



LIMPOPO

PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS PAPER 2

JUNE 2024

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MARKS: 150

TIME: 3 HOURS



MEMATHP2



This question paper consists of 10 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.



1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. ANSWERS ONLY will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write legibly and present your work neatly.



QUESTION 1

In a survey, a student asked his friends and family how many marketing phone calls they receive per month. The following table represent the data:

AGE OF FRIENDS AND FAMILY	FREQUENCY	CUMULATIVE FREQUENCY
$20 < x \leq 30$	7	7
$30 < x \leq 40$		27
$40 < x \leq 50$	25	
$50 < x \leq 60$		64
$60 < x \leq 70$		72
$70 < x \leq 80$	4	
$80 < x \leq 90$		80

- 1.1 Complete the Cumulative frequency table in the ANSWER BOOK. (3)
- 1.2 How many people took part in the survey? (1)
- 1.3 Write down the modal class. (1)
- 1.4 Draw the ogive in the ANSWER BOOK. (3)
- 1.5 Determine the percentage of marketing calls a person older than 54 will receive. (3)

[11]

QUESTION 2

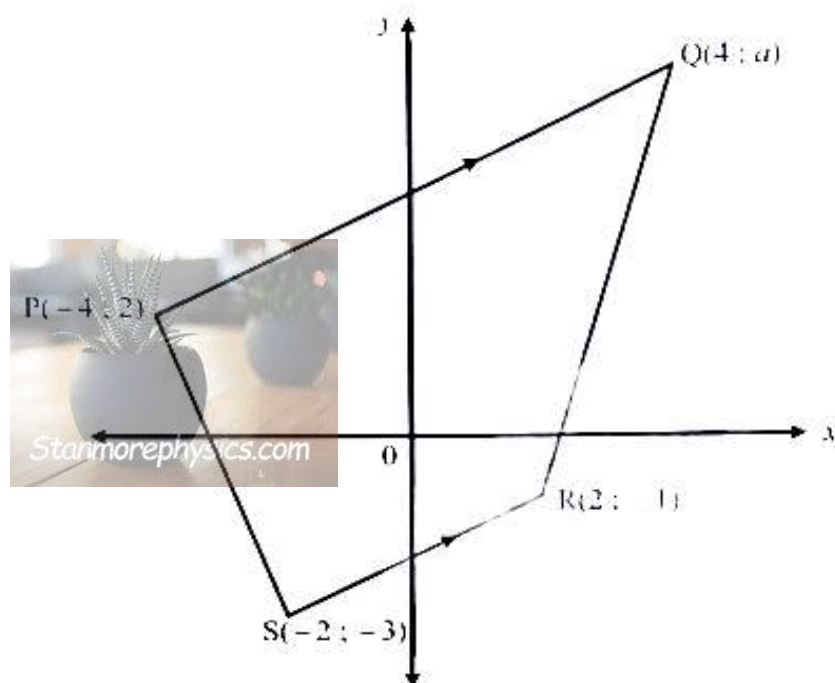
The numbers {10, 20, 45, 58, 80} are the 5 number summary of a set of data with nine numbers. The second and third number of the data set are the same. The eighth number is 3 times the first quartile. The fourth number is equal to the interquartile range. The mean for the data set is 42.

- 2.1 Draw the box and whisker diagram in the ANSWER BOOK. (2)
- 2.2 Comment on the skewness of the data. (1)
- 2.3 Write down a possible list of nine numbers which will result in the above box and whisker plot. (6)

[9]

QUESTION 3

In the diagram $P(-4; 2)$, $Q(4; a)$, $R(2; -1)$ and $S(-2; -3)$ are the vertices of trapezium PQRS with $PQ \parallel SR$.

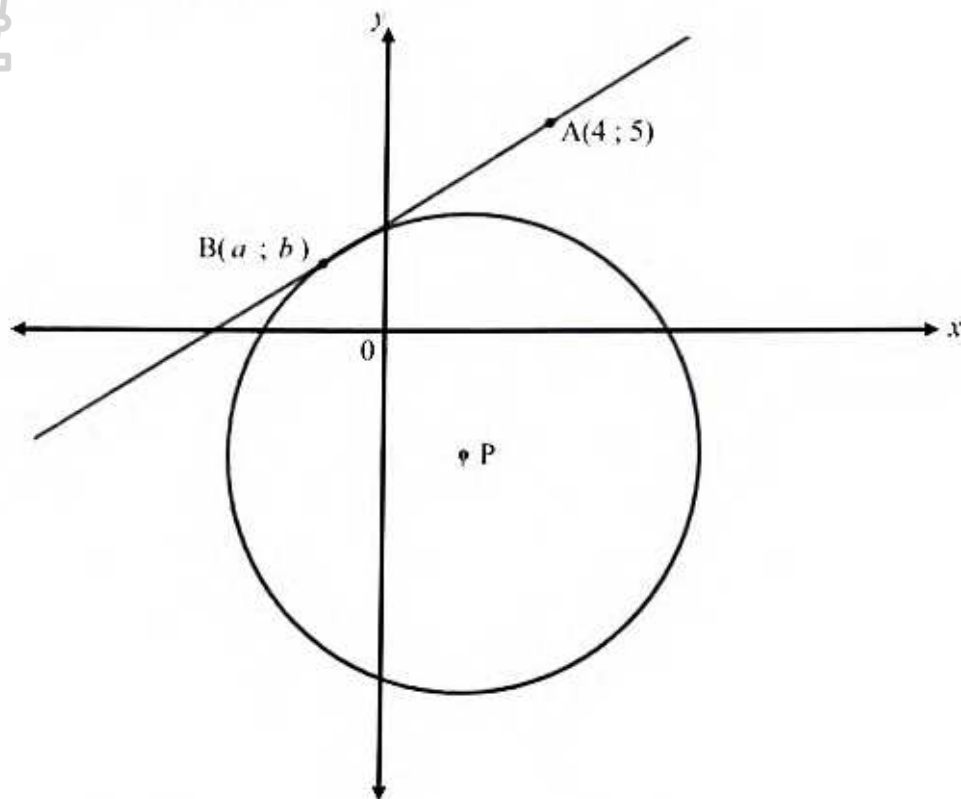


- 3.1 Calculate the value of a . (4)
- 3.2 Find the equation of the line passing through P and R. (3)
- 3.3 Find the size of $\angle PRQ$. (5)
- 3.4 If $A(-1; t)$ and P, A and R are collinear, calculate the value of t . (2)

[14]

QUESTION 4

In the diagram the circle P with equation $(x-2)^2 + (y+3)^2 = 34$ is drawn. The point A(4 ; 5) is a point on the tangent defined by $5y = 3x + 13$. B(a ; b) is the point on the circumference where the tangent touches the circle.



- 4.1 Determine the coordinates of P and the value of the radius of the circle. (2)
- 4.2 Write down the gradient of the radius PB in terms of a and b . (2)
- 4.3 Determine the gradient of the tangent AB in terms of a and b . (1)
- 4.4 Hence or otherwise, show that the values of $a = -1$ and $b = 2$. (6)
- 4.5 Calculate the length of AB. Leave the answer in surd form. (2)
- 4.6 A second tangent is drawn from A to touch the circle at C(d ; 0). Determine the value of d . (4)
- 4.7 Prove that $\angle BPC = 90^\circ$. (3)
- 4.8 If AB intersects the y -axis at D and CP produced intersects the y -axis at E, calculate the length of DE. (5)

[25]

QUESTION 5

5.1 If $\tan 35^\circ = m$, determine the value of the following in terms of m :

5.1.1 $\sin 215^\circ$ (3)

5.1.2 $\sin 70^\circ$ (3)

5.2 $\tan \alpha = \frac{1}{3}$ and $\tan \beta = \frac{1}{7}$, where α and β are acute angles.

Determine the value of: $50 \times \sin(2\alpha + \beta)$. (6)

5.3 Simplify: $\frac{\cos(-x) \tan(180^\circ - x) \cos(90^\circ - x)}{\sin(540^\circ + x) \sin(180^\circ - x)}$ (6)

5.4 5.4.1 Prove that: $\frac{\sin x}{1 - \sin x} + \frac{\sin x}{1 + \sin x} = \frac{2 \tan x}{\cos x}$ (4)

5.4.2 For which value(s) of x in the interval $[0^\circ; 180^\circ]$ will the identity in 5.3.1 not be defined. (2)

[24]

QUESTION 6

$f(x) = \cos(x - 30^\circ)$ and $g(x) = \sin x$, $x \in [-90^\circ; 90^\circ]$

6.1 If $x \in [-90^\circ; 90^\circ]$, solve the equation $f(x) = g(x)$. (4)

6.2 6.2.1 Draw the graphs of $f(x)$ and $g(x)$ on the same set of axes provided in the ANSWER BOOK. (4)

Hence use the graphs to answer the following questions:

6.2.2 Write down the range of f . (2)

6.2.3 For which values of x will $f(x)$ decrease if x increase. (2)

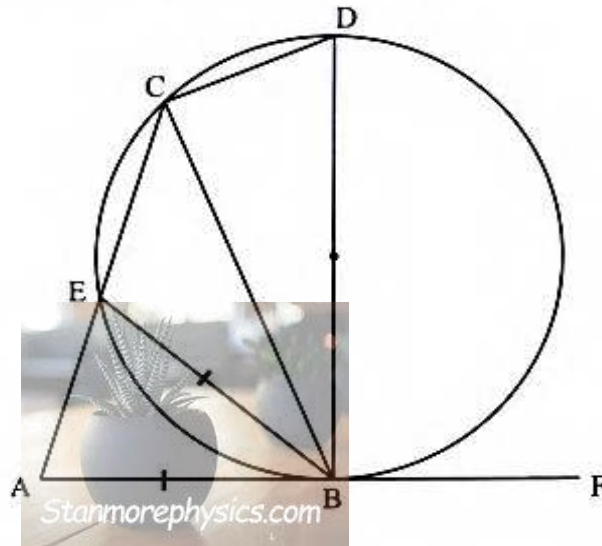
6.2.4 If $x \in [-90^\circ; 0^\circ]$, for which values of x will $f(x) \cdot g(x) \geq 0$? (3)

6.2.5 Write down the value for of x where $g(x) > f(x)$. (2)

[17]

QUESTION 7

In the diagram AF is a tangent. AEC is a line intersecting the circle at E. BD is a diameter. EB, CB and CD are joined. $\hat{ABE} = 2x$ and $AB = EB = h$.

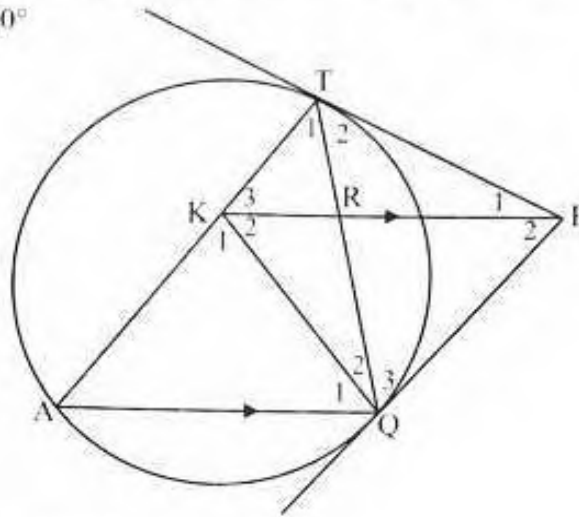


- 7.1 7.1.1 Write down, with reasons, the size of $\hat{A}$ in terms of x . (2)
- 7.1.2 Write down, with reasons, the size of $\hat{ACB}$ in terms of x . (2)
- 7.2 Show that $BC = \frac{h}{2\sin x}$. (3)
- 7.3 Hence show with reasons that $BD = \frac{h}{\sin 2x}$. (3)

[10]

QUESTION 8

In the diagram PT and PQ are tangents to the circle at T and Q. $KP \parallel QA$. Chords TQ and PK intersect at R. $\hat{T}_2 = 50^\circ$



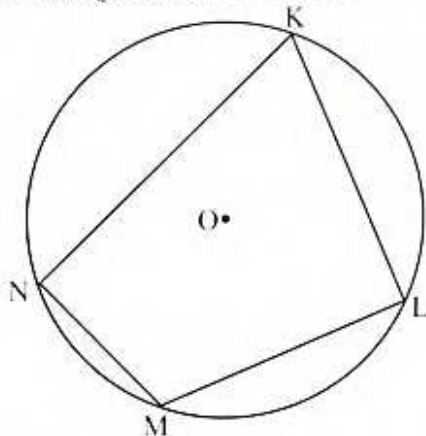
Find, with reasons, the sizes of:

- | | | |
|-----|---|-----|
| 8.1 | $\hat{A}$ | (2) |
| 8.2 | $\hat{K}_3$ | (2) |
| 8.3 | $\hat{Q}_3$ | (2) |
| 8.4 | Give a reason why TKQP a cyclic quadrilateral is. | (1) |
| 8.5 | $\hat{Q}_1$ | (3) |

[10]

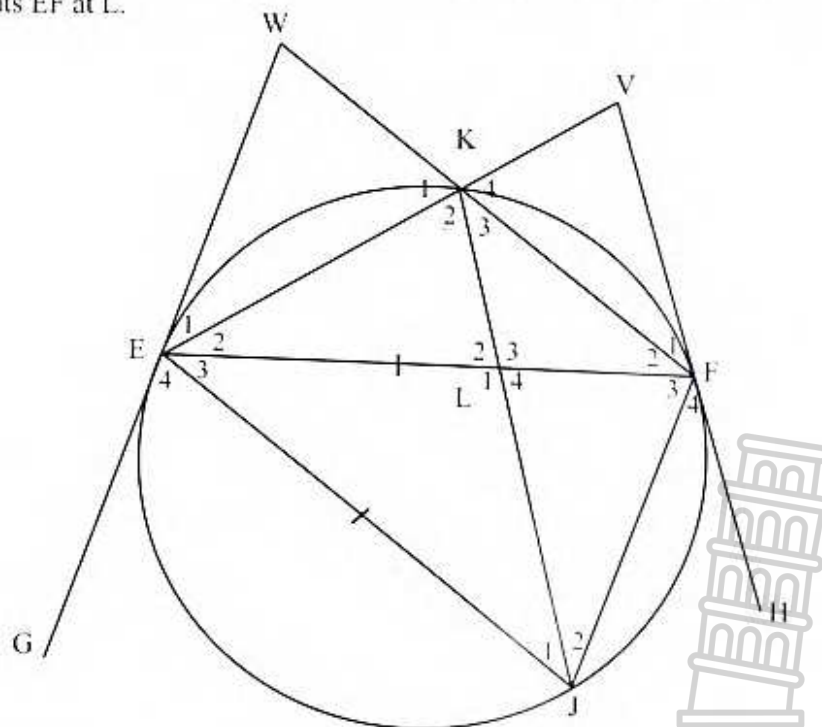
QUESTION 9

- 9.1 In the diagram O is the centre of the circle and KLMN is a cyclic quadrilateral. Prove the theorem stating that $\hat{K} + \hat{M} = 180^\circ$.



(5)

- 9.2 In the diagram below, WEG and VFH are tangents to the circle. EJ is drawn such that $EJ = EF$. WF cuts the circle at K. EK produced meets VH at V. JK cuts EF at L.



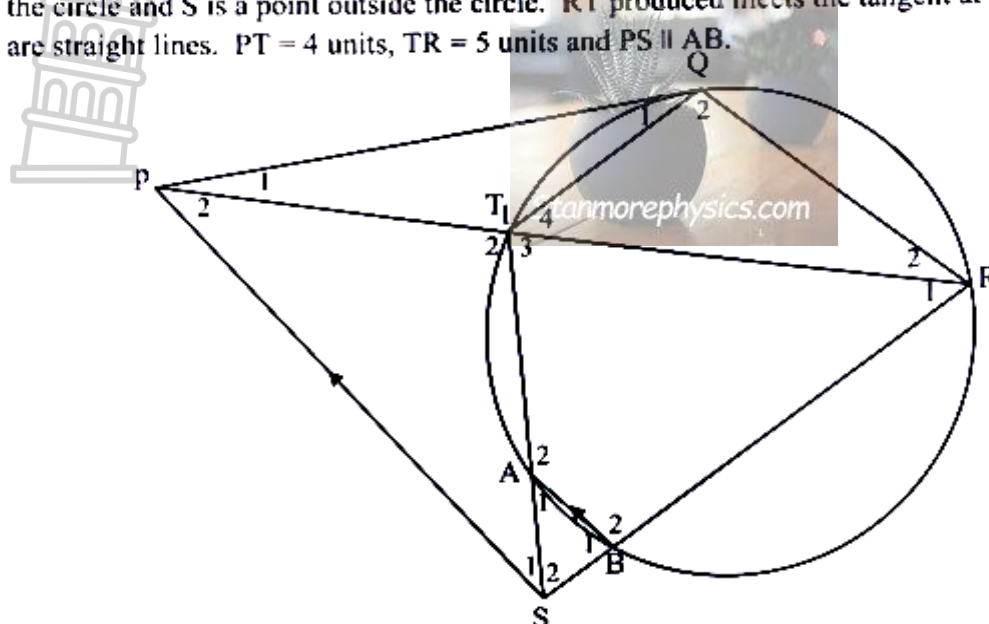
Prove that:

- 9.2.1 $\hat{EJF} = \hat{K}$ (4)
 9.2.2 $\hat{K}_1 = \hat{K}_2$ (2)
 9.2.3 KLFV is a cyclic quadrilateral. (3)

[14]

QUESTION 10

In the diagram PQ is a tangent to the circle at Q. T, R, A and B are points in the circumference of the circle and S is a point outside the circle. RT produced meets the tangent at P. TAS and RBS are straight lines. $PT = 4$ units, $TR = 5$ units and $PS \parallel AB$.



- 10.1 If $\hat{S}_1 = x$, name with reasons, 2 other angles equals to x . (4)
- 10.2 Prove that $\triangle PQT \sim \triangle PRQ$. (3)
- 10.3 Calculate the length of PQ. (3)
- 10.4 Prove that $\triangle PTS \sim \triangle PSR$. (3)
- 10.5 Prove that $PQ = PS$. (3)

[16]

GRAND TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1} ; r \neq 1$$

$$S_\infty = \frac{a}{1 - r} ; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\bar{x} = \frac{\sum f_x}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\hat{y} = a + bx$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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MATHEMATICS P2/*WISKUNDE 2*

JUNE 2024

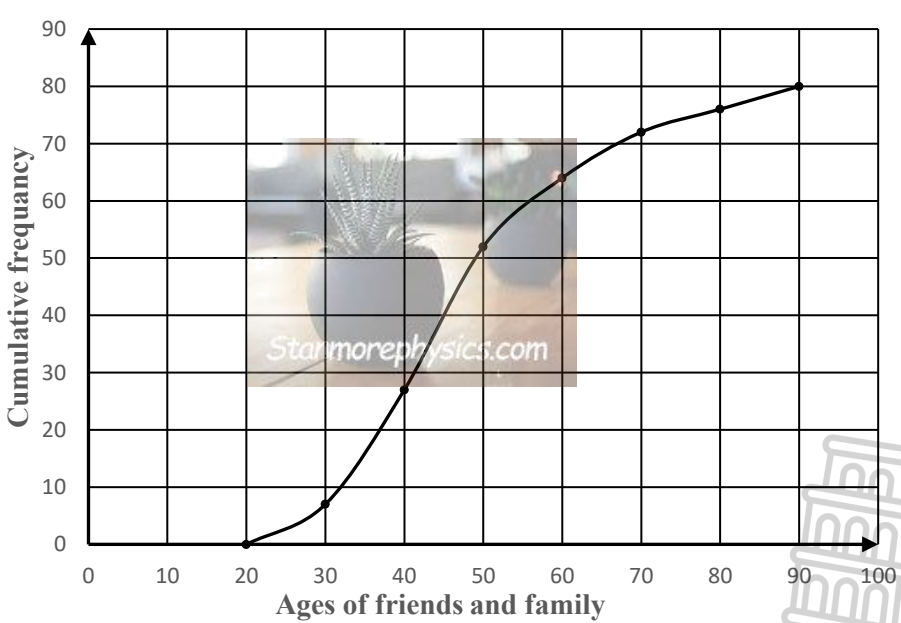
MARKING GUIDELINES/*NASIENRICGLYNE*

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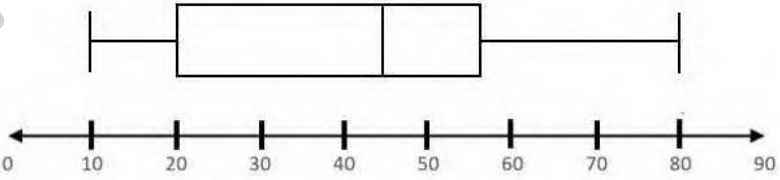
MARKS/*PUNTE*: 150

This marking guidelines consist of 19 pages/*Hierdie nasienriglyne bestaan uit 19 bladsye.*

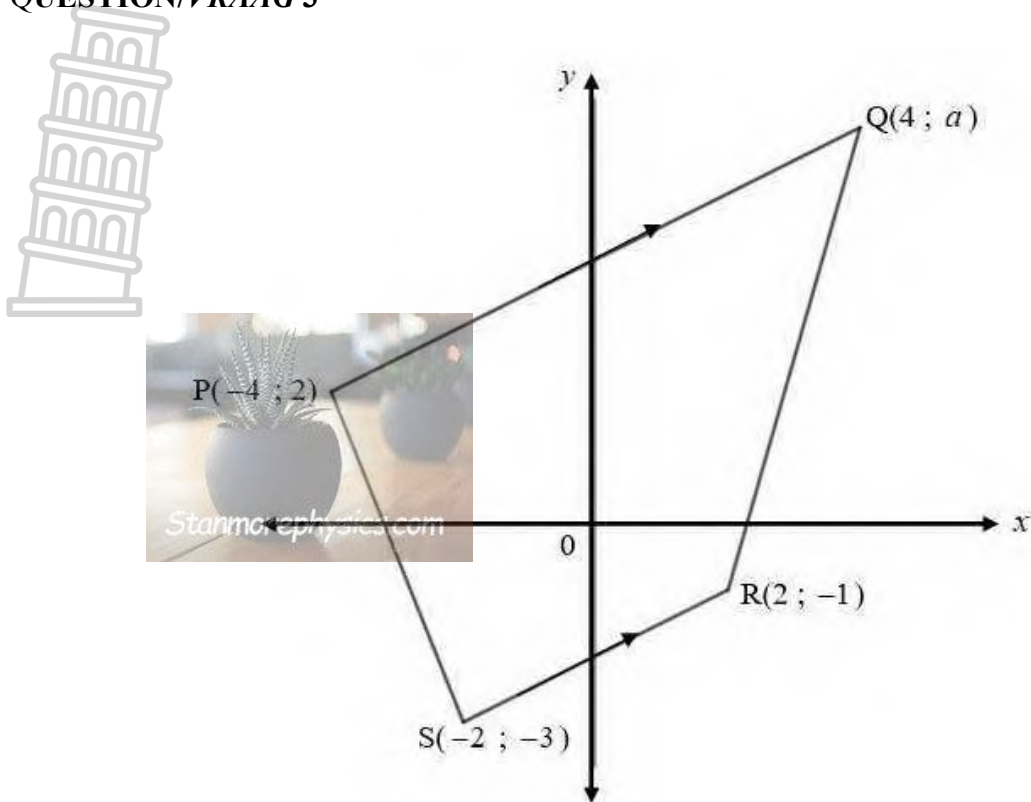
QUESTION/VRAAG 1

1.1	<table><thead><tr><th>AGE OF FRIENDS AND FAMILY</th><th>FREQUENCY</th><th>CUMULATIVE FREQUENCY</th></tr></thead><tbody><tr><td>$20 < x \leq 30$</td><td>7</td><td>7</td></tr><tr><td>$30 < x \leq 40$</td><td>20</td><td>27</td></tr><tr><td>$40 < x \leq 50$</td><td>25</td><td>52</td></tr><tr><td>$50 < x \leq 60$</td><td>12</td><td>64</td></tr><tr><td>$60 < x \leq 70$</td><td>8</td><td>72</td></tr><tr><td>$70 < x \leq 80$</td><td>4</td><td>76</td></tr><tr><td>$80 < x \leq 90$</td><td>4</td><td>80</td></tr></tbody></table>	AGE OF FRIENDS AND FAMILY	FREQUENCY	CUMULATIVE FREQUENCY	$20 < x \leq 30$	7	7	$30 < x \leq 40$	20	27	$40 < x \leq 50$	25	52	$50 < x \leq 60$	12	64	$60 < x \leq 70$	8	72	$70 < x \leq 80$	4	76	$80 < x \leq 90$	4	80	✓ 20 & 52 ✓ 12 & 8 ✓ 76 & 4	(3)
AGE OF FRIENDS AND FAMILY	FREQUENCY	CUMULATIVE FREQUENCY																									
$20 < x \leq 30$	7	7																									
$30 < x \leq 40$	20	27																									
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$60 < x \leq 70$	8	72																									
$70 < x \leq 80$	4	76																									
$80 < x \leq 90$	4	80																									
1.2	80	✓ answer	(1)																								
1.3	$40 < x \leq 50$	✓ answer	(1)																								
1.4	Ages of family and friends 	✓ grounding ✓ end point ✓ 2 points	(3)																								
1.5	$\frac{21}{80} \times 100 = 26,25\%$	✓ accept 20 – 22 ✓ $\times 100$ ✓ accept 25 – 27,5	(3)																								
			[11]																								

QUESTION/VRAAG 2

2.1		✓✓ plotting values	(2)
2.2	Skew to the left OR negatively skew	✓ answer	(1)
2.3	2nd = 20 4th = 38 8th = 60 7th = 56 6th : $\frac{10 + 20 + 20 + 38 + 45 + x + 56 + 60 + 80}{9} = 42$ $\frac{329 + x}{9} = 42$ $329 + x = 378$ $x = 49$	✓ 20 ✓ 32 ✓ 60 ✓ 56 ✓ $\frac{329 + x}{9} = 42$ ✓ answer	(6)
			[9]

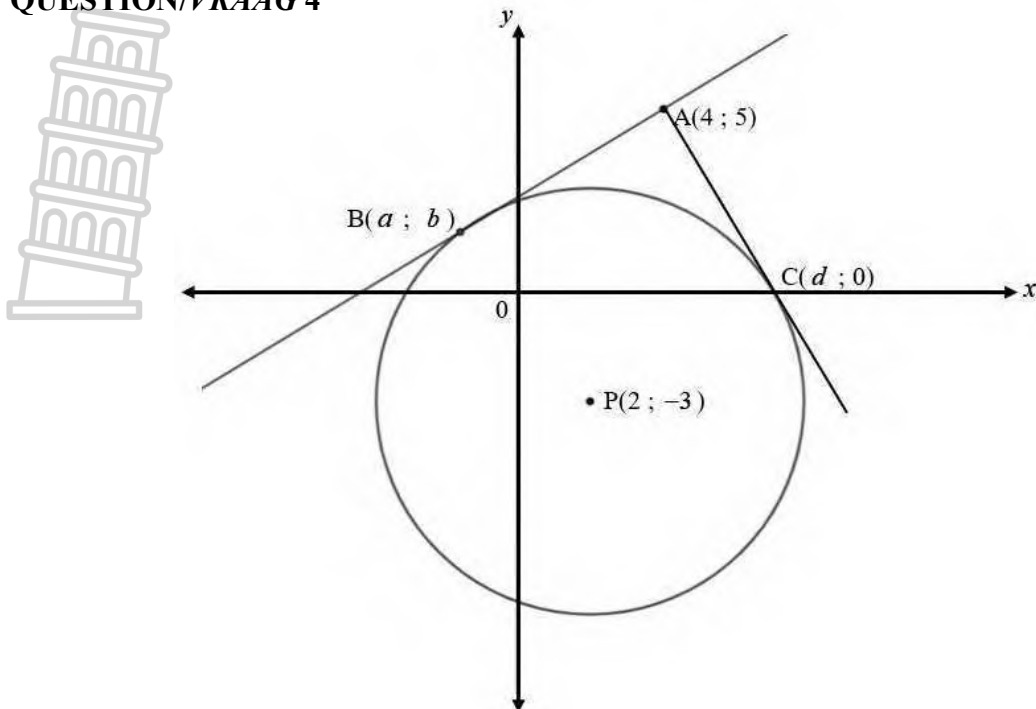
QUESTION/VRAAG 3



3.1	$m(PQ) = m(SR)$ $\frac{2-a}{-4-4} = \frac{-3+1}{-2-2}$ $\frac{2-a}{-8} = \frac{1}{2}$ $4-2a = -8$ $-2a = -12$ $a = 6$	✓ $m(PQ)$ ✓ $m(SR)$ ✓ equating ✓ answer	(4)
3.2	$m(PR) = \frac{2+1}{-4-2} = -\frac{1}{2}$ $y = mx + c$ $2 = \left(-\frac{1}{2}\right)(-4) + c$ $c = 0$ $y = -\frac{1}{2}x$ OR	✓ $m(PR)$ ✓ subst gradient and point ✓ answer	(3)

	$m(PR) = \frac{2+1}{-4-2} = -\frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - 2 = \left(-\frac{1}{2}\right)(x + 4)$ $y - 2 = -\frac{1}{2}x - 2$ $y = -\frac{1}{2}x$	✓ $m(PR)$ ✓ subst gradient and point ✓ answer	(3)
3.3	$m(PR) = -\frac{1}{2} \qquad m(QR) = \frac{6+1}{4-2} = \frac{7}{2}$ $\tan \alpha = -\frac{1}{2} \qquad \tan \beta = \frac{7}{2}$ $\alpha = 180^\circ - 26,57^\circ \qquad \beta = 74,05^\circ$ $\alpha = 153,43^\circ$ $\therefore \hat{P}RQ = 79,38^\circ$	✓ tan ratio α ✓ $\alpha = 153,43^\circ$ ✓ $m(QR)$ ✓ $\beta = 74,05^\circ$ ✓ $\hat{P}RQ$	(5)
3.4	$y = -\frac{1}{2}x$ $(-1; t): t = -\frac{1}{2}(-1)$ $t = \frac{1}{2}$ OR $m(PA) = m(PR)$ $\frac{2-t}{-4+1} = -\frac{1}{2}$ $\frac{2-t}{-3} = -\frac{1}{2}$ $4 - 2t = 3$ $-2t = -1$ $t = \frac{1}{2}$	✓ equation ✓ answer ✓ equating ✓ answer	(2)
			[14]

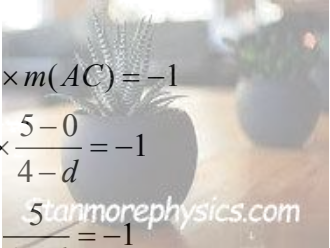
QUESTION/VRAAG 4



4.1	$P(2; -3)$ Radius = $\sqrt{34}$	✓ centre ✓ radius	(2)
4.2	$m(PB) = \frac{b+3}{a-2}$	✓ subst in formula ✓ answer	(2)
4.3	$m(AB) = -\frac{a-2}{b+3}$ OR $m(AB) = \frac{b-5}{a-4}$	✓ answer	(1)
4.4	Eq AB: $5y = 3x + 13$ through $B(a; b)$ $5b = 3a + 13$ $b = \frac{3}{5}a + \frac{13}{5}$ 1 Eq PB: $y = mx + c$ $m = -\frac{5}{3}$ and $P(2; -3)$ $-3 = -\frac{5}{3}(2) + c$ $c = \frac{1}{3}$ $y = -\frac{5}{3}x + \frac{1}{3}$ through $B(a; b)$ $b = -\frac{5}{3}a + \frac{1}{3}$ 2	✓ subst $B(a; b)$ in line equation ✓ $m(PB) = -\frac{5}{3}$ ✓ subst gradient and $P(2; -3)$ ✓ equation PB	

	Subs 1 in 2: $\frac{3}{5}a + \frac{13}{5} = -\frac{5}{3}a + \frac{1}{3}$ $9a + 39 = -25a + 5$ $34a = -34$ $a = -1$ $b = 2$ OR Eq AB: $5y = 3x + 13$ through B(a ; b) $5b = 3a + 13$ $b = \frac{3}{5}a + \frac{13}{5}$ 1 $m(PB) \times m(BA) = -1$ $\frac{b+3}{a-2} \times \frac{b-5}{a-4} = -1$ $(b+3)(b-5) = -1(a-2)(a-4)$ 2 Subs 1 in 2: $\left(\frac{3}{5}a + \frac{13}{5} + 3\right)\left(\frac{3}{5}a + \frac{13}{5} - 5\right) = -1(a-2)(a-4)$ $\left(\frac{3}{5}a + \frac{28}{5}\right)\left(\frac{3}{5}a + \frac{12}{5}\right) = -a^2 + 6a - 8$ $\frac{9}{25}a^2 + \frac{48}{25}a - \frac{336}{25} = -a^2 + 6a - 8$ $34a^2 - 102a - 136 = 0$ $a^2 - 3a - 4 = 0$ $(a-4)(a+1) = 0$ $a = 4$ or/of $a = -1$ NA $b = 2$	✓ equate equations ✓ simplification ✓ subst B(a ; b) in line equation ✓ $m(PB) \times m(BA) = -1$ ✓ subst gradients in formula ✓ subst b value in eq ✓ simplification ✓ simplification	(6) (6)
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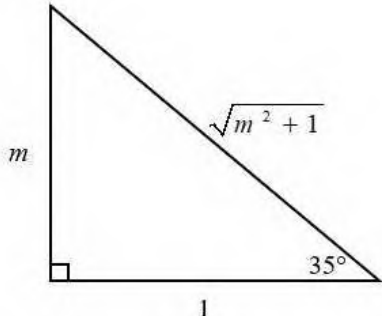
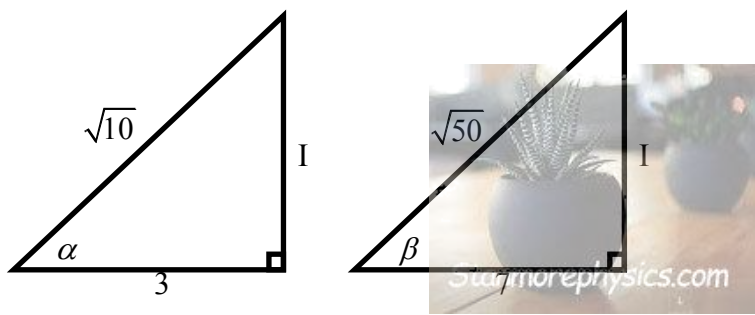
	OR Subst B(a ; b) in $5y = 3x + 13$: $5b = 3a + 13$ 1 $y = \frac{3}{5}x + \frac{13}{5}$ $\therefore \frac{3}{5} = -\frac{a-2}{b+3}$ $3(b+3) = -5(a-2)$ $3b+9 = -5a+10$ $3b = -5a+1$ $b = -\frac{5}{3}a + \frac{1}{3}$ 2 Subst 2 in 1: $5\left(-\frac{5}{3}a + \frac{1}{3}\right) = 3a + 13$ $-\frac{25}{3}a + \frac{5}{3} = 3a + 13$ $-25a + 5 = 9a + 39$ $-34a = 34$ $a = -1$ $b = 2$	✓ subst B(a ; b) in line equation ✓ equate gradients ✓ simplification ✓ equation 2 ✓ subst ✓ simplification	(6)
4.5	$AB = \sqrt{(-1-4)^2 + (2-5)^2}$ $AB = \sqrt{34}$	✓ subst in dist formula ✓ answer	(2)
4.6	$(x-2)^2 + (y+3)^2 = 34$ through (d ; 0) $(d-2)^2 + (0+3)^2 = 34$ $(d-2)^2 + 9 = 34$ $(d-2)^2 = 25$ $d-2 = \pm 5$ $d = 7 \quad \text{or} \quad d = -3$ NA	✓ Subst in formula ✓ simplification ✓ factors ✓ answers	(4)

	OR $(x-2)^2 + (y+3)^2 = 34 \text{ through } (d ; 0)$ $(d-2)^2 + (0+3)^2 = 34$ $d^2 - 4d + 4 + 9 = 34$ $d^2 - 4d - 21 = 0$ $(d-7)(d+3) = 0$ $d = 7 \quad \text{or} \quad d = -3$ NA OR  $m(PC) \times m(AC) = -1$ $\frac{-3-0}{2-d} \times \frac{5-0}{4-d} = -1$ $\frac{-3}{2-d} \times \frac{5}{4-d} = -1$ $\frac{-15}{8-6d+d^2} = -1$ $-15 = -8 + 6d - d^2$ $d^2 - 6d - 7 = 0$ $(d-7)(d+1) = 0$ $d = 7 \quad \text{or} \quad d = -1$ NA	✓ subst in formula ✓ standard form ✓ factors ✓ select answer	(4)
4.7	$m(PB) = \frac{2+3}{-1-2} = -\frac{5}{3}$ $m(PC) = \frac{-3-0}{2-7} = \frac{3}{5}$ $\therefore m(PB) \times m(PC) = -\frac{5}{3} \times \frac{3}{5} = -1$ $\therefore \hat{BPC} = 90^\circ$	✓ $m(PB)$ ✓ $m(PC)$ ✓ subst in formula	(3)

4.8	y-intercept of AB: Let $x = 0$ in $5y = 3x + 13$ $5y = 13$ $y = \frac{13}{5}$ $D\left(0; \frac{13}{5}\right)$ Equation of CP: $y = mx + c$ $-3 = \frac{3}{5}(2) + c$ $c = -\frac{21}{5}$ $y = \frac{3}{5}x - \frac{21}{5}$ $E\left(0; -\frac{21}{5}\right)$ $DE = \frac{34}{5} = 6,8$	✓ y - value ✓ subst m and point ✓ equation of CP ✓ y - value of E ✓ answer	(5)
			[25]



QUESTION/VRAAG 5

5.1.1	$\sin 215^\circ$ $= -\sin 35^\circ$ $= -\frac{m}{\sqrt{m^2 + 1}}$ 	$\checkmark -\sin 35^\circ$ $\checkmark \sqrt{m^2 + 1}$ $\checkmark$ answer	(3)
5.1.2	$\sin 70^\circ = \sin 2 \times 35^\circ$ $= 2 \sin 35^\circ \cos 35^\circ$ $= 2 \left(\frac{m}{\sqrt{m^2 + 1}} \right) \left(\frac{1}{\sqrt{m^2 + 1}} \right)$ $= \frac{2m}{m^2 + 1}$	$\checkmark$ double $\angle$ $\checkmark$ subst $\checkmark$ answer	(3)
5.2	 $50 \times \sin(2\alpha + \beta)$ $= 50 [\sin 2\alpha \cos \beta + \cos 2\alpha \sin \beta]$ $= 50 \left[2 \sin \alpha \cos \alpha \cos \beta + (2 \cos^2 \alpha - 1) \sin \beta \right]$ $= 50 \left[2 \left(\frac{1}{\sqrt{10}} \right) \left(\frac{3}{\sqrt{10}} \right) \left(\frac{7}{\sqrt{50}} \right) + \left(2 \left(\frac{3}{\sqrt{10}} \right)^2 - 1 \right) \left(\frac{1}{\sqrt{50}} \right) \right]$ $= 50 \left[\frac{42}{10\sqrt{50}} + \frac{8}{10} \left(\frac{1}{\sqrt{50}} \right) \right]$ $= 50 \left[\frac{42}{10\sqrt{50}} + \frac{8}{10\sqrt{50}} \right]$ $= 50 \left[\frac{50}{10\sqrt{50}} \right]$ $= 25\sqrt{2}$	$\checkmark \sqrt{10} \text{ \& } \sqrt{50}$ $\checkmark$ compound $\angle$ $\checkmark$ double $\angle$'s $\checkmark$ subst $\checkmark$ simplification $\checkmark$ answer	(6)

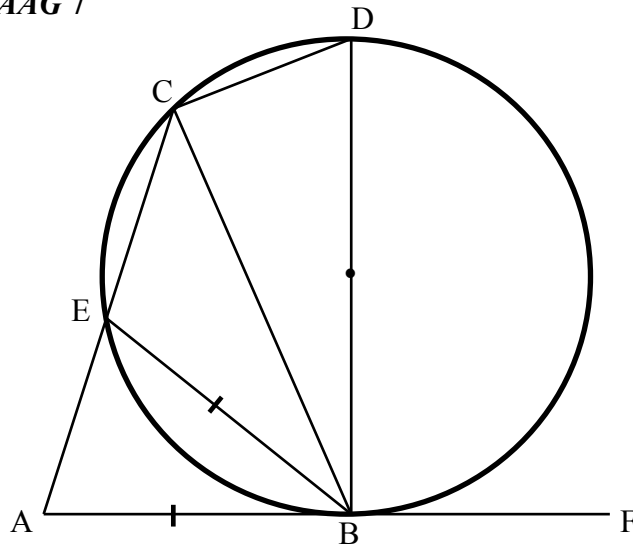
5.3	$\frac{\cos(-x) \tan(180^\circ - x) \cos(90^\circ - x)}{\sin(540^\circ + x) \sin(180^\circ - x)}$ $= \frac{(\cos x)(-\tan x)(\sin x)}{(-\sin x)(\sin x)}$ $= \frac{-\tan x}{-\tan x}$ $= 1$	✓ $\cos x$ ✓ $-\tan x$ ✓ $\sin x$ ✓ $-\sin x$ ✓ $\sin x$ ✓ answer	(6)
5.4.1	$\frac{\sin x}{1 - \sin x} + \frac{\sin x}{1 + \sin x} = \frac{2 \tan x}{\cos x}$ $\text{LHS} = \frac{\sin x}{1 - \sin x} + \frac{\sin x}{1 + \sin x}$ $= \frac{\sin x(1 + \sin x) + \sin x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$ $= \frac{\sin x + \sin^2 x + \sin x - \sin^2 x}{1 - \sin^2 x}$ $= \frac{2 \sin x}{\cos^2 x}$ $= \frac{2 \sin x}{\cos x \cos x}$ $= \frac{2 \tan x}{\cos x} = \text{RHS}$	✓ LCM ✓ simplify numerator ✓ $1 - \sin^2 x$ ✓ $\cos^2 x$	(4)
5.4.2	$x = 90^\circ$ and $x = 270^\circ$	✓ 90° ✓ 270°	(2)
			[24]



QUESTION/VRAAG 6

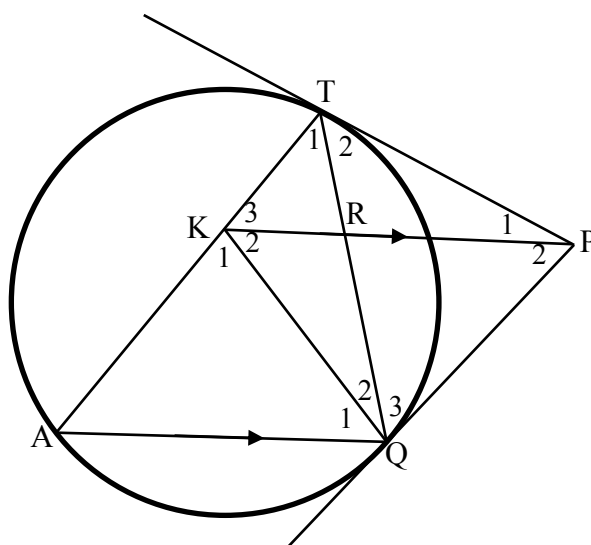
6.1	$\cos(x - 30^\circ) = \sin x$ $\cos(x - 30^\circ) = \cos(90^\circ - x)$ $x - 30^\circ = 90^\circ - x + k.360^\circ$ or $x - 30^\circ = 360^\circ - (90^\circ - x) + k.360^\circ$ $2x = 120^\circ + k.360^\circ$ $x - 30^\circ = 360^\circ - 90^\circ + x + k.360^\circ$ $x = 60^\circ + k.180^\circ, k \in \mathbb{Z}$ NA $x = 60^\circ$	✓ co-function ✓ both equations ✓ $x = 60^\circ + k.180^\circ,$ $k \in \mathbb{Z}$ ✓ $x = 60^\circ$	(4)
6.2.1		✓✓ f ✓✓ g	(4)
6.2.2	$y \in [-1 ; 1]$	✓ critical values ✓ notation	(2)
6.2.3	$x \in (30^\circ ; 90^\circ]$	✓ critical values ✓ notation	(2)
6.2.4	$x \in [-90^\circ ; -60^\circ], x = 0^\circ$	✓✓ $x \in [-90^\circ ; -60^\circ]$ ✓ $x = 0^\circ$	(3)
6.2.5	$x \in (60^\circ ; 90^\circ]$	✓ critical values ✓ notation	(2)
			[17]

QUESTION/VRAAG 7



7.1.1	$\hat{A} = \hat{AEB}$ $\angle$'s opp equal sides $2\hat{A} = 180^\circ - 2x$ $\angle$'s of Δ $\hat{A} = 90^\circ - x$	✓ S ✓ R	(2)
7.1.2	$\hat{ACB} = 2x$ tan- chord theorem	✓ S ✓ R	(2)
7.2	In ΔECB : $\frac{BC}{\sin(90^\circ + x)} = \frac{h}{\sin 2x}$ $BC = \frac{h \cos x}{2 \sin x \cos x}$ $BC = \frac{h}{2 \sin x}$	✓ subst in sine rule ✓ co-function ✓ double $\angle$	(3)
7.3	In ΔBCD : $\hat{C} = 90^\circ$ $\frac{BD}{\sin 90^\circ} = \frac{h}{\sin(90^\circ - x)}$ $BD = \frac{h}{2 \sin x \cos x}$ $BD = \frac{h}{\sin 2x}$ OR $\sin(90^\circ - x) = \frac{h}{BD}$ $BD = \frac{h}{2 \sin x \cos x} = \frac{h}{\sin 2x}$	✓ $\hat{C} = 90^\circ$ ✓ subst in sine rule ✓ co-function ✓ $\hat{D} = 90^\circ - x$ ✓ subst in sine rule ✓ co-function	(3)
			[10]

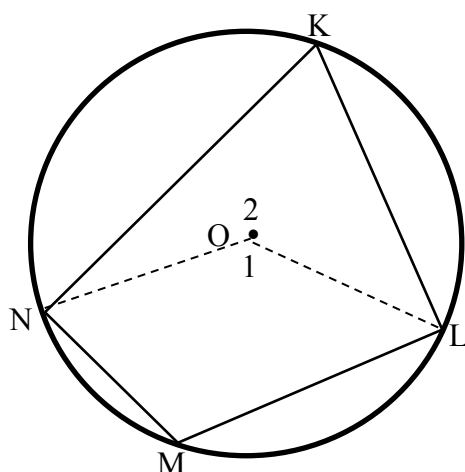
QUESTION/VRAAG 8



8.1	$\hat{T}_2 = \hat{A} = 50^\circ$	tan-chord theorem	✓ S ✓ R	(2)
8.2	$\hat{A} = \hat{K}_3 = 50^\circ$	corresp $\angle$'s , $AQ \parallel KP$	✓ S ✓ R	(2)
8.3	$\hat{A} = \hat{Q}_3 = 50^\circ$	tan-chord theorem	✓ S ✓ R	(2)
	OR			
	$\hat{T}_2 = \hat{Q}_3 = 50^\circ$	$\angle$'s opp equal tangents	✓ S ✓ R	(2)
8.4	Line subtend equal $\angle$'s OR converse $\angle$'s in the same segment		✓ R	(1)
8.5	$\hat{T}_2 = \hat{K}_2 = 50^\circ$	$\angle$'s in same segment	✓ S ✓ R	
	$\hat{K}_2 = \hat{Q}_1 = 50^\circ$	alt $\angle$'s , $AQ \parallel KP$	✓ R	(3)
				[10]

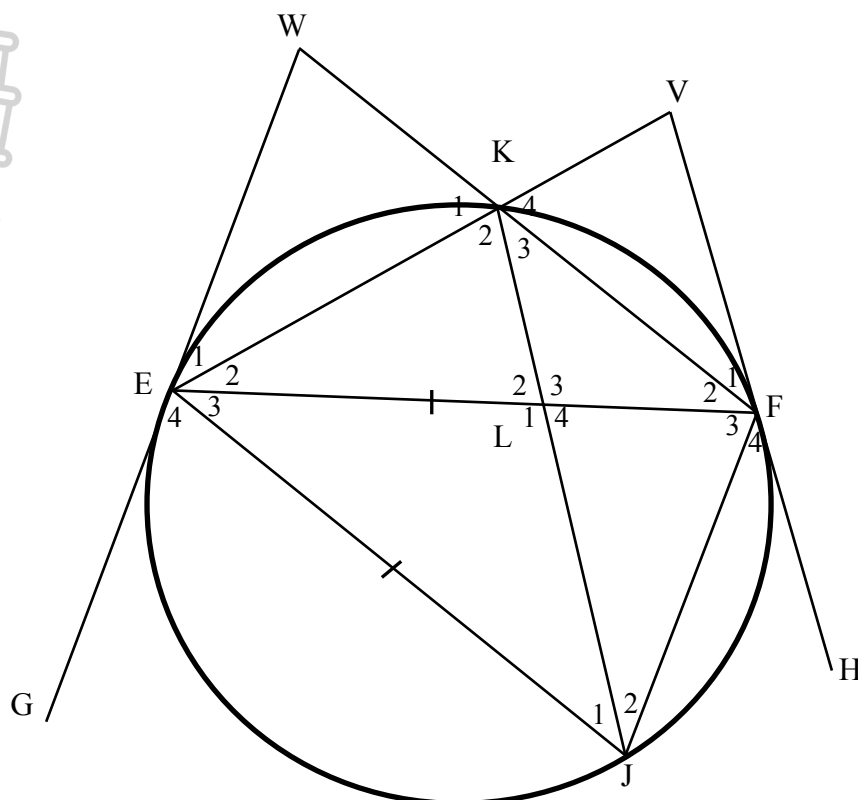


QUESTION/VRAAG 9



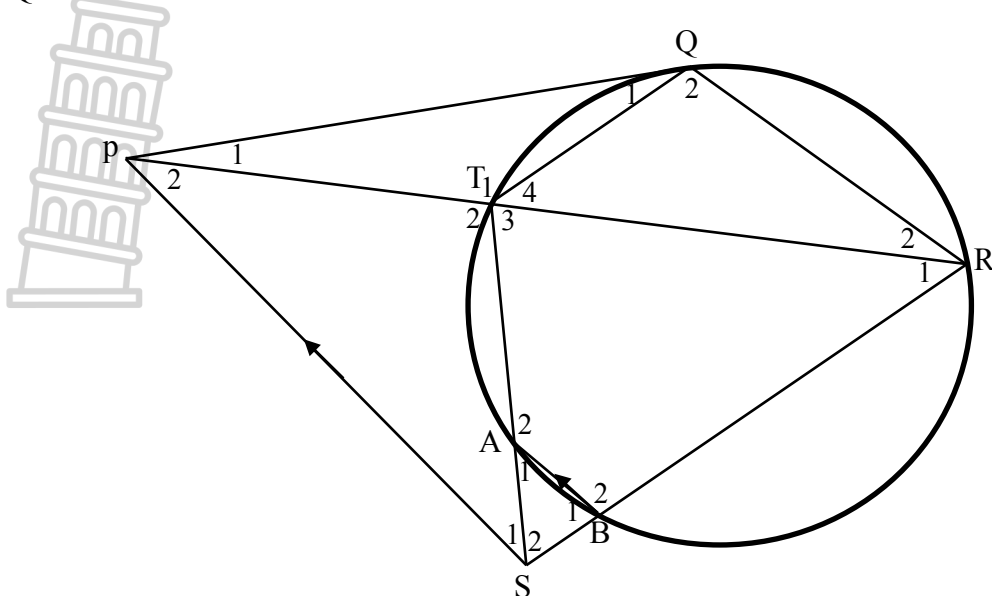
9.1	Construction: Draw radii ON and OL	✓ Construction	
	$\hat{O}_1 = 2 \times \hat{K}$ midpt $\angle = 2 \times \text{circumf } \angle$	✓ S ✓ R	
	$\hat{O}_2 = 2 \times \hat{M}$ midpt $\angle = 2 \times \text{circumf } \angle$	✓ S/R	
	$\hat{O}_1 + \hat{O}_2 = 360^\circ$ revolution	✓ S/R	
	$2\hat{K} + 2\hat{M} = 360^\circ$		
	$\hat{K} + \hat{M} = 180^\circ$		(5)





9.2.1	$\hat{J} = \hat{F}_3$ $\hat{F}_3 = \hat{K}_2$ $\therefore \hat{J} = \hat{K}_2$	$\angle$'s opp equal sides $\angle$'s in the same segment	$\checkmark$ S $\checkmark$ R $\checkmark$ S $\checkmark$ R	(4)
9.2.2	$\hat{K}_1 = \hat{J}$ $\hat{J} = \hat{K}_2$ $\hat{K}_1 = \hat{K}_2$	ext $\angle$ of cyclic quad proven	$\checkmark$ S $\checkmark$ R	(2)
9.2.3	$\hat{F}_1 + \hat{F}_2 = \hat{J}$ $\hat{J} = \hat{K}_2$ $\hat{F}_1 + \hat{F}_2 = \hat{K}_2$ $\therefore$ KLFV is a cyclic quad	tan-chord theorem proven converse ext $\angle$ of cyclic quad	$\checkmark$ S $\checkmark$ R $\checkmark$ $\checkmark$ R	(3)
				[14]

QUESTION/VRAAG 10



10.1	$\hat{S}_1 = \hat{A}_1 = x$ $\hat{A}_1 = \hat{R}_1 = x$	alt $\angle$'s, PS $\parallel$ AB ext $\angle$ of cyclic quad	✓ S ✓ R ✓ S ✓ R	(4)
10.2	In $\triangle PQT$ and $\triangle PRQ$: (i) $\hat{P}_1 = \hat{P}_1$ (ii) $\hat{Q}_1 = \hat{R}_2$ $\therefore \triangle PQT \parallel \triangle PRQ$	common tan-chord theorem 3 $\angle$'s	✓ S ✓ S/R ✓ R	(3)
	OR In $\triangle PQT$ and $\triangle PRQ$: (i) $\hat{P}_1 = \hat{P}_1$ (ii) $\hat{Q}_1 = \hat{R}_2$ (iii) $\hat{T}_1 = \hat{PQR}$ $\therefore \triangle PQT \parallel \triangle PRQ$	common tan-chord theorem 3 rd $\angle$ 3 $\angle$'s	✓ S ✓ S/R ✓ S	(3)
10.3	$\frac{PQ}{PR} = \frac{PT}{PQ}$ $\frac{PQ}{9} = \frac{4}{PQ}$ $PQ^2 = 36$ $\therefore PQ = 6$	$\parallel \Delta$'s	✓ R ✓ subst ✓ answer	(3)

10.4	In $\triangle PTS$ and $\triangle PSR$: (i) $\hat{P}_2 = \hat{P}_2$ common (ii) $\hat{S}_1 = \hat{R}_1 = x$ proven $\therefore \triangle PTS \parallel \triangle PSR$ 3 $\angle$'s OR In $\triangle PTS$ and $\triangle PSR$: (i) $\hat{P}_2 = \hat{P}_2$ common (ii) $\hat{S}_1 = \hat{R}_1 = x$ proven (iii) $\hat{P}_2 = \hat{S}$ 3rd $\angle$ $\therefore \triangle PTS \parallel \triangle PSR$ 3 $\angle$'s	✓ S ✓ S ✓ R ✓ S ✓ S/R ✓ S	(3)
10.5	$\frac{PT}{PS} = \frac{PS}{PR}$ $\parallel \triangle$'s $\therefore PS^2 = PT.PR$ $PQ^2 = PT.PR$ from 10.3 $PS^2 = PQ^2$ $\therefore PS = PQ$	✓ S/R ✓ S ✓ S	(3)
			[16]

TOTAL/TOTAAL : 150

