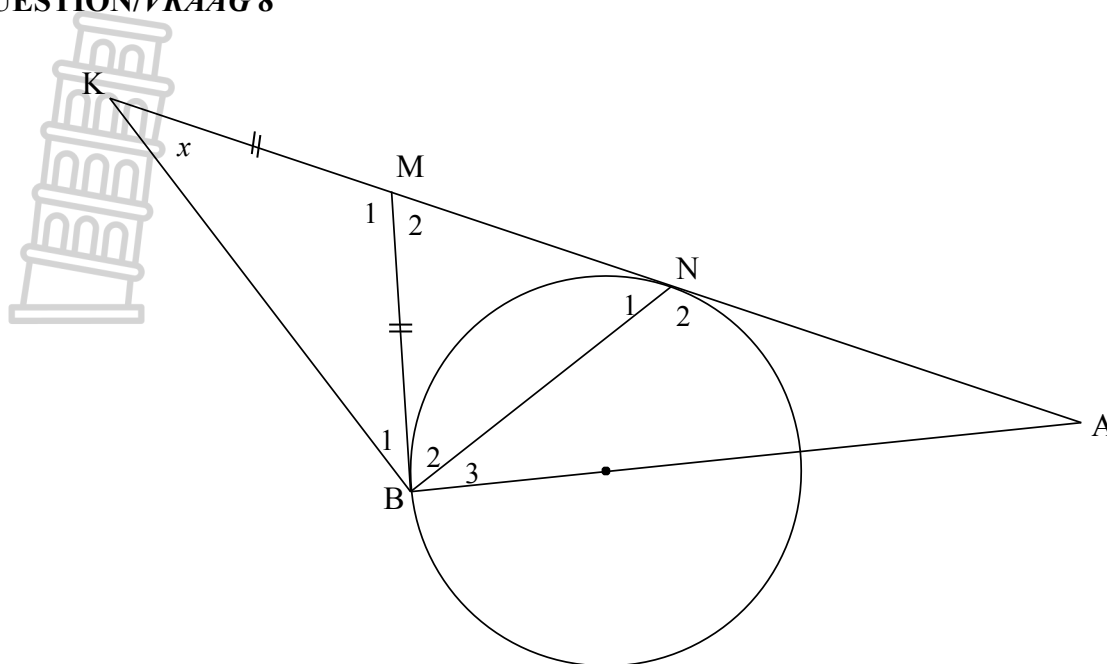
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| 7.1.1(a) | $\angle M\hat{O}S = 62^\circ$ [ $\angle$ at centre = $2 \times \angle$ at circumf/middelpts $\angle = 2 \times$ omtreks $\angle$ ]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | ✓ S ✓ R<br>(2)                                                                                                     |
| 7.1.1(b) | $\hat{L} = 31^\circ$ [equal chords; equal $\angle$ s / = koorde; = $\angle$ e]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | ✓ S ✓ R<br>(2)                                                                                                     |
| 7.1.2    | <p><math>LN = NP</math> and <math>LO = OM</math></p> <p><math>\therefore ON = \frac{1}{2} PM</math> [midpoint theorem/middelpuntstelling]</p> <p><math>\therefore ON = \frac{1}{2} MS</math> [<math>PM = MS</math>]</p> <p><b>OR</b></p> <p><math>\hat{N}_1 = 90^\circ</math> [line from centre to midpt chord/lyn v midpt na midpt kd]</p> <p><math>\hat{P} = 90^\circ</math> [<math>\angle</math> in semi-circle/<math>\angle</math> in halfsirkel]</p> <p><math>\hat{L}</math> is common/gemeen</p> <p><math>\therefore \triangle NLO \cong \triangle PLM</math> (<math>\angle \angle \angle</math>)</p> <p><math>\frac{NL}{PL} = \frac{NO}{PM} = \frac{1}{2}</math></p> <p><math>\therefore ON = \frac{1}{2} PM</math></p> <p><math>\therefore ON = \frac{1}{2} MS</math> [<math>PM = MS</math>]</p> | <p>✓ <math>LO = OM</math></p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S R</p> <p>✓ S/R</p> <p>✓ S</p> <p>✓ S</p> <p>(4)</p> |

7.2

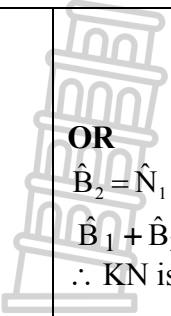


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| 7.2.1 | $\frac{AN}{AM} = \frac{AK}{AB}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel BM</math>/<br/>lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $\frac{AN}{AM} = \frac{3y}{5y} = \frac{3}{5}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | <p>✓ R</p> <p>✓ S</p> <p>(2)</p>                                                        |
| 7.2.2 | $\frac{AM}{MC} = \frac{10x}{23x}$ <p>[given]</p> $AM = 5y = 10x \quad \therefore y = 2x$ $\frac{LC}{KL} = \frac{MC}{NM}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel LM</math>/<br/>lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$ <p><b>OR</b></p> $\frac{AM}{MC} = \frac{10x}{23x}$ <p>[given]</p> $\frac{AN}{MN} = \frac{3y}{2y} = \frac{6x}{4x}$ $\frac{LC}{KL} = \frac{MC}{NM}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math> <b>OR</b> prop theorem; <math>KN \parallel LM</math>/<br/>lyn <math>\parallel</math> sy van <math>\Delta</math> <b>OR</b> eweredigheidst; <math>KN \parallel BM</math>]</p> $= \frac{23x}{2y} = \frac{23x}{4x} = \frac{23}{4}$ | <p>✓ S</p> <p>✓ R</p> <p>✓ S</p> <p>(3)</p> <p>✓ S</p> <p>✓ R</p> <p>✓ S</p> <p>(3)</p> |
|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | [13]                                                                                    |

### QUESTION/VRAAG 8

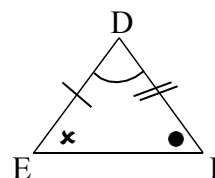
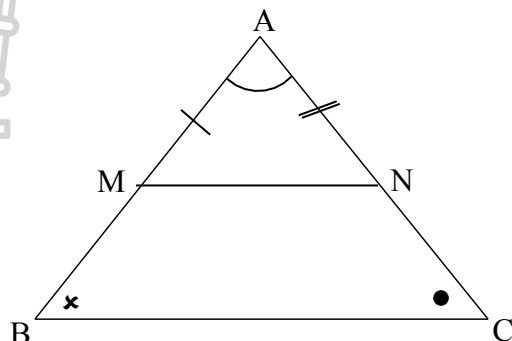


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|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8.1 | $\hat{B}_1 = x$ [ $\angle$ 's opp = sides/ $\angle e$ teenoor = sye ]<br>$\hat{M}_2 = 2x$ [ext $\angle$ of $\Delta$ ] <b>OR</b> $\hat{M}_1 = 180^\circ - 2x$ [ $\angle$ s of $\Delta$ ]<br>BM = MN [ 2 tans from a common point/ <i>raaklyne vanuit dieselfde punt</i> ]<br>$\hat{N}_1 = \frac{180^\circ - 2x}{2} = 90^\circ - x$ [ $\angle$ 's opp = sides/ $\angle e$ teenoor = sye ]<br><b>OR</b><br>NM = BM [ 2 tans from a common point/ <i>raaklyne vanuit dieselfde punt</i> ]<br>$\hat{B}_2 = \hat{N}_1$ [ $\angle$ 's opp = sides/ $\angle e$ teenoor = sye ]<br>$\hat{B}_1 = x$ [ $\angle$ 's opp = sides/ $\angle e$ teenoor = sye ]<br>In $\Delta$ KBN:<br>$x + x + \hat{B}_2 + \hat{N}_1 = 180^\circ$ [sum of $\angle$ 's of $\Delta$ ]<br>$2x + 2\hat{N}_1 = 180^\circ$<br>$x + \hat{N}_1 = 90^\circ$<br>$\hat{N}_1 = 90^\circ - x$ | $\checkmark S$<br>$\checkmark S \checkmark R$<br>$\checkmark S \checkmark R$<br>$\checkmark$ answer<br><br>$\checkmark S \checkmark R$<br>$\checkmark S \checkmark R$<br>$\checkmark S$<br><br>$\checkmark$ answer<br>(6) |
| 8.2 | $M \hat{B} A = \hat{B}_2 + \hat{B}_3 = 90^\circ$ [tangent $\perp$ diameter/ <i>raaklyn <math>\perp</math> middellyn</i> ]<br>$\hat{B}_3 = 90^\circ - \hat{B}_2$<br>$\quad = 90^\circ - (90^\circ - x) = x$<br>$\hat{B}_3 = \hat{K} = x$<br>$\therefore$ AB is a tangent/ <i>raaklyn</i> converse tan-chord theorem/ <i>omgekeerde raakl koordst</i> ]                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | $\checkmark S \checkmark R$<br>$\checkmark S$<br>$\checkmark S$<br>$\checkmark R$<br>(5)                                                                                                                                  |

|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                                                                              |
|--|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
|  |  <p><b>OR</b><br/> <math>\hat{B}_2 = \hat{N}_1</math><br/> <math>\hat{B}_1 + \hat{B}_2 = x + (90^\circ - x) = 90^\circ</math><br/> <math>\therefore</math> KN is diameter/<i>middel lyn</i> [converse <math>\angle</math> in semi-circle/<br/> <i>omgekeerde <math>\angle</math> in halfsirkel</i>]<br/> <math>M \hat{B} A = \hat{B}_2 + \hat{B}_3 = 90^\circ</math> [tangent <math>\perp</math> diameter]<br/> <math>\therefore</math> AB is a tangent/<i>raaklyn</i> converse tan-chord theorem/<br/> <i>omgekeerde raakl koordst</i>]]</p> | <p>✓ S<br/>                 ✓ R<br/> <br/>                 ✓ S   ✓ R<br/>                 ✓ R</p> <p>(5)</p> |
|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | [11]                                                                                                         |



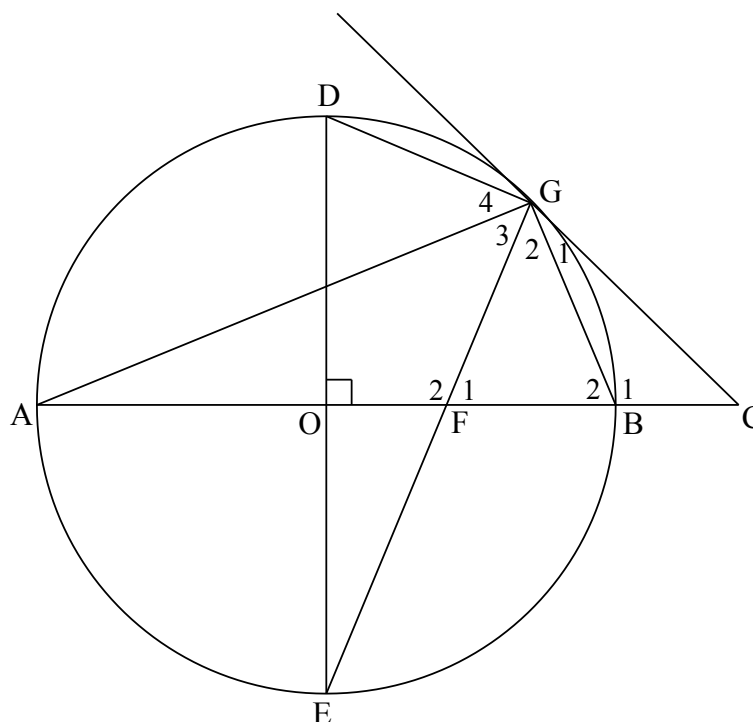
## 9.1



|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                           |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9.1 | <p>Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN.</p> <p>Konst: Merk M en N op AB en AC onderskeidelik af sodanig dat AM = DE en AN = DF. Verbind MN.</p> <p>Proof:</p> <p>In Δ AMN and Δ DEF</p> <p>AM = DE [Constr]</p> <p>AN = DF [Constr]</p> <p><math>\hat{A} = \hat{D}</math> [Given]</p> <p>∴ Δ AMN ≅ Δ DEF(SAS)</p> <p>∴ <math>\hat{M} = \hat{E}</math></p> <p>MN    BC [corresp ∠'s are equal/ooreenkomstige ∠e =]</p> $\frac{AB}{AM} = \frac{AC}{AN}$ <p>[line    one side of Δ OR prop theorem; MN   BC]</p> $\therefore \frac{AB}{DE} = \frac{AC}{DF}$ <p>[AM = DE and AN = DF]</p> | <p>✓ Constr / Konst</p><br><br><br><br><p>✓ Δ AMN ≅ Δ DEF</p> <p>✓ SAS</p> <p>✓ MN    BC and R</p> <p>✓ <math>\frac{AB}{AM} = \frac{AC}{AN}</math> ✓R</p> |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|

(6)

9.2



|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                                                     |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------|
| 9.2.1(a) | <p> <math>\hat{D}OB = 90^\circ</math><br/> <math>\hat{D}GF = \hat{G}_3 + \hat{G}_4 = 90^\circ</math> [ <math>\angle</math> in semi-circle / <math>\angle</math> in halfsirkel ]<br/> <math>\hat{D}OB + \hat{D}GF = 180^\circ</math><br/> <math>\therefore</math> DGFO is a cyclic quad. [converse: opp <math>\angle</math>s of cyclic quad / omgekeerde teenoorst <math>\angle</math>e v koordevh]<br/>                     OR<br/> <math>\angle</math>s of quad = <math>180^\circ</math> / <math>\angle</math>e van koordevh = <math>180^\circ</math> </p> <p> <b>OR</b><br/> <math>\hat{E}OB = 90^\circ</math><br/> <math>\hat{D}GF = \hat{G}_3 + \hat{G}_4 = 90^\circ</math> [ <math>\angle</math> in semi-circle / <math>\angle</math> in halfsirkel ]<br/> <math>\hat{E}OB = \hat{D}GF</math><br/> <math>\therefore</math> DGFO is a cyclic quad. [converse: ext <math>\angle</math> = opp int <math>\angle</math> / omgekeerde buite <math>\angle</math> = teenoorst <math>\angle</math>]<br/>                     OR<br/>                     ext <math>\angle</math> of quad = opp int <math>\angle</math> / buite <math>\angle</math> v <math>vh</math> = teenoorst <math>\angle</math> </p> | <p>✓ S ✓ R</p> <p>✓ R</p> <p>(3)</p>                |
| 9.2.1(b) | <p> <math>\hat{F}_1 = \hat{D}</math> [ext <math>\angle</math> of cyclic quad / buite <math>\angle</math> v koordevh]<br/> <math>\hat{G}_1 + \hat{G}_2 = \hat{D}</math> [tan-chord theorem / raakl koordst]<br/> <math>\therefore \hat{F}_1 = \hat{G}_1 + \hat{G}_2</math><br/> <math>\therefore GC = CF</math> [sides opp equal <math>\angle</math>s / sye teenoor = <math>\angle</math>e]                 </p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | <p>✓ S ✓ R</p> <p>✓ S ✓ R</p> <p>✓ R</p> <p>(5)</p> |

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                  |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 9.2.2(a) | $AB = DE = 14$<br>$\therefore OB = 7 \text{ units}$<br>$\therefore BC = OC - OB = 11 - 7$<br>$= 4 \text{ units}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;">Answer only: full marks</div>                                                                                                                                                                                                                                                                                                                                                                                                                                                             | <div style="display: flex; justify-content: space-between;"> <div>             ✓ S<br/>             ✓ S<br/>             ✓ S           </div> <div>(3)</div> </div>                                                                                                                                                              |
| 9.2.2(b) | <p>In <math>\triangle CGB</math> and <math>\triangle CAG</math></p> <p><math>\hat{G}_1 = \hat{A} = x</math> [tan-chord theorem/<i>raakl koordst</i>]<br/> <math>\hat{C} = \hat{C}</math> [common]<br/> <math>\triangle CGB \parallel \triangle CAG</math> [<math>\angle, \angle, \angle</math>]<br/> <math>\frac{CG}{CA} = \frac{CB}{CG}</math><br/> <math>\frac{CG}{18} = \frac{4}{CG}</math><br/> <math>CG^2 = 72</math><br/> <math>CG = \sqrt{72} \text{ or } 6\sqrt{2} \text{ or } 8,49 \text{ units}</math></p>                                                                                                                                                                                    | <div style="display: flex; justify-content: space-between;"> <div>             ✓ S/R<br/>             ✓ S<br/>             ✓ S<br/>             ✓ CA = 18<br/>             ✓ answer           </div> <div>(5)</div> </div>                                                                                                       |
| 9.2.2(c) | <p>OF = OC - FC<br/> <math>= 11 - \sqrt{72}</math><br/> <math>\tan E = \frac{OF}{OE}</math><br/> <math>= \frac{11 - \sqrt{72}}{7} = 0,36</math><br/> <math>\hat{E} = 19,76^\circ</math></p> <p><b>OR</b></p> <p>OF = OC - FC<br/> <math>= 11 - \sqrt{72}</math><br/> <math>FE^2 = OE^2 + OF^2</math><br/> <math>= 7^2 + (11 - \sqrt{72})^2</math><br/> <math>FE = 7,437.. = 7,44</math><br/> <math>\cos E = \frac{OE}{FE}</math><br/> <math>= \frac{7}{7,44} = 0,94</math><br/> <math>\hat{E} = 19,76^\circ</math></p> <p style="text-align: center;">OR</p> <p><math>\sin E = \frac{OF}{FE}</math><br/> <math>= \frac{11 - \sqrt{72}}{7,44} = 0,338</math><br/> <math>\hat{E} = 19,76^\circ</math></p> | <div style="display: flex; justify-content: space-between;"> <div>             ✓ OF<br/>             ✓ trig ratio<br/>             ✓ substitution<br/>             ✓ answer<br/>             ✓ OF<br/>             ✓ trig ratio<br/>             ✓ substitution<br/>             ✓ answer           </div> <div>(4)</div> </div> |

|  |                         |
|--|-------------------------|
|  | <b>TOTAL/TOTAL: 140</b> |
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