



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE/ NATIONAL SENIOR CERTIFICATE

KEREITI 12

DIPALO P2

LOETSE 2021(2)

MATSHWAO: 150

NAKO: Dihora tse 3

Pampiri ena ena le maqephe a 15 le leqephe le 1 la tlhahisoleseding.



DITAELO LE TLHAHISOLESERING

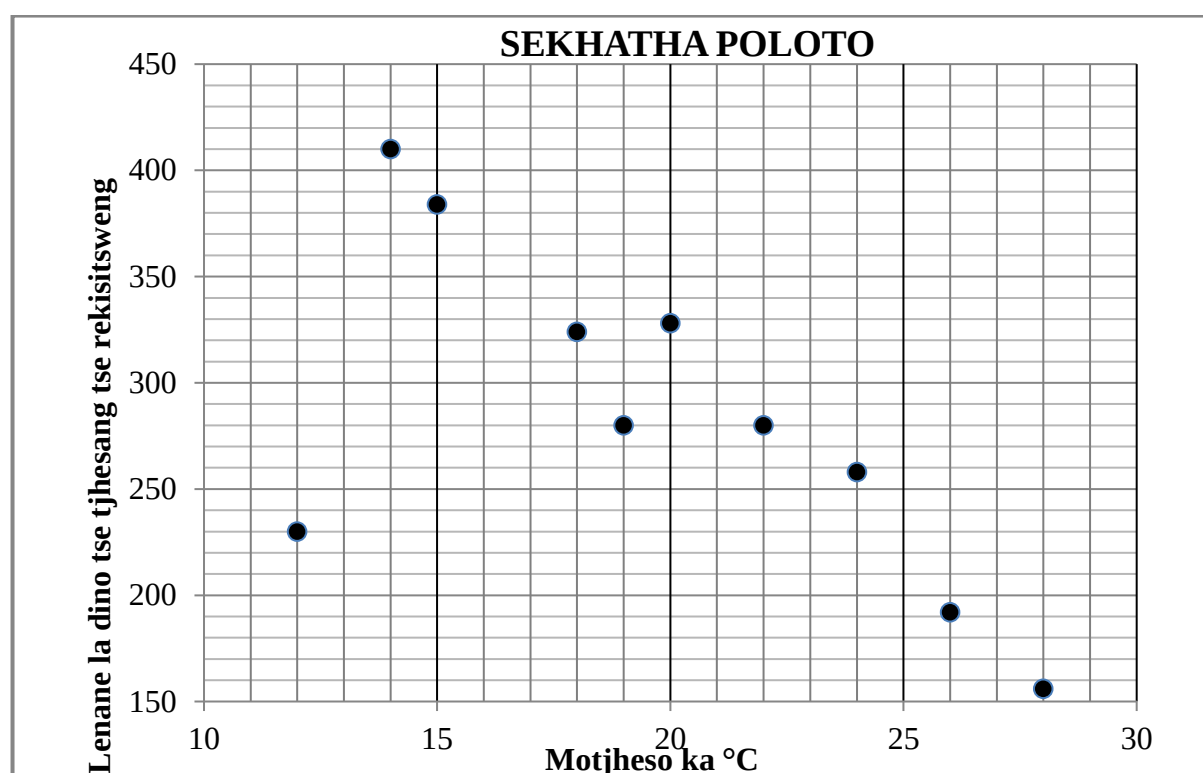
Bala ditaelo tse latelang ka hloko.

1. Ena pampiri ena le dipotso tse 10.
2. Araba dipotso TSOHLE ho BUKANA E IKGETHILENG YA HO ARABELA.
3. Ka makgethe BONTSHA tsohle dipalo, ditshwantsho, dikerafo, jwalojwalo. tseo o di sebedisitseng ho fumana dikarabo tsa hao.
4. Dikarabo feela di KEKE tsa fumana matshwao kaofela ka nako tsohle.
5. Oka sebedisa khalukhetara e dumelletsweng ya saense (esa porogeremuang ebile ena le dikerafo), kante le haeba ho boletswe ka tsela e nngwe.
6. Moo ho hlokahalang teng, atametsa dikarabo tsa hao ho desimale tse PEDI, kante le haeba ho boletswe ka tsela e nngwe.
7. HAHO bolele hore ditshwantsho tsohle di takuwe ho latela ditekanyo tse nepahetseng.
8. Leqephe la tlhahisoleding le nang le di-fomula le teng moo pampiri e fellang teng.
9. Ngola ka makgethe leka tsela e bonahalang.

POTSO 1

Mokete wa selemo wa dipapadi o tshwerwe nako ya matsatsi a 11. Lebenkele le rekisa dino tse tjhesang moketeng ona. Ho letsatsi le leng le leng ho matsatsi a pele a 10, monga lebenkele o ile a ngola mofuthu ka 13:00 le palo ya dino tse tjhesang tse rekisitsweng. Tlhahisoleseding ena e bontshitswe tafoleng leho sekhatha poloto ka tlase.

Mofuthu (in °C)	14	24	26	18	20	28	22	15	12	19
Palo ya dino tse tjhesang tse rekisitsweng	410	258	192	324	328	156	280	384	230	280

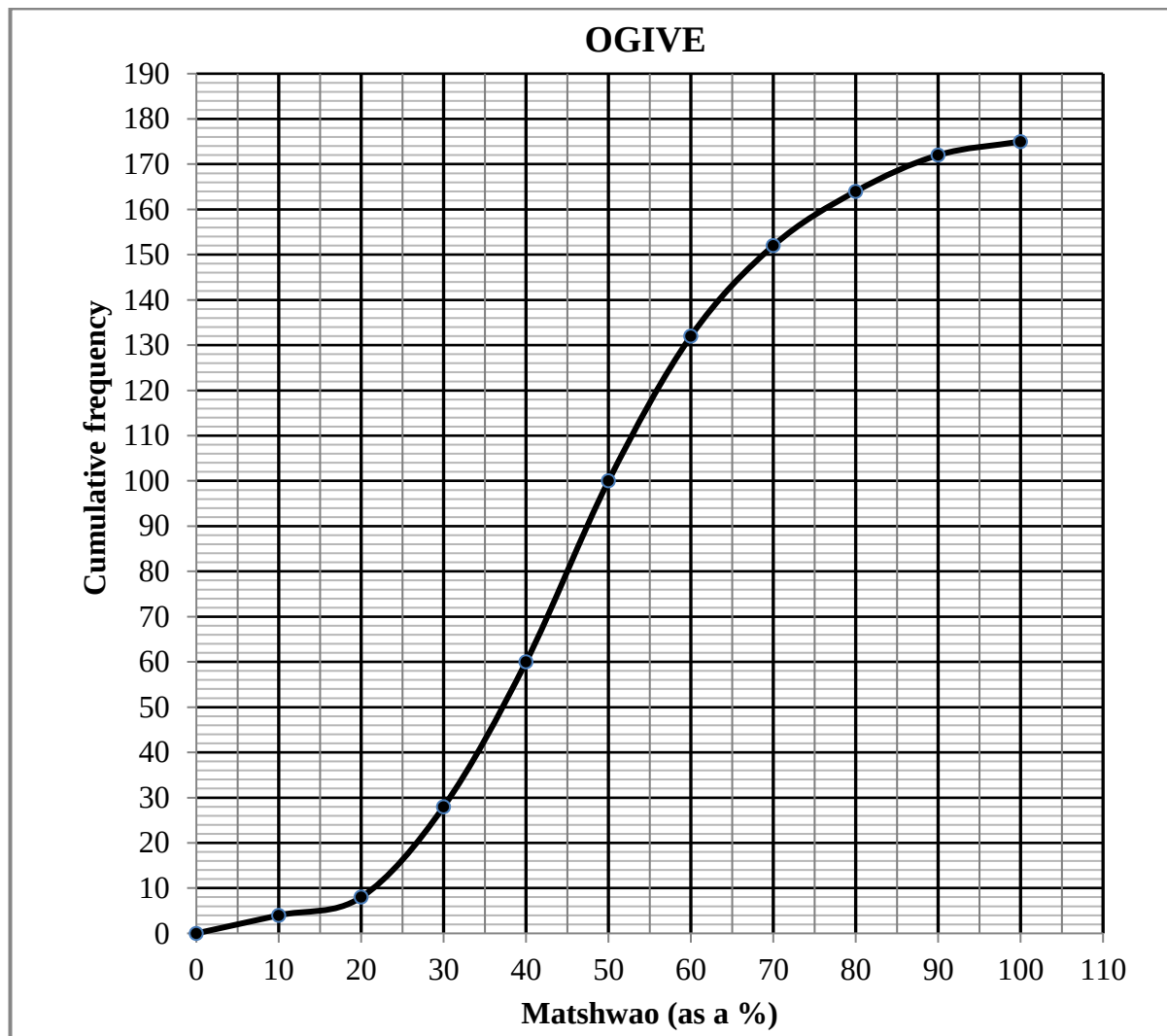


- 1.1 Hlalosa mokgwa oo data eo bontshang. (1)
- 1.2 Fumana ekweshini ya mola wa disekwere tse nyane tsa rekereshene tsa datha. (3)
- 1.3 Monga lebenkele o hlokometse hore o sebedisitse litara ele nngwe ya lebeso ho dikopi tse 8 tsa dino tse tjhesang tse rekisitsweng. Haeba mofuthu ka 13:00 ka letsatsi labo 11 ene lebelletswe hoba 17 °C, akanya lenane la lebokoso la litara ele nngwe ya lebeso monga lebenkele a tlamehang ho e reka ka letsatsi la bo 11. (3)
- 1.4 Kgetha outlier ho datha. (1)

[8]

POTSO2

- 2.1 Baithuti hotswa dikolong tse fapaneng ba ngotse teko tsa mahlale hore ba kgone hore ba khwalifaye ho fumana basari. Matshwao a bona (ka peresente) a bontshitswe ho ogive (cumulative frequency kerafo) ka fatshe.



- 2.1.1 Ho ngotse baithuti ba bakae teko? (1)
- 2.1.2 Ngola modal class ya data. (1)
- 2.1.3 Matshwao a tlase a ho khwalifayela basari ke 75%. Ke ba bake baithuti batla khwalifayela basari? (2)

- 2.2 Tafole e latelang e bontsha matshwao a baithuti ba 15 hotswa sekolong se itseng a fumanweng ho teko ya mahlale.

Matshwao (ka%)	62	58	78	85	74	48	74	84	100	46	80	92	60	90	92
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Fumana:

2.2.1 Letshwao la mini le fumanweng ke baithuti. (2)

2.2.2 Setandade devieshene ya matshwao a baithuti. (1)

2.2.3 Lenane la baithuti bao matshwao a bona a fumanehang ho feta setandathe devieshene sele seng ka hodimo ho mini. (2)

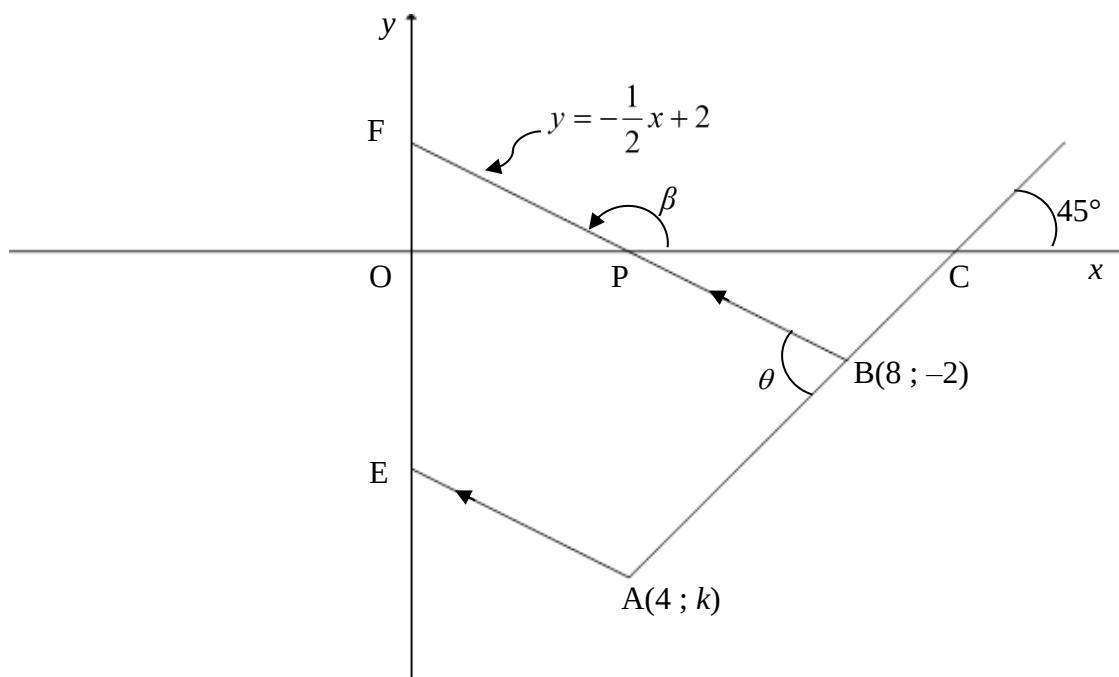
- 2.3 Matshwao a Kereiti 11 (ka persente) aho qetela a fumanweng ke baithuti a lekotswe. Setandathe devieshene sele seng ho inthavale ya mini e fumanwe ele (82,7; 94,1).

Fumana setandathe devieshene ya matshwao aho qetela a Kereiti 11. (3)
[12]



POTSO 3

Ho setshwantsho se latelang, mola wa BF o takilwe hotswa ho $B(8; -2)$ ho kgaola x -axis ho P le y -axis ho F. Engele ya inclination BF ke β le ekweshene ya BF $y = -\frac{1}{2}x + 2$. Hotswa ho $A(4; k)$, mola o mong o takilwe o pharalele ho BF le ho kgaola y -axis ho E. Mola o fetang ho A le ho B ena le inclination ya 45° le ho kgaola x -axis ho C. $\hat{ABF} = \theta$.

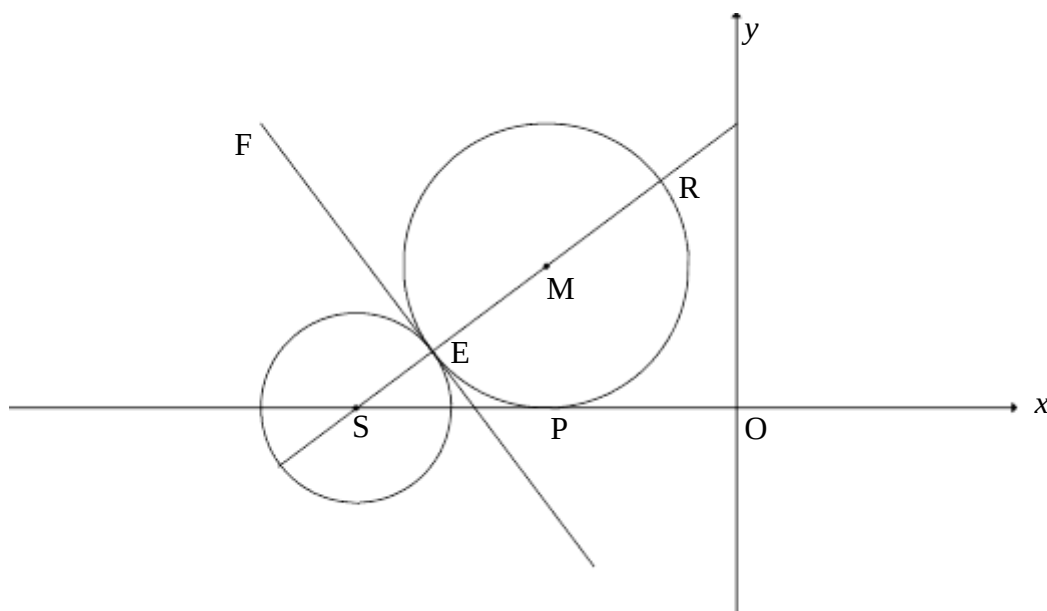


- 3.1 Batla keradiante ya AB. (1)
- 3.2 Bontsha hore velu ya k ke -6 . (2)
- 3.3 Fumana ekweshini ya EA ka tsela ya $y = mx + c$. (3)
- 3.4 Fumana:
 - 3.4.1 Boholo ba θ (3)
 - 3.4.2 Bolelele ba BF (3)
 - 3.4.3 Eria ya $\triangle ABF$ (4)
- 3.5 Haeba G ke ntlha e khwaderanteng ya bone hore APBG ebe pharalelokereme. Fumana boholo ba $\hat{PAG}$. (4)

[20]

POTSO 4

Ho setshwantsho se latelang, S ke ntlha ho x -axis. Sedikadikwe se nang le bohare S le sedikadikwe se nang le bohare M di takilwe. Didikadikwe tse pedi di kopana kante ho E. FE ke thanjente ya didikadikwe tse pedi ho E. Sedikadikwe sa bohare ba M, ena le ER ele diametha, e kopana le x -axis ho P.

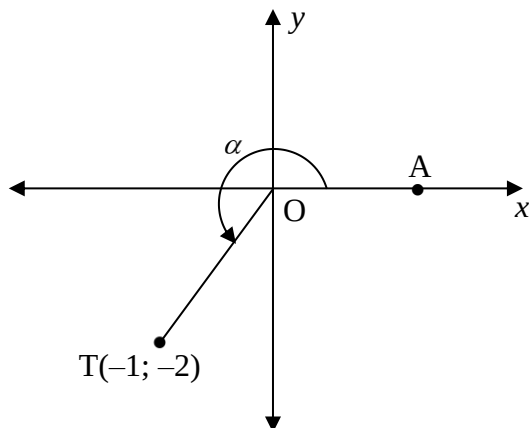


- 4.1 Ekweshini ya sedikadikwe se nang le bohare S ke $(x+8)^2 + y^2 = 4$.
- 4.1.1 Fumana dikhoodinetse tsa S. (2)
- 4.1.2 Bontsha hore diametha ya sedikadikwe sa bohare ba S ke diyuniti tse 4. (1)
- 4.2 Haeba re boela re nehwa hore $SR = 8$ diyuniti le $R\left(-\frac{8}{5}; \frac{24}{5}\right)$, bala::
- 4.2.1 Bolelele ba EM (2)
- 4.2.2 Keradiante ya thanjente FE (3)
- 4.2.3 Khoodinetse tsa M (4)
- 4.2.4 Khoodinetse tsa E (2)
- 4.3 Sedikadikwe sa bohare ba $M(-4; 3)$ e shifotile uniti e 1 hoya ka letsohong le leqele le ho refolekotiwa ho x -axis ho fumana sedikadikwe se setjha ho K. Fumana hore ntlha $(-8; 0)$ eka hare kapa kante ho sedikadikwe ho K. Bontsha mesebetsi YOHLE. (5)

[19]

POTSO 5

- 5.1 Ntlha $T(-1; -2)$ re e nehlwe setshwantshong se ka fatshe. A ke ntlha ho x -axis hore reflex $\widehat{AOT} = \alpha$.



Fumana, **ntle le tshebediso ya khalukhetara**, velu ya tse latelang:

5.1.1 $\tan \alpha$ (1)

5.1.2 $\cos \alpha$ (2)

5.1.3 $\cos(\alpha + 45^\circ)$ ho foromo e bobebe. (4)

- 5.2 Fumana, **ntle le tshebediso ya khalukhetara**, velu ya expression tse latelang:

$2\sin(-20^\circ) \cdot \sin 160^\circ - \cos 40^\circ$ (4)

- 5.3 Ela hloko: $3\cos x \cdot \sin x + \tan x \cdot \cos^2(180^\circ - x)$

5.3.1 Etsa expression ena ebe bobebe hoyla ho trigonometric reshio ele nngwe. (4)

5.3.2 Ebe, o ngola renje ya:

$f(x) = 3\cos x \cdot \sin x + \tan x \cdot \cos^2(180^\circ - x)$ (2)

5.4 Pruva identity: $\frac{\cos 3x}{\cos x} = 4\cos^2 x - 3$ (5)

- 5.5 Fumana jeneral solushene ya x ho ekweshini e latelang:

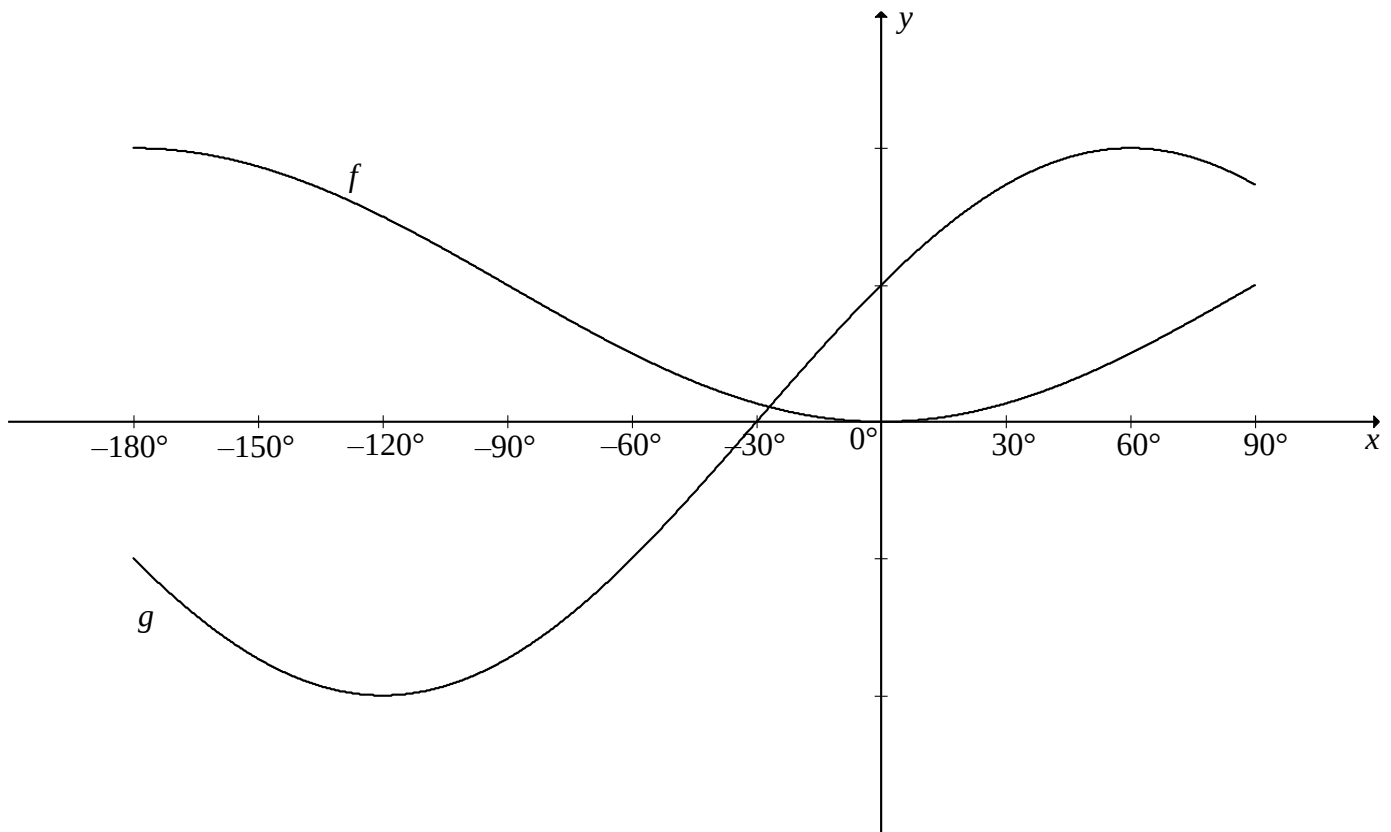
$3^{2\tan x} - 3^{\tan x + 1} = 54$ (5)

[27]



POTSO 6

Setshwantshong se latelang, kerafo ya $f(x) = -\cos x + 1$ le $g(x) = 2\sin(x + 30^\circ)$ di takilwe ho inthavale $x \in [-180^\circ; 90^\circ]$.



6.1 Ke velu efe ya x , $x \in [-180^\circ; 90^\circ]$, moo:

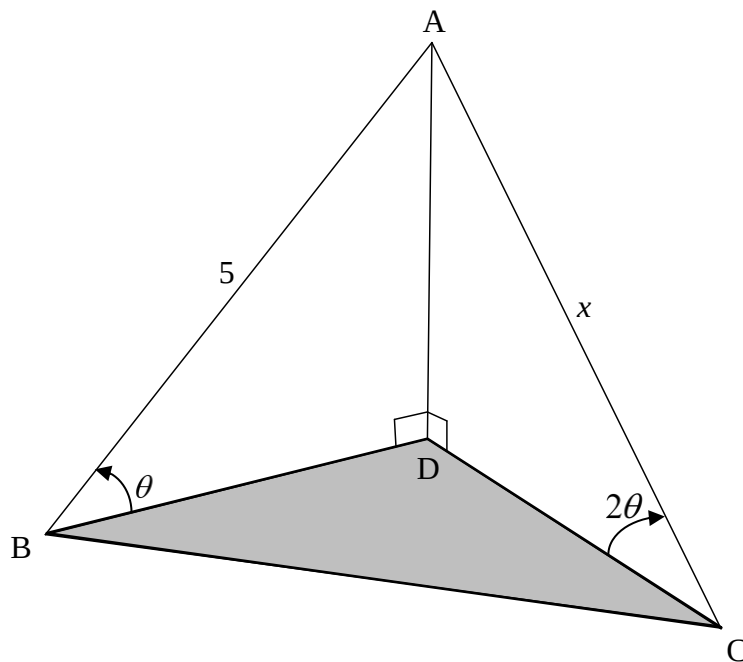
6.1.1 $f(x) \cdot g(x) \geq 0$ (2)

6.1.2 $g(x) = -1$ (2)

6.2 y -axis e tsamaisitswe 90° hoya ka letsohong le letona. Fumana ekweshini e ntjha ya kerafo eo qalong e ile ya bitswa f , ho foromo e bobebe. (2)
[6]

POTSO 7

Setshwantshong se latelang, B, C and D di sebakeng se tshwanang se tshekaletseng. AD ke palo e tsepameng e tsheheditsweng ke dithapo tse pedi, AB and AC. Engele tsa eleveishene ho qala ho B le C ho ya ho A, tsullung ya palo, ke θ le 2θ ka ho latellana. $AB = 5$ diyuniti le $AC = x$ diyuniti.

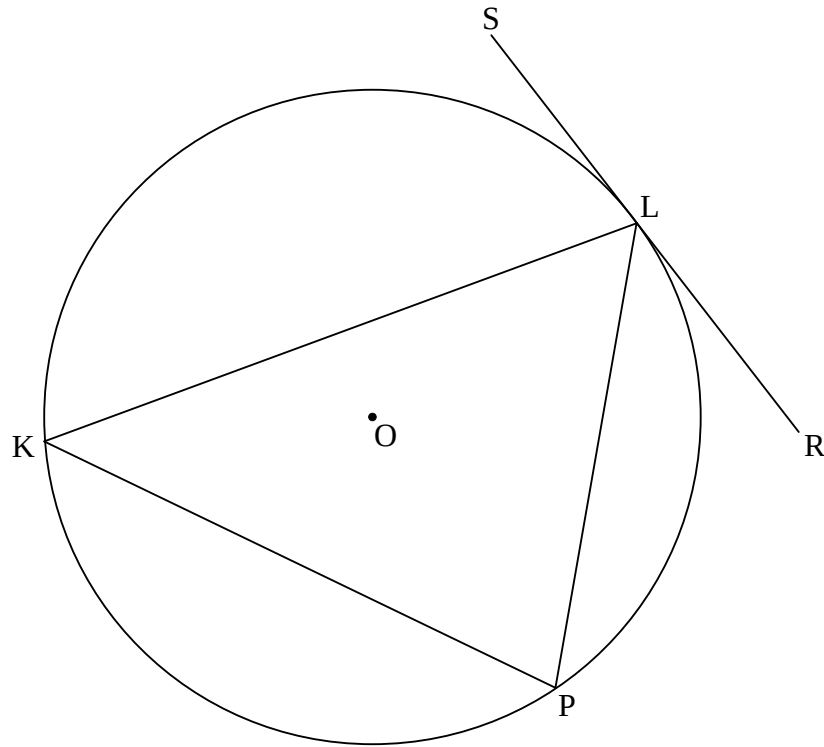


7.1 Bontsha hore $x = \frac{5}{2 \cos \theta}$ (5)

7.2 Bala bolelele ba BC haeba re nehilwe hore $\hat{BAC} = 112^\circ$ le $\theta = 30^\circ$. (3)
[8]

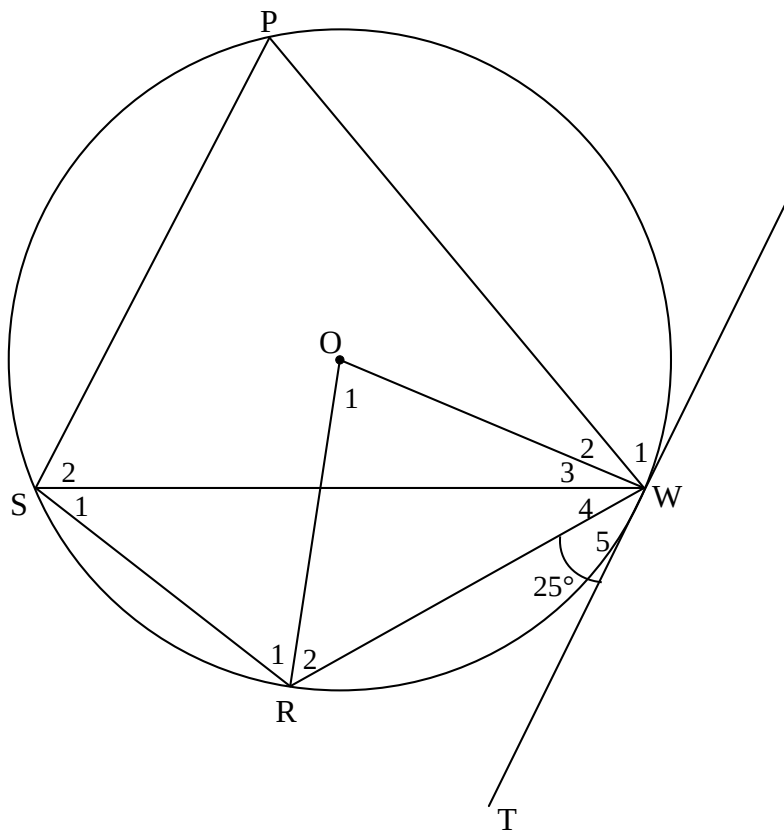
POTSO 8

- 8.1 Setshwantshong se latelang, dikhodo KL, LP and KP di takilwe ka hare ho sedikadikwe, se nang le bohare ba O. SLR ke thanjente ya sedikadikwe ho L.



Pruva theoreme e bolelang hore engele pakeng tsa thanjente SLR le khodo KL e lekana le engele e fumanehang ho althanete segmente, ka hoo pruva hore $\hat{SLK} = \hat{P}$. (6)

- 8.2 Ho setshwantsho se latelang, $PWRS$ ke saetliliki khwaderilatherale ka hare ho sedikadikwe, bohareng ho O . $\triangle PSW$ ke ekhwilateralerae. TW ke thanjente ya sedikadikwe ho W . Radii OR le OW di takilwe. $\hat{W}_5 = 25^\circ$.



8.2.1 Fumana, o fana ka mabaka, boholo ba:

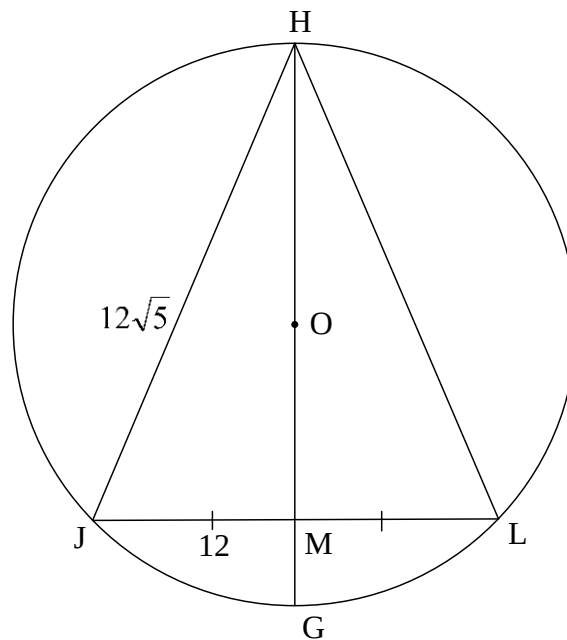
(a) $\hat{S}_1$ (2)

(b) $\hat{O}_1$ (2)

(c) $\hat{R}_1$ (5)

8.2.2 Pruva hore $SP \parallel TW$. (3)

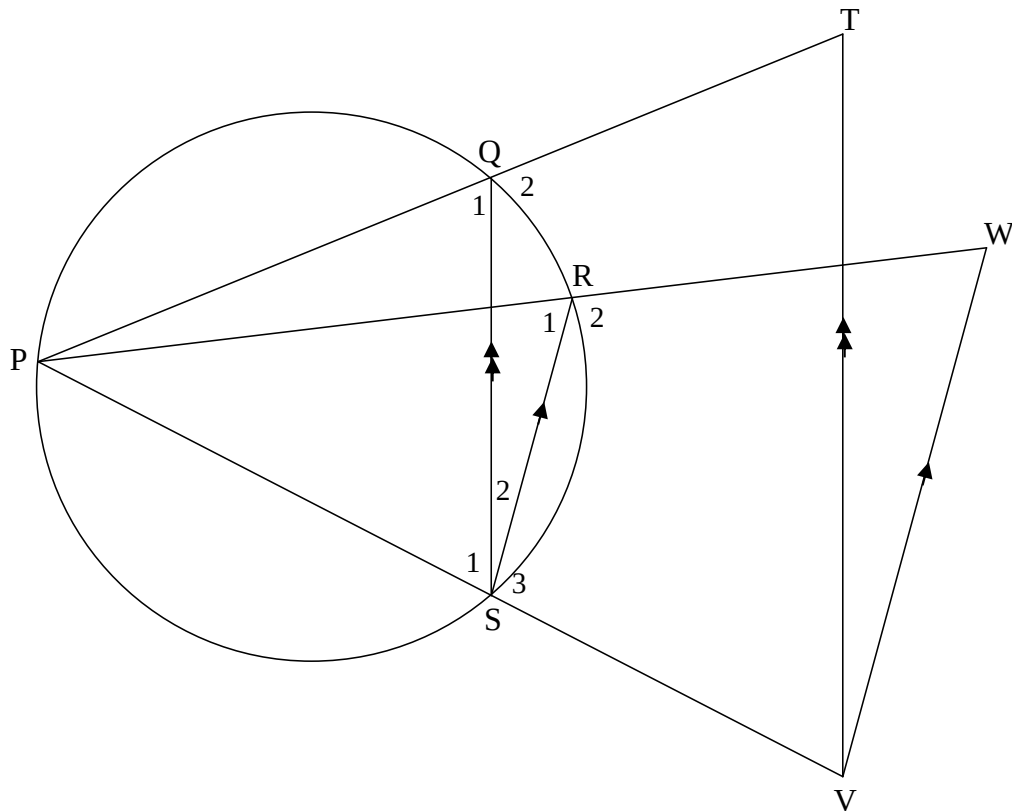
- 8.3 Setshwantshong se latelang, sedikadikwe se nang le bohare O se takilwe. H, J, G and L ke dintlha tse hodima sedikadikwe. $\triangle HJL$ e takilwe. HOG e kgaola JL mahareng ho M.
 $HJ = 12\sqrt{5}$ diyuniti and $JM = 12$ diyuniti.



- 8.3.1 Haeba $MG = 6$ diyuniti le $OM = x$, ngola HM ka mokgwa wa x . (2)
- 8.3.2 Bala, o fana ka mabaka, bolelele ba rediase ya sedikadikwe. (5)
- [25]

POTSO 9

Setshwantshong se latelang, P, Q, R and S ke dintlha tse hodima sedikadikwe. PS, PQ le PR di lelefaditswe hoja ho V, T le W ka ho latelanang. $VT \parallel SQ$ le $SR \parallel VW$.



Pruva, o fana ka mabaka, hore:

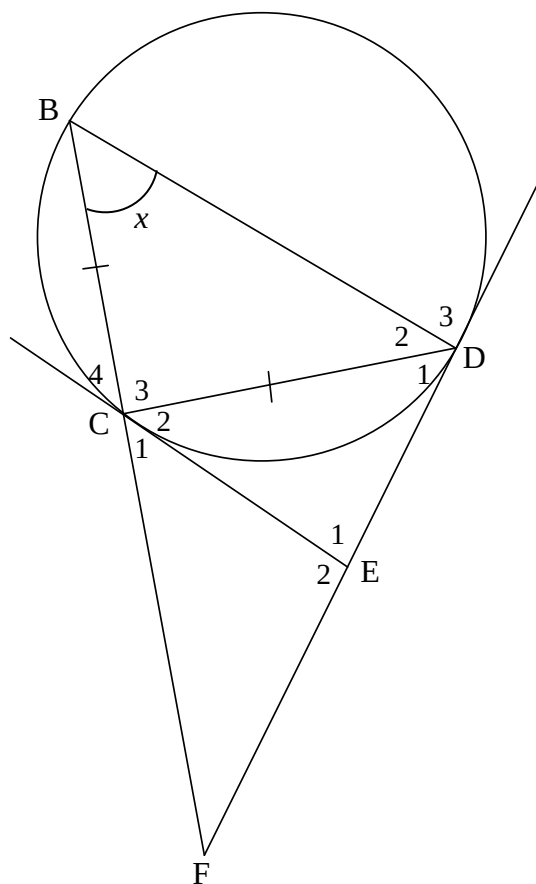
$$9.1 \quad \frac{TQ}{QP} = \frac{WR}{RP} \quad (3)$$

$$9.2 \quad \text{TPVW ke setliliki khwadilatherale} \quad (5)$$

[8]

POTSO 10

Setshwantshong se latelang, B, C le D ke dintlha tse hodima sedikadikwe hore $BC = CD$. EC le ED ke di thanjente hodima sedikadikwe ho C le D ka ho latellana. BC e lelefaditswe ho kopana le thanjente ho DE e lelefaditsweng hoya ho F. $\hat{B} = x$.



10.1 Pruva, o neha mabaka, hore:

10.1.1 $\hat{E}_1 = 180^\circ - 2x$ (5)

10.1.2 $\triangle ECD \parallel \triangle CBD$ (3)

10.2 Pruva, o fan aka mabaka, hore:

10.2.1 $CD^2 = CE \cdot BD$ (3)

10.2.2 $\frac{CF^2}{EF^2} = \frac{BD}{DE}$ (6)
[17]

KAOFELA: 150

a



LEQEPHE LA TLHAHISOLESING

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2}ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

