



education

MPUMALANGA PROVINCE  
REPUBLIC OF SOUTH AFRICA

NATIONAL  
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

JUNE 2023

MARKS: 150

TIME: 3 hours

Stanmorephysics

This question paper consists of 13 pages and 1 information sheet  
and an answer book is provided.

**INSTRUCTIONS AND INFORMATION**

Read the following instructions carefully before answering the questions.

1. The question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.



**QUESTION 1**

An estate agent did a survey on the salaries of the people renting homes in a complex. She selected 12 homes where two salary earners live for her survey. The data collected is recorded in the table below.



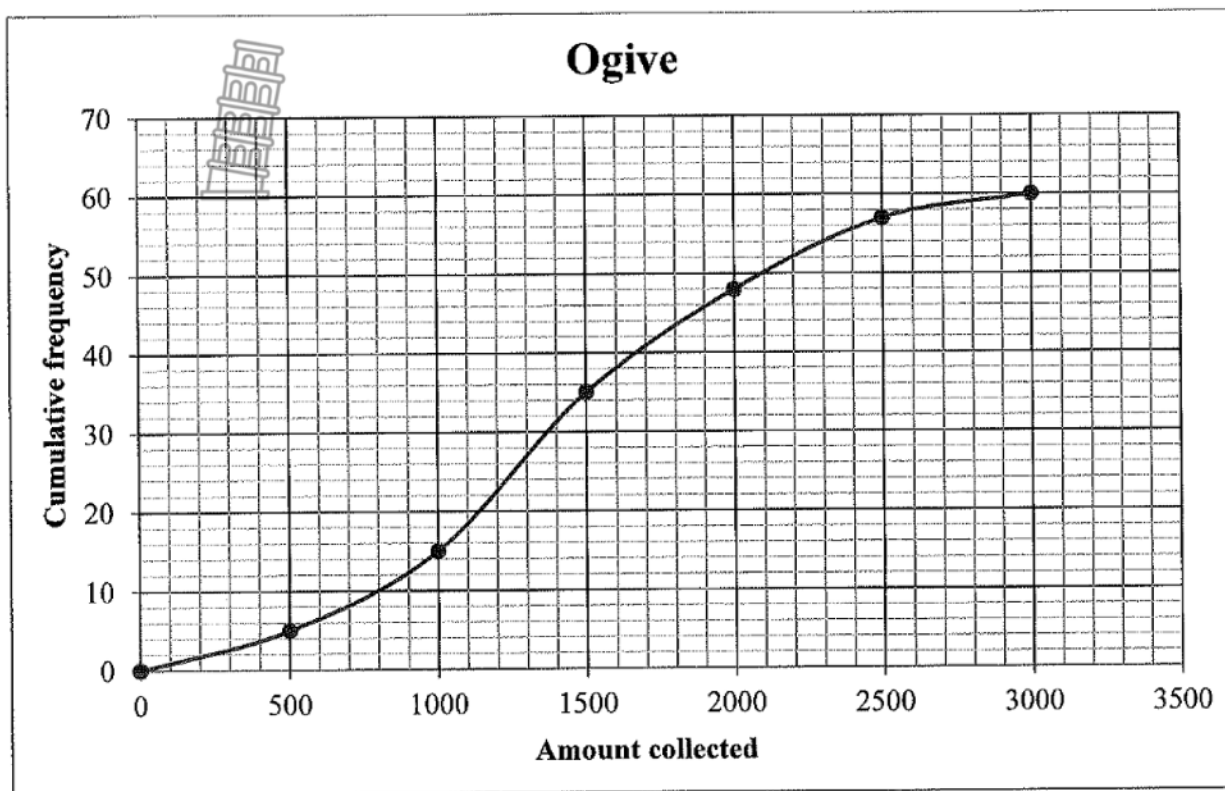
|                    |      |      |      |      |      |      |      |      |       |       |      |       |
|--------------------|------|------|------|------|------|------|------|------|-------|-------|------|-------|
| Salary of person 1 | 3000 | 2100 | 5100 | 3560 | 6250 | 7400 | 4210 | 3200 | 2600  | 1000  | 4100 | 8000  |
| Salary of person 2 | 4500 | 8320 | 6500 | 3500 | 1500 | 4200 | 6420 | 3520 | 10500 | 11000 | 7800 | 19350 |

- 1.1 Determine the median salary for person 1 in this data. (2)
- 1.2 Determine the mean income for person 2 in this data. (2)
- 1.3 Determine the number of salaries for person 2 that are above ONE standard deviation from the mean. (3)
- 1.4 Determine the equation of the least squares regression line for the given data in the table. (3)
- 1.5 Draw a scatter plot and the least squares regression line for this data on the grid provided in the ANSWER BOOK. (4)
- 1.6 Explain what will happen to the least squares regression line if the income of the person receiving an income of R19350 decreases by 50%. (2)

**[16]**

## QUESTION 2

The ogive below shows the money collected by parents during a fundraising event at a school.



- 2.1 Determine the modal class of this data. (1)
- 2.2 Use the ogive to determine the total number of parents who raised R1000 or more. (3)
- 2.3 Use the ogive to determine the values of  $a$ ,  $b$  and  $c$  in the five number summary, given in the table below.

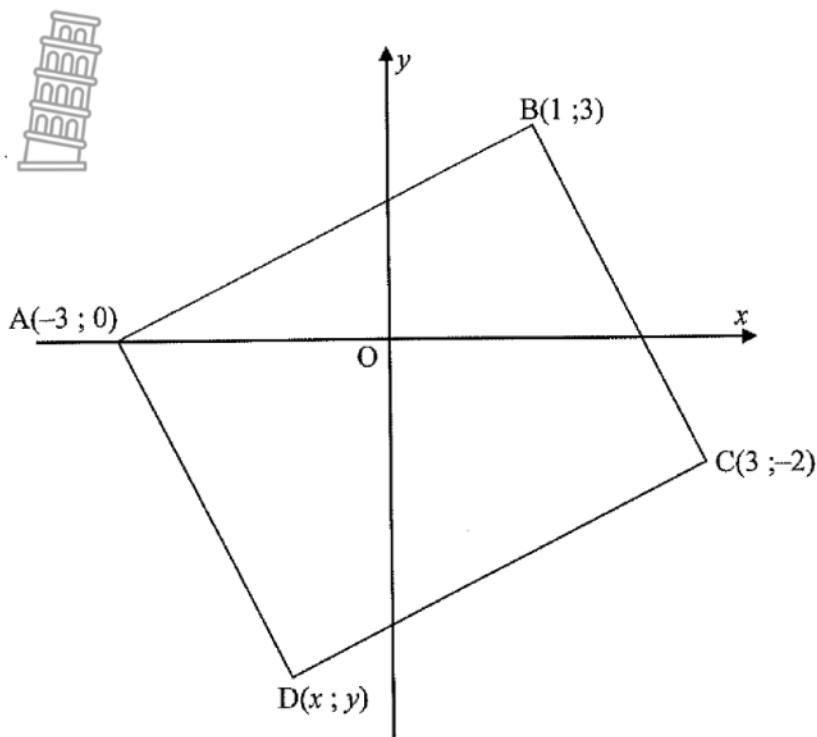
|    |     |     |     |       |
|----|-----|-----|-----|-------|
| R0 | $a$ | $b$ | $c$ | R3000 |
|----|-----|-----|-----|-------|

(4)  
[8]



**QUESTION 3**

In the diagram below ABCD is a parallelogram with  $A(-3; 0)$ ,  $B(1; 3)$ ,  $C(3; -2)$  and  $D(x; y)$ .

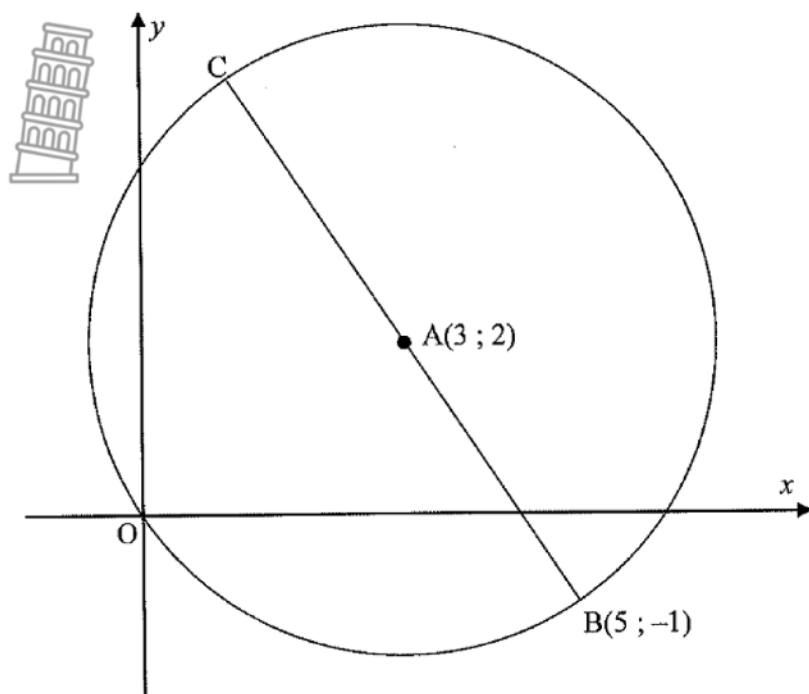


- 3.1 Write down the coordinates of D. (2)
- 3.2 Calculate the gradients of AB and BC and state if ABCD is a rectangle. Give a reason for your answer. (4)
- 3.3 Determine the coordinates of M, the midpoint of AB. (2)
- 3.4 Find the equation of line MN passing through M, which is perpendicular to AB. (3)
- 3.5 Calculate the size of  $\hat{BCD}$ . (5)
- 3.6 Determine the area of  $\triangle BCD$ . (5)
- [21]**



**QUESTION 4**

- 4.1  $A(3; 2)$  is the centre of the circle through B and C. BC is a diameter of the circle.

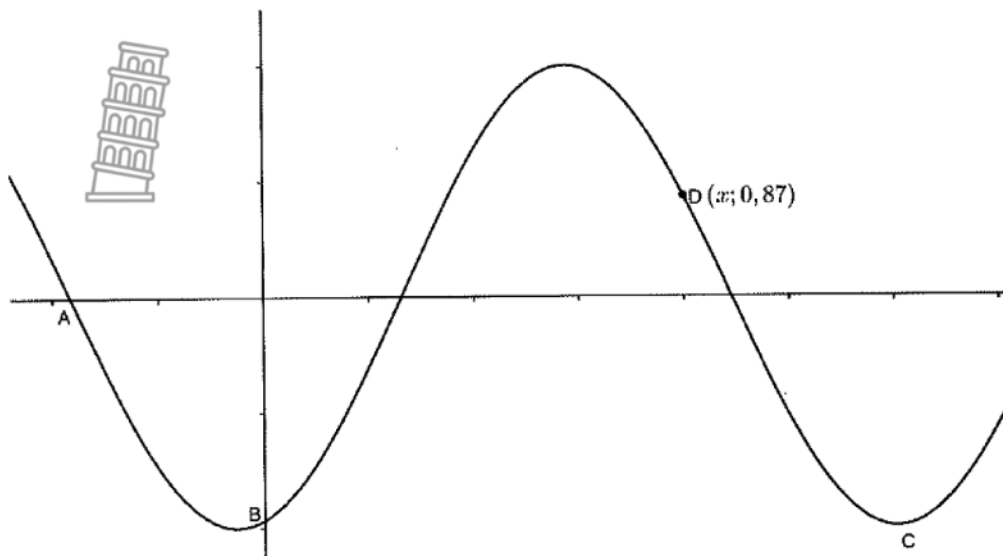


- 4.1.1 Write down the equation of the circle in the form  $(x - a)^2 + (y - b)^2 = r^2$  (4)
- 4.1.2 determine the coordinates of C. (2)
- 4.1.3 Determine the equation of the tangent to the circle at C. (4)
- 4.1.4 Determine the equations of the tangents to the circle which gradients are zero. Give your answers in the simplest surd form. (2)
- 4.1.5 A new circle is drawn. A point  $P(x; y)$  on the circumference of the new circle, is such that it is always 4 units from the circumference of the original circle, and outside the original circle. Determine the equation of this new circle. (2)
- 4.2 Determine whether the circles with equations A:  $(x + 3)^2 + y^2 = 4$  and B:  $x^2 + y^2 + 4y + 3 = 0$  will intersect or not. Show ALL calculations. (6)

**[20]**

## QUESTION 6

The graph of  $f(x) = -2 \cos(x + 15^\circ)$  is given.



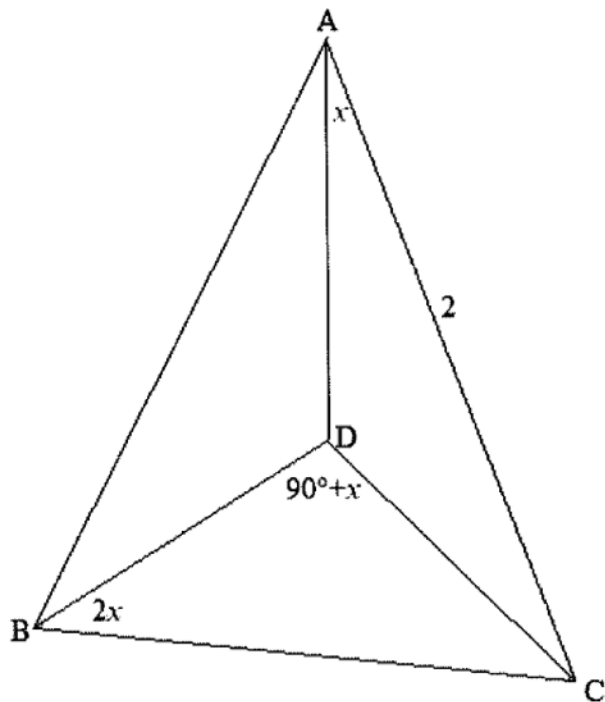
- 6.1 Give the amplitude of  $f(x)$ . (1)
- 6.2 Give the period of  $f(x)$ . (1)
- 6.3 If  $g(x) = f(x) + 2$ , give the range of  $g(x)$ . (2)
- 6.4 Give the coordinates of:
- 6.4.1 A, an  $x$  - intercept of  $f$  (2)
- 6.4.2 B, the  $y$  - intercept of  $f$  (2)
- 6.4.3 C, a turning-point of  $f$ . (2)
- 6.4.4 The  $x$ - coordinate of D, if D has coordinates  $(x; 0,87)$ . Show your calculations (3)
- [13]



## QUESTION 7

AD is a vertical pole and points B and C are in the same horizontal plane as D, the foot of the tower., and

$$\hat{DAC} = x, \hat{CBD} = 2x, \hat{BDC} = 90^\circ + x \text{ and } AC = 2$$



7.1 Show that  $BC = 1$ .

(6)

7.2 Show that  $BD = \frac{\cos 3x}{\cos x}$

(3)

[9]

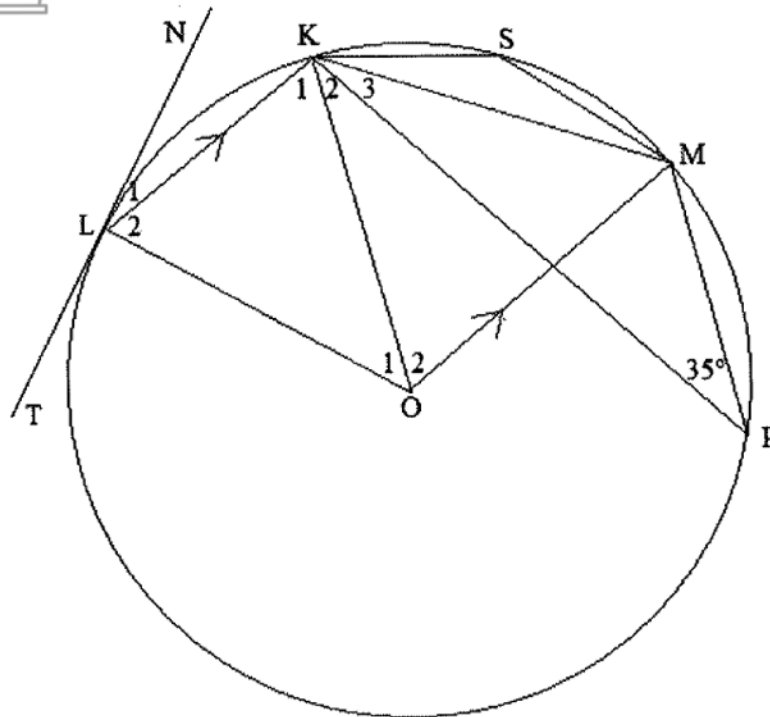


Give reasons for your statements and calculations in QUESTIONS 8, 9 and 10.

**Question 8**

8.1 In the diagram, O is the centre of the circle.  $KL \parallel OM$ , NLT is a tangent to the circle at L.

$$\angle KPM = 35^\circ$$



Determine, giving reasons, the sizes of the following angles:

8.1.1  $\hat{O}_2$  (2)

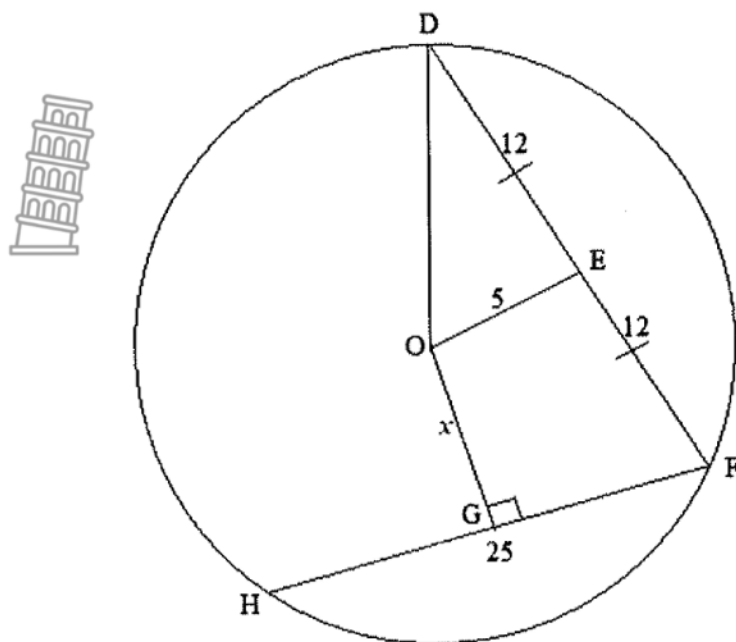
8.1.2  $\hat{O}_1$  (3)

8.1.3  $\hat{L}_1$  (2)

8.1.4  $\hat{S}$  (2)



8.2 In the circle below with centre O,  $DE = EF = 12$ ,  $FH = 25$ ,  $OG \perp FH$ ,  $OE = 5$  and  $OG = x$ .



Determine, giving reasons, the length of:

8.2.1 OD.

(4)

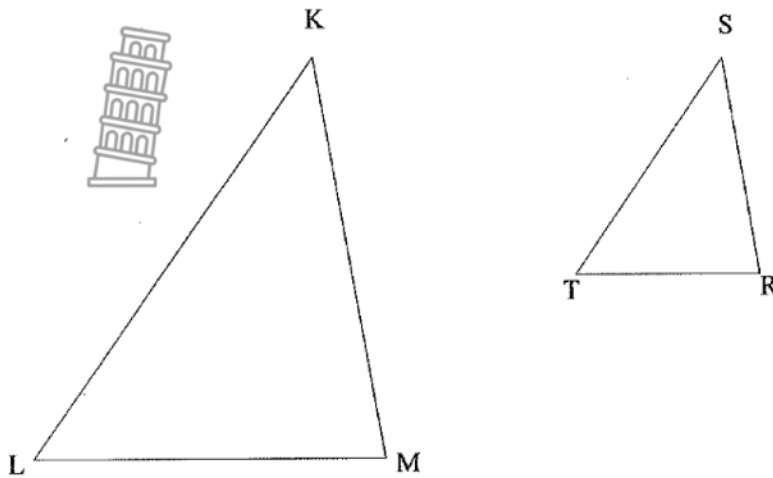
8.2.2 GH.

(2)

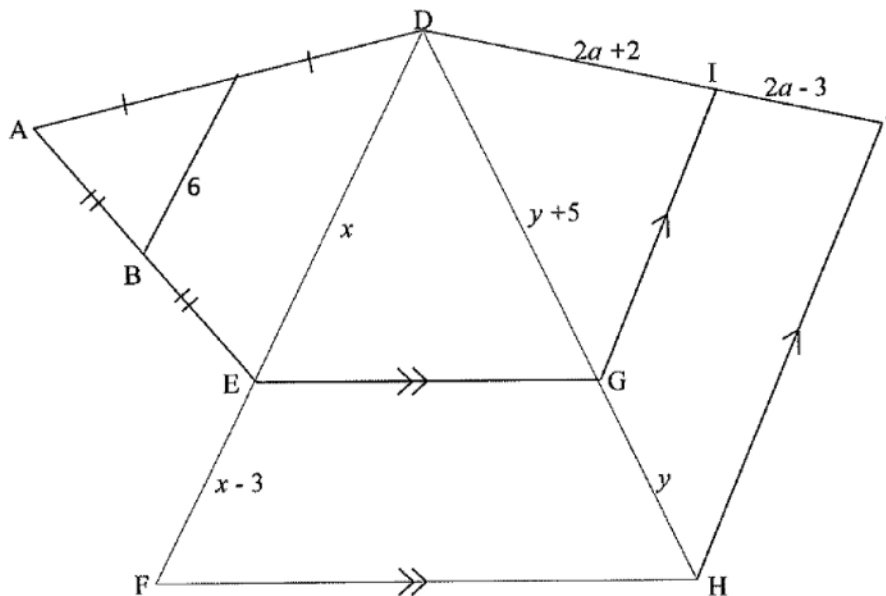
[15]

**QUESTION 9**

9.1 In  $\triangle KLM$  and  $\triangle STR$ ,  $\hat{K} = \hat{S}$ ,  $\hat{L} = \hat{T}$ ,  $\hat{M} = \hat{R}$ . Prove that  $\frac{ST}{KL} = \frac{SR}{KM}$ . (6)



9.2 In the sketch  $EG \parallel FH$  and  $GI \parallel HJ$ .  $AB = BE$  and  $AC = CD$ .  $BC = 6$ ,  $DE = x$ ,  $EF = x - 3$ ,  $DG = y + 5$ ,  $GH = y$ ,  $DI = 2a + 2$  and  $IJ = 2a - 3$ .



Calculate the values of :

9.2.1  $x$

(2)

9.2.2  $y$

(3)

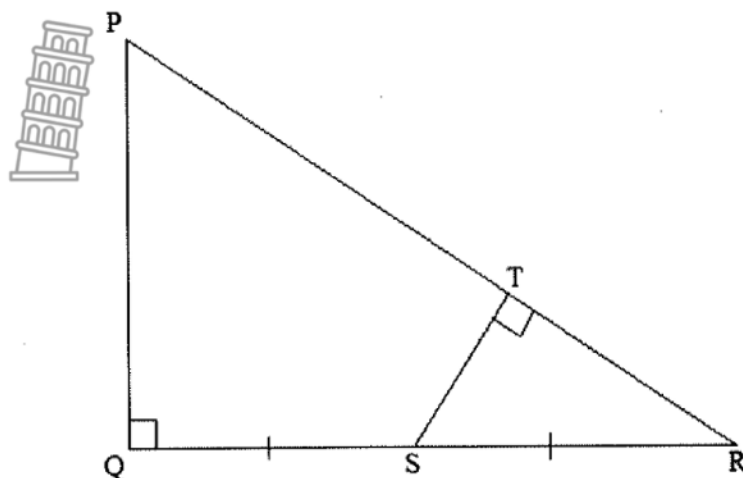
9.2.3  $a$

(3)

[8]

## QUESTION 10

In the sketch,  $QS = SR$  and  $\hat{PQT} = \hat{STR} = 90^\circ$ .



10.1 Prove that  $\triangle PQR \sim \triangle STR$

(4)

10.2 Hence, prove that  $PR \cdot RT = SQ \cdot RQ$

(2)

[16]

**TOTAL: 150**



**FORMULA SHEET**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$T_n = a + (n - 1)d$$

$$T_n = ar^{n-1}$$



$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c \quad y - y_1 = m(x - x_1)$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \sin \beta$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= 1 - 2\sin^2 \alpha$$

$$= 2\cos^2 \alpha - 1$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$\hat{y} = a + bx$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

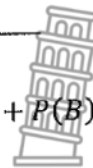
$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



## QUESTION/VRAAG 5

|       |                                                                                       |     |
|-------|---------------------------------------------------------------------------------------|-----|
| 5.1.1 |      |     |
| 5.1.2 |                                                                                       | (3) |
| 5.2   |  | (2) |
|       |                                                                                       | (5) |



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**MARKING GUIDELINE**

**MARKS: 150 marks**

**This question paper consists of 13 pages and an information sheet.**



**NOTE:**

1. If a candidate answered a question TWICE, mark only the FIRST attempt.
2. If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
3. Consistent accuracy applies to ALL aspects of the marking memorandum.
4. Assuming values/answers in order to solve a problem is unacceptable.



**LET WEL:**

5. As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
6. As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
7. Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
8. Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.
9. Write neatly and legibly.

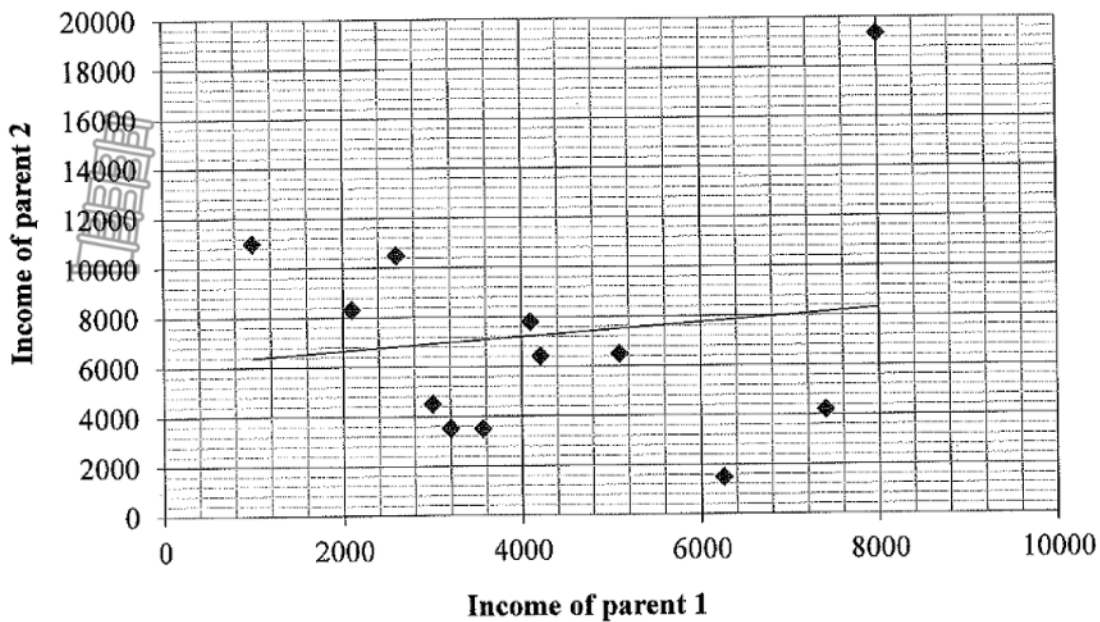
**QUESTION/VRAAG 1**

|     |                                                                                                                              |                                                                   |
|-----|------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------|
| 1.1 | $\frac{87110}{12} = R 7259,17$                                                                                               | ✓ R 7259,17<br>(2)                                                |
| 1.2 | SD = R4579,26<br>Above ONE standard deviation = mean + 1SD<br>= R7259,17 + R4579,26<br>= R11838,43<br><br>Only ONE household | ✓ SD<br>✓ boundary<br><br>✓ answer<br>(3)                         |
| 1.3 | $y = a + bx$<br>$a = 6102,11$<br>$b = 0,27$<br>$y = 6102,11 + 0,27x$                                                         | ✓ $a = 6102,11$<br>✓ $b = 0,27$<br>✓ $y = 6102,11 + 0,27x$<br>(3) |



1.4

### Scatter plot of income per household



✓✓ points correctly plotted

✓✓ regression line

(4)

1.5

The gradient of the line will become smaller, causing the data to be more symmetrical about the regression line, and there will be no outlier.

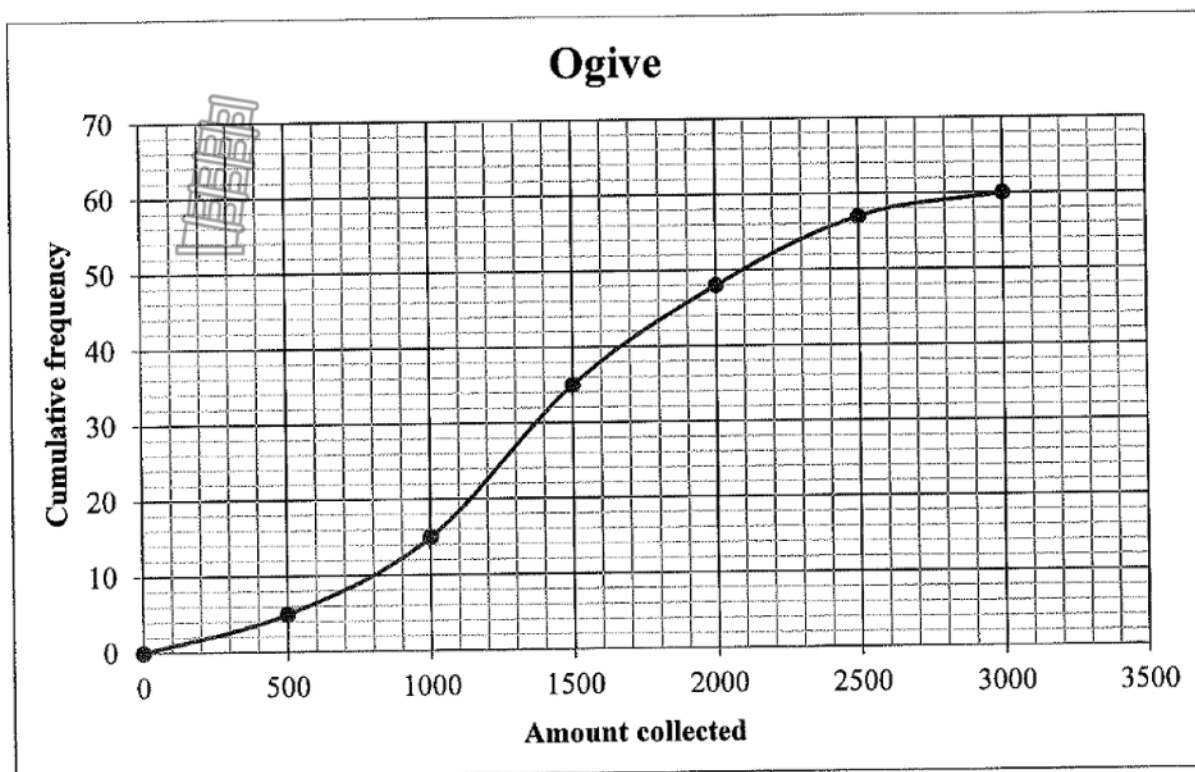
✓✓ answer

(2)

[14]

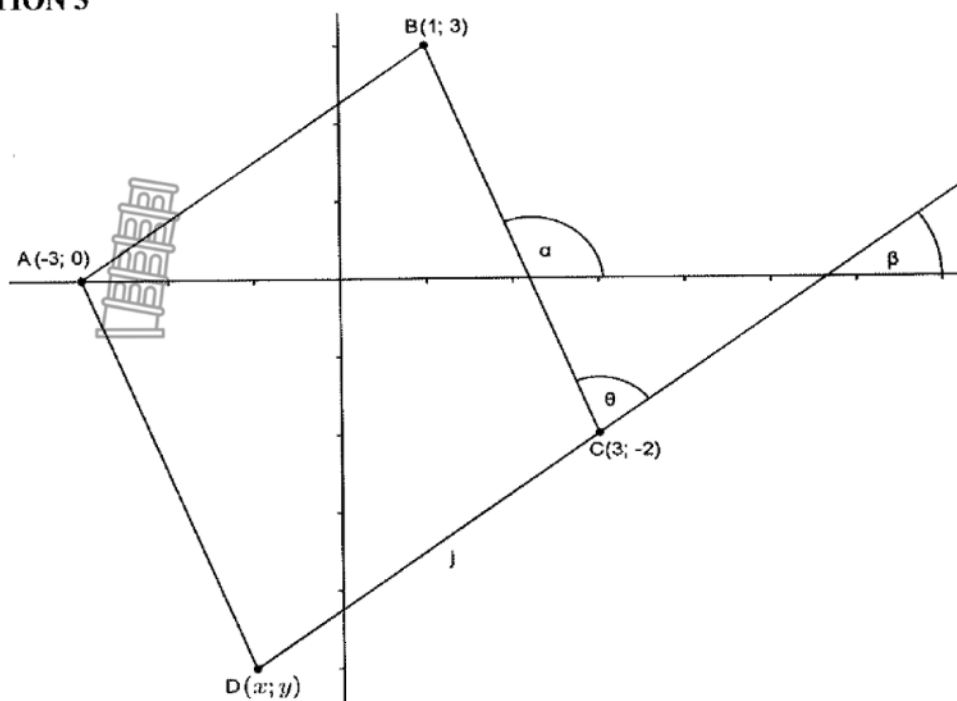


QUESTION/VRAAG 2



|     |                                                                                                        |                                                |
|-----|--------------------------------------------------------------------------------------------------------|------------------------------------------------|
| 2.1 | $1000 < x \leq 1500$                                                                                   | ✓ answer<br>(1)                                |
| 2.2 | $60 - 15 = 45$ parents                                                                                 | ✓ 15 and 60<br>✓ answer<br>(3)                 |
| 2.2 | $a = \text{lower quartile} = 1000$<br>$b = \text{median} = 1350$<br>$c = \text{upper quartile} = 1850$ | ✓ method<br>✓ Q1<br>✓ Q2<br>✓ Q3<br>(4)<br>[7] |
|     |                                                                                                        |                                                |
|     |                                                                                                        |                                                |

## QUESTION 3



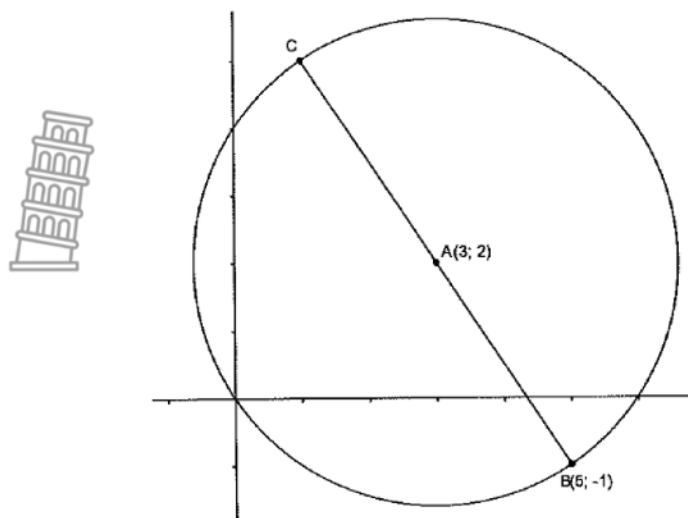
|     |                                                                                                                                                                                                                                                               |                                                                |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| 3.1 | $D(-1; -5)$                                                                                                                                                                                                                                                   | ✓✓ (2)                                                         |
| 3.2 | $m_{AB} = \frac{3-0}{1-(-3)} = \frac{3}{4}$ $m_{BC} = \frac{3-(-2)}{1-3} = \frac{5}{-2}$ $\frac{3}{4} \times \frac{5}{-2} = \frac{15}{-8} \neq -1$ <p><math>\therefore</math> AB not perpendicular to BC, <math>\therefore</math> ABCD is not a rectangle</p> | $m_{AB}$ ✓<br>$m_{BC}$ ✓<br>$\neq -1$ ✓<br>Not rectangle ✓ (4) |
| 3.3 | $M_{AB} = \left(\frac{-3+1}{2}; \frac{0+3}{2}\right) = \left(-1; \frac{3}{2}\right)$                                                                                                                                                                          | ✓✓ (2)                                                         |
| 3.4 | $m_{AB} = \frac{3}{4}, \therefore m_{\perp} = -\frac{4}{3}$ $y - \frac{3}{2} = -\frac{4}{3}(x - (-1))$ $y = -\frac{4}{3}x - \frac{4}{3} + \frac{3}{2}$ $= -\frac{4}{3}x + \frac{1}{6}$                                                                        | $m_{\perp}$ ✓<br>Subst ✓<br>✓<br>(3)                           |
| 3.5 | $\tan \alpha = -\frac{5}{2}$ $\alpha = -68,198 \dots^{\circ} + 180^{\circ} = 111,80^{\circ}$ $\tan \beta = \frac{3}{4} \quad (AB \parallel CD)$ $\beta = 36,87^{\circ}$                                                                                       | ✓<br>✓<br>✓                                                    |

|     |                                                                                                                                                                                                                                                                                                                    |                                                                                                                                           |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
|     | $\therefore \theta = 111,80^\circ - 36,87^\circ$<br>$= 74,93^\circ$ (ext $\angle$ of $\Delta$ )<br>$\hat{BCD} = 105,07^\circ$ (angles on straight line)                                                                                                                                                            | $\checkmark$<br>$\checkmark$<br>(5)                                                                                                       |
| 3.6 | $BC^2 = (1-3)^2 + (3-(-2))^2$<br>$= 4 + 25 = 29$<br>$BC = \sqrt{29}$<br>$CD^2 = (3-(-1))^2 + (-2-(-5))^2$<br>$= 16 + 9 = 25$<br>$CD = 5$<br>$\text{Area } \Delta BCD = \frac{1}{2} BC \cdot CD \cdot \sin \hat{BCD}.$<br>$= \frac{1}{2} \cdot \sqrt{29} \cdot 5 \cdot \sin 105,07^\circ$<br>$= 13 \text{ units}^2$ | Subst in formula $\checkmark$<br><br>$BC \checkmark$<br><br>$CD \checkmark$<br><br>Subst in sine form<br>$\checkmark$<br>$\checkmark$ (5) |
|     |                                                                                                                                                                                                                                                                                                                    | [21]                                                                                                                                      |



QUESTION 4

4.1



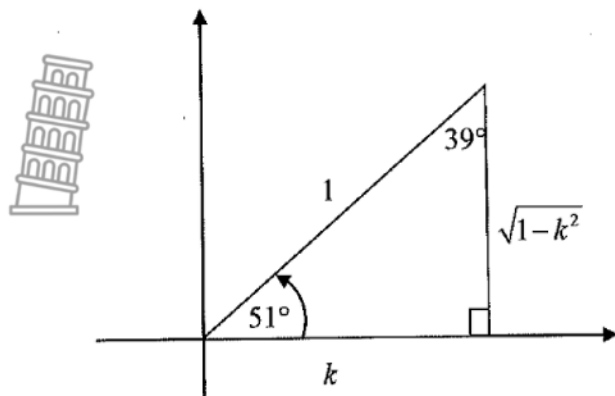
|       |                                                                                                                                                                           |                              |     |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------|-----|
| 4.1.1 | $r^2 = (5 - 3)^2 + (-1 - 2)^2$ $= 4 + 9 = 13$ $\therefore (x - 3)^2 + (y - 2)^2 = 13$                                                                                     | ✓<br>✓<br>✓✓                 | (4) |
| 4.1.2 | $C = (1; 5)$ (symmetry)                                                                                                                                                   | ✓✓                           | (2) |
| 4.1.3 | $m_{AC} = \frac{5-2}{1-3} = \frac{3}{-2}$<br>$m_{\text{tangent}} = \frac{2}{3}$<br>$y - 5 = \frac{2}{3}(x - 1)$ $y = \frac{2}{3}x - \frac{2}{3} + 5$ $= x + 4\frac{1}{3}$ | ✓<br><br>✓<br><br>✓<br><br>✓ | (4) |
| 4.1.4 | $r = \sqrt{13}$ and horizontal lines $\sqrt{13}$ from centre.<br>$\therefore y = 2 + \sqrt{13}$ and<br>$y = 2 - \sqrt{13}$                                                | ✓<br>✓                       | (2) |
| 4.1.5 | $(x - 3)^2 + (y - 2)^2 = (4 + \sqrt{13})^2$<br>Or $(x - 3)^2 + (y - 2)^2 = 57,84$                                                                                         | ✓ centre<br>✓ radius         | (2) |
| 4.2   | $x^2 + y^2 + 4y + 3 = 0$ $x^2 + y^2 + 4y + 2^2 = -3 + 2^2$ $x^2 + (y + 2)^2 = 1$                                                                                          | ✓                            |     |

|  |                                                                                                                                                                                                                                                                                                                                    |                                                  |
|--|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------|
|  | <p>Centres: A: (3; 0) and B: (0; -2)</p> <p>Distance between centres <math>AB = \sqrt{(3 - 0)^2 + (2 - 0)^2} = \sqrt{13}</math><br/> <math>= 3,61</math></p> <p>Radii = 1 and 2</p> <p>Sum of radii = 3</p> <p>Sum of radii <math>\leq</math> Distance between centres</p> <p><math>\therefore</math> Circles do not intersect</p> | <p>✓</p> <p>✓</p> <p>✓</p> <p>✓</p> <p>✓ (6)</p> |
|  |                                                                                                                                                                                                                                                                                                                                    | [20]                                             |



QUESTION 5

5.1  $\cos 51^\circ = \frac{k}{1}$



|       |                                                                                                                                                                                                                                                                |                                                                                 |
|-------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------|
| 5.1.1 | $y^2 = 1^2 - k^2$ $y = \sqrt{1 - k^2}$<br>$\tan 219^\circ = \tan(180^\circ + 39^\circ)$ $= \tan 39^\circ$ $= \frac{k}{\sqrt{1 - k^2}}$                                                                                                                         | ✓ Pythagoras<br><br>✓ $-\tan 39^\circ$<br>✓ answer<br>(3)                       |
| 5.1.2 | $\sin(-411^\circ) = \sin(-411^\circ + 360^\circ)$ $= \sin(-51^\circ)$ $= -\sin 51^\circ$ $= -\frac{\sqrt{1 - k^2}}{1}$                                                                                                                                         | ✓ reduction<br>✓ answer<br>(2)                                                  |
| 5.1.3 | $\cos 9^\circ = \cos(60^\circ - 51^\circ)$ $= \cos 60^\circ \cos 51^\circ + \sin 60^\circ \sin 51^\circ$ $= \left(\frac{1}{2}\right)(k) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{1 - k^2}}{1}\right)$ $= \frac{1k}{2} + \frac{\sqrt{3(1 - k^2)}}{2}$ | ✓ compound angles<br>✓ expansion<br><br>✓ substitution<br>✓ substitution<br>(4) |



|     |                                                                                                                                                                                                                                                                                                                                            |                                                                                                                                                                         |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.2 | $\sin(45^\circ + x) \cdot \sin(45^\circ - x)$ $= (\sin 45 \cos x + \cos 45 \sin x)(\sin 45 \cos x - \cos 45 \sin x)$ $= \left( \frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x \right) \left( \frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x \right)$ $= \frac{1}{2} \cos^2 x - \frac{1}{2} \sin^2 x$ $= \frac{1}{2} \cos 2x$ | ✓ expansion<br>✓ expansion<br>✓ $\frac{\sqrt{2}}{2}$<br>✓ simplification<br>✓ answer<br>(5)                                                                             |
| 5.3 | $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$ $LHS = \frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x}$ $= \frac{\sin x + 2 \sin x \cos x}{1 + \cos x + 2 \cos^2 x - 1}$ $= \frac{\sin x + 2 \sin x \cos x}{\cos x + 2 \cos^2 x}$ $= \frac{\sin x(1 + 2 \cos x)}{\cos x(1 + 2 \cos x)}$ $= \tan x = RHS$                            | ✓ $2 \sin x \cos x$<br>✓ $2 \cos^2 x - 1$<br><br>✓ simplification of denominator<br><br>✓ common factor<br>✓ common factor<br>✓ $\frac{\sin x}{\cos x} = \tan x$<br>(6) |
| 5.4 | $\sin 15^\circ = \sin(45^\circ - 30^\circ)$ $= \sin 45 \cos 30 - \cos 45 \sin 30$ $= \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) - \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right)$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$                                                                                 | ✓ $\sin(45^\circ - 30^\circ)$<br>✓ expansion<br><br>✓ substitution<br><br>✓ simplification $\sqrt{6}$<br>(4)                                                            |

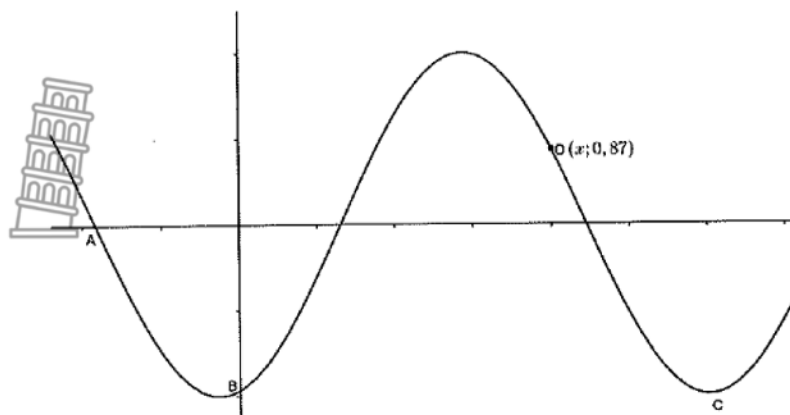


|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                 |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5.5 | $\frac{(\sin x - \cos x)^2 - 1}{\sin^2 x - 1} = 2$ $\sin^2 x - 2\sin x \cos x + \cos^2 x - 1 = 2(\sin^2 x - 1)$ $\sin^2 x - 2\sin x \cos x + \cos^2 x - 1 = 2\sin^2 x - 2$ $-2\sin^2 x - 2\sin x \cos x + 2 = 0$ $-\sin^2 x - \sin x \cos x + 1 = 0$ $-\sin^2 x - \sin x \cos x + \sin^2 x + \cos^2 x = 0$ $\cos^2 x - \sin x \cos x = 0$ $\cos x(\cos x - \sin x) = 0$ $\cos x = 0 \quad \text{or} \quad \cos x - \sin x = 0$ $x = \pm 90^\circ + 360k \quad \cos x = \sin x$ $\tan x = 1$ $x = 45^\circ + 180k, k \in \mathbb{Z}$ | $\checkmark 2\sin^2 x - 2$<br><br>$\checkmark \sin^2 x + \cos^2 x$<br>$\checkmark$ standard form<br>$\checkmark$ common factor<br><br>$\checkmark x = \pm 90 + 360k \quad k \in \mathbb{Z}$<br><br>$\checkmark \tan x = 1$<br>$\checkmark x = 45^\circ + 180k$<br><br><div style="text-align: right;">(7)</div> |
|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | [31]                                                                                                                                                                                                                                                                                                            |



# QUESTION 6

The graph of  $f(x) = -2 \cos(x + 15^\circ)$  is given.

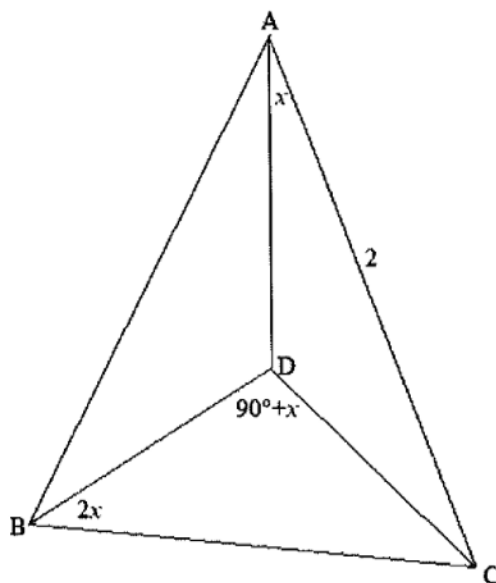


|       |                                                                                                                                                                                                        |                 |     |
|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------|-----|
| 6.1   | Amplitude = 2                                                                                                                                                                                          | ✓               | (1) |
| 6.2   | Period = $360^\circ$                                                                                                                                                                                   | ✓               | (1) |
| 6.3   | Range of $g(x)$ : $y \in [0; 4]$                                                                                                                                                                       | ✓✓              | (2) |
| 6.4.1 | $\therefore A = (-105^\circ; 0)$                                                                                                                                                                       | ✓✓              | (2) |
| 6.4.2 | $B = (0, -1,93)$                                                                                                                                                                                       | ✓✓              | (2) |
| 6.4.3 | $C = (165^\circ; -2)$                                                                                                                                                                                  | ✓✓              | (2) |
| 6.4.4 | $-2 \cos(x + 15^\circ) = 0,87$<br>$\cos(x + 15^\circ) = -0,435$<br>$x + 15^\circ = 115,785 \dots^\circ + 360k, k \in \mathbb{Z}$<br>$x = 100,79^\circ + 360k$<br>$\therefore D = (100,79^\circ; 0,87)$ | ✓<br><br>✓<br>✓ | (3) |
|       |                                                                                                                                                                                                        | [13]            |     |



## QUESTION 7

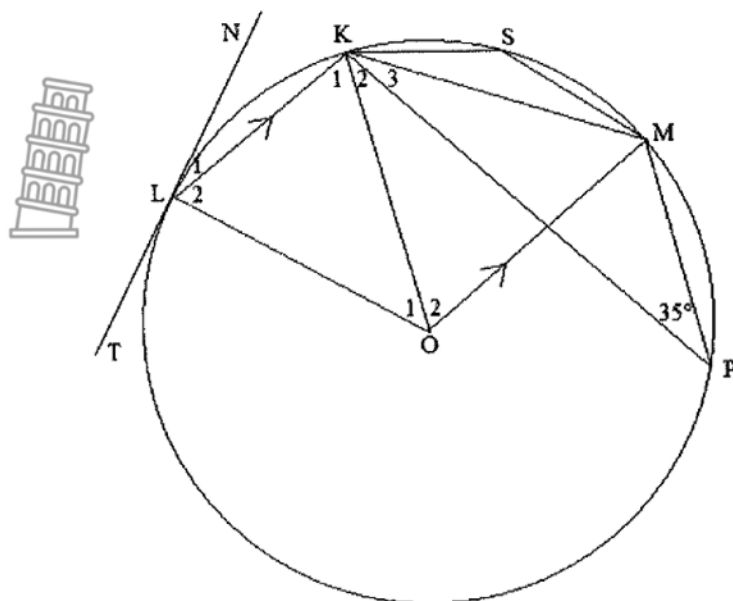
AD is a vertical pole and points B and C are in the same horizontal plane as D, the foot of the tower.  $\widehat{DAC} = x$ ,  $\widehat{BDC} = 2x$ ,  $\widehat{BDC} = 90^\circ + x$  and  $AC = 2$ .



|     |                                                                                                                                                                                                                                                             |                                                                                                                     |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| 7.1 | $\sin x = \frac{CD}{2}$ $CD = 2 \sin x$ $\frac{BC}{\sin(90^\circ + x)} = \frac{CD}{\sin 2x}$ $\frac{BC}{\cos x} = \frac{2 \sin x}{2 \sin x \cos x}$ $BC = \frac{2 \sin x \cdot \cos x}{2 \sin x \cos x}$ $= 1$                                              | <p>✓</p> <p>✓</p> <p>Subst in Sin-rule ✓</p> <p>Cos x ✓</p> <p>2 sinx cosx ✓</p> <p>Simplification ✓</p> <p>(6)</p> |
| 7.2 | $\widehat{BCD} = 180^\circ - (90^\circ + x) - 2x$ $= 90^\circ - 3x$ <p>In <math>\triangle BCD</math>:</p> $\frac{BD}{\sin(90^\circ - 3x)} = \frac{1}{\sin(90^\circ + x)}$ $\frac{BD}{\cos 3x} = \frac{1}{\cos x}$ $\frac{\cos 3x}{\cos x} = 2 \cos 2x - 1.$ | <p>✓</p> <p>✓</p> <p>Subst in sine-form</p> <p>Co-functions ✓ (3)</p> <p>[9]</p>                                    |

**Question 8**

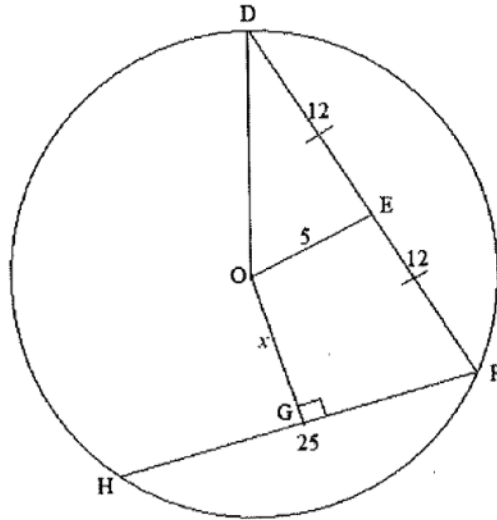
8.1 In the diagram, O is the centre of the circle.  $KL \parallel OM$ , NLT is a tangent to the circle at L.



|       |                                                                            |                                                                                            |                                   |
|-------|----------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|-----------------------------------|
| 8.1.1 | $\hat{O}_2 = 70^\circ$                                                     | $\angle$ at centre circle = $2 \times \angle$ at circumf                                   | S ✓ R ✓ (2)                       |
| 8.1.2 | $\hat{K}_1 = 70^\circ$<br>$\hat{L}_2 = 70^\circ$<br>$\hat{O}_1 = 40^\circ$ | alt. $\angle$ s, $KL \parallel MO$<br>$\angle$ s opp = radii<br>int $\angle$ s of $\Delta$ | S + R ✓<br>S + R ✓<br>S + R ✓ (3) |
| 8.1.3 | $\hat{L}_1 = 20^\circ$                                                     | radius $\perp$ tangent                                                                     | S ✓ R ✓ (2)                       |
| 8.1.4 | $\hat{S} = 135^\circ$                                                      | opp $\angle$ of cyclic quad                                                                | S ✓ R ✓ (2)                       |



8.2

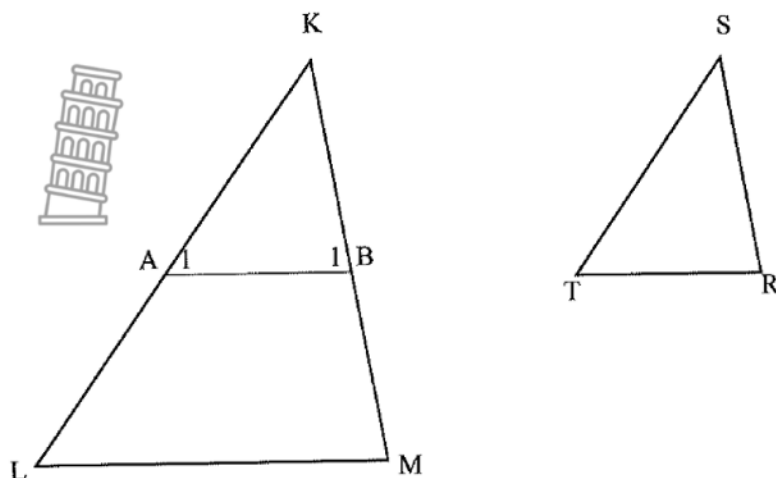


|       |                                                                        |                                                       |                   |      |
|-------|------------------------------------------------------------------------|-------------------------------------------------------|-------------------|------|
| 8.2.1 | $\angle OED = 90^\circ$<br>$OD^2 = 12^2 + 5^2$<br>$= 169$<br>$OD = 13$ | Centre circle midpoint chord<br>Theorem of Pythagoras | S ✓ R ✓<br>✓<br>✓ | (4)  |
| 8.2.2 | $FG = 12,5$                                                            | Centre circle $\perp$ chord                           | S ✓ R ✓           | (2)  |
|       |                                                                        |                                                       |                   | [15] |



## QUESTION 9

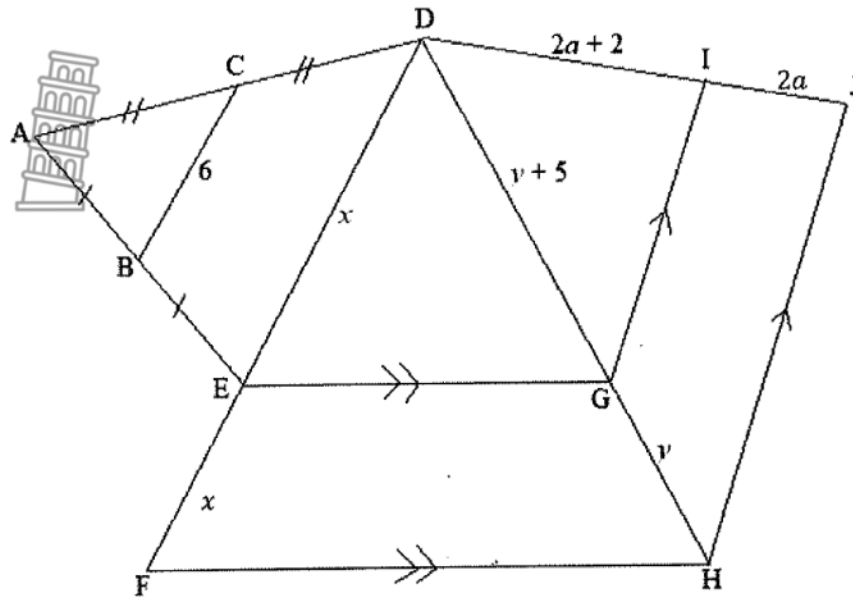
9.1 In  $\triangle KLM$  and  $\triangle STR$ ,  $\hat{K} = \hat{S}$ ,  $\hat{L} = \hat{T}$ ,  $\hat{M} = \hat{R}$ . Prove that  $\frac{ST}{KL} = \frac{SR}{KM}$ . (6)



|     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                  |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| 9.1 | <p>Construction</p> <p>In <math>\triangle KAB</math> and <math>\triangle STR</math>:</p> <p><math>AK = ST</math> construction</p> <p><math>KB = SR</math> construction</p> <p><math>\hat{K} = \hat{S}</math> given</p> <p><math>\therefore \triangle KAB \equiv \triangle STR</math> SAS</p> <p><math>\therefore \hat{A}1 = \hat{T}</math></p> <p>But <math>\hat{T} = \hat{L}</math> given</p> <p><math>\therefore \hat{A}1 = \hat{L}</math></p> <p><math>\therefore AB \parallel LM</math> corresponding angles =</p> <p><math>\therefore \frac{KA}{KL} = \frac{KB}{KM}</math> line // one side of <math>\triangle</math></p> <p><math>\therefore \frac{ST}{KL} = \frac{SR}{KM}</math> <math>AK = ST</math> and <math>KB = SR</math></p> | <p>✓</p> <p>✓</p> <p>✓</p> <p>✓</p> <p>✓</p> <p>✓</p> <p>(6)</p> |
|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|

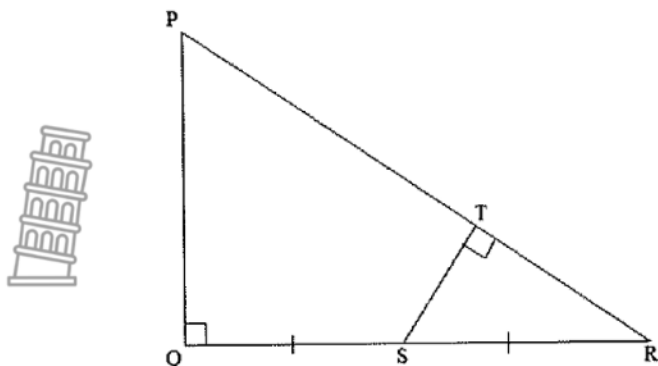


9.2 In the sketch  $EG \parallel FH$  and  $GI \parallel HJ$ .  $AB = BE$  and  $AC = CD$ .  $BC = 6$ ,  $DE = x$ ,  $EF = x - 3$ ,  $DG = y + 5$ ,  $GH = y$ ,  $DI = 2a + 2$  and  $IJ = 2a - 3$ .



|     |                                                                                                                            |                                 |                  |      |
|-----|----------------------------------------------------------------------------------------------------------------------------|---------------------------------|------------------|------|
| 9.1 | $x = 12$                                                                                                                   | midpt theorem                   | S ✓ R ✓          | (2)  |
| 9.2 | $\frac{x}{x-3} = \frac{y+5}{y}$ $\frac{12}{9} = \frac{y+5}{y}$ $12y = 9y + 45$ $3y = 45$ $y = 15$                          | line // to one side of $\Delta$ | S ✓ R ✓<br><br>✓ | (3)  |
| 9.3 | $\frac{2a+2}{2a-3} = \frac{y+5}{y}$ $\frac{2a+2}{2a-3} = \frac{20}{15} = \frac{4}{3}$ $6a + 6 = 8a - 12$ $2a = 18$ $a = 9$ | line // to one side of $\Delta$ | S ✓ R ✓<br><br>✓ | (3)  |
|     |                                                                                                                            |                                 |                  | [14] |

## QUESTION 10



|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                                                                                                |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| 10.1 | <p>In <math>\triangle PQR</math> and <math>\triangle STR</math></p> <p><math>\hat{Q} = \hat{T}</math> <span style="margin-left: 100px;"><math>90^\circ</math> given</span></p> <p><math>\hat{R} = \hat{R}</math> <span style="margin-left: 100px;">common <math>\angle</math></span></p> <p><math>\therefore \hat{P} = \hat{S}</math> <span style="margin-left: 100px;">int <math>\angle</math>s of <math>\Delta</math></span></p> <p><math>\triangle PQR \parallel \triangle STR</math> <span style="margin-left: 100px;">A A A</span></p> | <p>S + R ✓</p> <p>S + R ✓</p> <p>S + R ✓</p> <p>R ✓ <span style="float: right;">(4)</span></p> |
| 10.2 | <p><math>\frac{PQ}{ST} = \frac{RQ}{RT} = \frac{PR}{RS}</math> <span style="margin-left: 100px;">similarity</span></p> <p><math>\therefore PR \cdot RT = RS \cdot RQ</math></p> <p>but <math>RS = QS</math></p> <p><math>\therefore PR \cdot RT = QS \cdot RQ</math></p>                                                                                                                                                                                                                                                                     | <p>✓</p> <p>✓ <span style="float: right;">(2)</span></p>                                       |
|      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | [10]                                                                                           |

TOTAL: 150



MATHEMATICS P2 JUNE 2021 GRID

|     |                    | 1 | 2 | 3 | 4 | TOTAL |
|-----|--------------------|---|---|---|---|-------|
| 1.1 | median             |   | 2 |   |   |       |
| 1.2 | mean               | 2 |   |   |   |       |
| 1.3 | standard deviation |   | 3 |   |   |       |
| 1.4 | regression line    | 3 |   |   |   |       |
| 1.5 | scatter plot       |   | 4 |   |   |       |
| 1.6 | interpret          |   |   |   | 2 |       |

|     |                 |   |   |   |  |  |
|-----|-----------------|---|---|---|--|--|
| 2.1 | modal class     | 1 |   |   |  |  |
| 2.2 | interpret ogive |   | 3 |   |  |  |
| 2.3 | interpret ogive |   |   | 4 |  |  |

|     |                  |   |   |   |  |  |
|-----|------------------|---|---|---|--|--|
| 3.1 | coordinates      | 2 |   |   |  |  |
| 3.2 | Rectangle        |   | 4 |   |  |  |
| 3.3 | midpoint         |   |   | 1 |  |  |
| 3.4 | equation of line |   |   | 2 |  |  |
| 3.5 | size of angle    |   |   | 3 |  |  |
| 3.6 | area triangle    |   | 3 |   |  |  |
|     |                  |   |   |   |  |  |
|     |                  |   |   |   |  |  |
|     |                  |   |   |   |  |  |

|       |                     |   |   |   |  |  |
|-------|---------------------|---|---|---|--|--|
| 4.1.1 | equation circle     |   | 4 |   |  |  |
| 4.1.2 | coordinates         | 2 |   |   |  |  |
| 4.1.3 | equation tangent    |   | 4 |   |  |  |
| 4.1.4 | horizontal tangents |   |   | 2 |  |  |
| 4.1.5 | new circle          |   |   | 2 |  |  |
| 4.2   | circles intersect   |   |   | 6 |  |  |

|       |                    |   |   |   |   |   |
|-------|--------------------|---|---|---|---|---|
| 5.1.1 | reduction          | 3 |   |   |   |   |
| 5.1.2 | reduction          | 2 |   |   |   |   |
| 5.1.3 | compound angle     |   | 4 |   |   |   |
| 5.2   | compound expansion |   |   | 5 |   |   |
| 5.3   | identities         |   |   | 6 |   |   |
| 5.4   | identities         |   |   | 4 |   |   |
| 5.5   | general solution   |   |   |   | 7 |   |
| 6.1   | amplitude          | 1 |   |   |   |   |
| 6.2   | period             | 1 |   |   |   | 4 |

|       |               |  |   |   |   |    |
|-------|---------------|--|---|---|---|----|
| 6.3   | range         |  | 2 |   |   |    |
| 6.4.1 | x-intercept   |  |   |   |   |    |
| 6.4.2 | y-intercept   |  | 2 |   |   |    |
| 6.4.3 | turning point |  | 2 |   |   |    |
| 6.4.4 | x-coordinate  |  |   | 3 |   |    |
| 7.1   | show length   |  |   |   | 5 | 12 |
| 7.2   | show that     |  |   |   |   |    |

|       |                     |  |   |   |  |  |
|-------|---------------------|--|---|---|--|--|
| 8.1.1 | angle at centre     |  | 3 |   |  |  |
| 8.1.2 | int angles triangle |  |   | 3 |  |  |

|              |                                   |           |           |           |           |           |
|--------------|-----------------------------------|-----------|-----------|-----------|-----------|-----------|
| 8.1.3        | tan perp radius                   | 3         |           |           |           |           |
| 8.1.4        | opp angles cycliq                 |           |           | 2         |           | 11        |
| 8.2.1        | Pyth                              |           |           |           |           |           |
| 8.2.2        | centre circle perp chord          | 5         |           |           |           |           |
| 9.2.1        | cyclic quad                       |           | 4         |           |           |           |
| 9.2.2        | congruency                        |           | 3         |           |           |           |
| 9.2.3        | angle at centre                   |           |           | 5         |           |           |
| 9.2.4        | same segment                      |           |           | 5         |           | 22        |
| 10.1         | properties of parm                | 1         |           |           |           |           |
| 10.2         | one line // to 3 line in triangle |           | 3         |           |           |           |
| 10.3         | area rule                         |           | 2         |           |           | 6         |
| 11.1         | similarity                        | 3         |           |           |           |           |
| 11.2         | ratio                             |           | 3         |           |           |           |
| 11.3         | ratio                             |           |           | 5         |           | 11        |
| <b>TOTAL</b> |                                   | <b>29</b> | <b>55</b> | <b>58</b> | <b>14</b> | <b>66</b> |

